(Sketch omitted.)

If the coordinates of P are (a, b), then by symmetry, the coordinates of Q are $(-a, -b)$.

C_1 is given by:

$$xy = 1$$

Differentiating both sides:

$$\frac{d}{dx}(xy) = 0$$

$$x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Hence, the equation of the tangent to C_1 at P is:

$$y = -\frac{b}{a}(x - a) + b = -\frac{b}{a}x + 2b \tag{1}$$

C_2 is given by:

$$x^2 - y^2 = 2$$

Differentiating both sides:

$$\frac{d}{dx}(x^2 - y^2) = 0$$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

Hence, the equation of the tangent to C_2 at Q is:

$$y = \frac{a}{b}(x + a) - b \tag{2}$$

M is the intersection of the two tangents, hence, at M,

$$\frac{a}{b}(x + a) - b = -\frac{b}{a}x + 2b$$

Let $x = -b$. Then:

$$\frac{a}{b}(-b + a) - b = -\frac{b}{a}(-b) + 2b$$

$$-a+\frac{a^2}{b}-b=\frac{b^2}{a}+2b$$

$ab=1$ since (a,b) is on C_1, hence, $b=a^{-1}$.
The LHS:

$$-a+\frac{a^2}{b}-b=-a+\frac{a^2}{a^{-1}}-a^{-1}=a^3-a-a^{-1}$$

$$\text{LHS}=a(a^2-a^{-2}-1)$$

The RHS:

$$\frac{b^2}{a}+2b=\frac{a^{-2}}{a}+2a^{-1}=a^{-3}+2a^{-1}=a^{-3}-a+a+2a^{-1}$$

$$\text{RHS}=a^{-1}(a^{-2}-a^2+2)+a$$

$a^2-b^2=2$ since (a,b) is on C_2, hence, $a^2+a^{-2}=2$. Thus,

$$\text{LHS}=a(a^2-a^{-2}-1)=a(2-1)=a$$

$$\text{RHS}=a^{-1}(a^{-2}-a^2+2)+a=a^{-1}(-2+2)+a=a$$

LHS = RHS = a, so $(-b,a)$ are the coordinates of M. By symmetry along $y=-x$, N must have the coordinates $(b,-a)$.

Thus,

$$\begin{aligned}P &= (+a,+b)\\ M &= (-b,+a)\\ Q &= (-a,-b)\\ N &= (+b,-a)\end{aligned}$$

We note that M is the image of P under a 90^o counterclockwise rotation about the origin. Likewise, Q is the image of M, N is the image of Q and P is the image of N under the same rotation. Therefore, PMQN forms a square centred on the origin.