Let $x = 2 - \cos\theta$. Then,

$$\theta = \cos^{-1}(2 - x)$$

$$\frac{dx}{d\theta} = sin\theta$$

Hence,

$$\int_{\frac{3}{2}}^{2} \left(\frac{x-1}{3-x}\right)^{\frac{1}{2}} dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{\frac{1}{2}} \sin\theta \, d\theta$$

$\cos\theta = \left(\cos\frac{1}{2}\theta\right)^2 - \left(\sin\frac{1}{2}\theta\right)^2$, hence:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{\frac{1}{2}} \sin\theta \, d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{2\left(\sin\frac{1}{2}\theta\right)^2}{2\left(\cos\frac{1}{2}\theta\right)^2}\right)^{\frac{1}{2}} \sin\theta \, d\theta$$

Since $0 \leq \theta < \pi$ in the domain we are considering, both $\sin\frac{1}{2}\theta$ and $\cos\frac{1}{2}\theta$ may be assumed to be positive:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{2\left(\sin\frac{1}{2}\theta\right)^2}{2\left(\cos\frac{1}{2}\theta\right)^2}\right)^{\frac{1}{2}} \sin\theta \, d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin\frac{1}{2}\theta}{\cos\frac{1}{2}\theta} \sin\theta \, d\theta$$

$\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$, hence:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin\frac{1}{2}\theta}{\cos\frac{1}{2}\theta} \sin\theta \, d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin\frac{1}{2}\theta}{\cos\frac{1}{2}\theta} \cdot 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta \, d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\left(\sin\frac{1}{2}\theta\right)^2 d\theta$$

And since $1 - \cos\theta = 2\left(\sin\frac{1}{2}\theta\right)^2$:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\left(\sin\frac{1}{2}\theta\right)^2 d\theta = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - \cos\theta) \, d\theta = [\theta - \sin\theta]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

Therefore:

$$\int_{\frac{3}{2}}^{2} \left(\frac{x-1}{3-x}\right)^{\frac{1}{2}} dx = \left(\frac{\pi}{2} - \frac{\pi}{3}\right) - \left(1 - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} - 1 + \frac{\sqrt{3}}{2} = \frac{2(\pi + 3\sqrt{3} - 6)}{12}$$

Now, let $x = A - B\cos\theta$ with constants A, B such that:

$$\begin{aligned} x - a &= N(1 - \cos\theta) \\ b - x &= N(1 + \cos\theta) \end{aligned}$$

Comparing coefficients,

$$N = B = A - a = b - A$$

Since $A - a = b - A$, $A = \frac{1}{2}(a+b)$, hence, $B = N = \frac{1}{2}(b-a)$. Thus:

$$x = \frac{1}{2}\left((a+b) - (b-a)\cos\theta\right)$$

$$\cos\theta = \frac{(a+b) - 2x}{(b-a)}$$

$$\frac{dx}{d\theta} = \frac{1}{2}(b-a)\sin\theta$$

Let p' be such that $p = \frac{1}{2}\left((a+b) - (b-a)\cos p'\right)$. Then:

$$\cos p' = \frac{(a+b) - 2p}{(b-a)} = \frac{(a+b) - \frac{1}{2}(3a+b)}{(b-a)} = \frac{\frac{1}{2}(b-a)}{(b-a)} = \frac{1}{2}$$

Hence $p' = \frac{\pi}{3}$.

Let q' be defined analogously for q. Then:

$$\cos q' = \frac{(a+b) - 2q}{(b-a)} = \frac{(a+b) - (a+b)}{(b-a)} = 0$$

Hence $p' = \frac{\pi}{2}$.

Therefore:

$$\int_p^q \left(\frac{x-a}{b-x}\right)^{\frac{1}{2}} dx = \int_{p'}^{q'} \left(\frac{\frac{1}{2}(b-a)(1-\cos\theta)}{\frac{1}{2}(b-a)(1-\cos\theta)}\right)^{\frac{1}{2}} \cdot \frac{1}{2}(b-a)\sin\theta\, d\theta$$

$$\int_p^q \left(\frac{x-a}{b-x}\right)^{\frac{1}{2}} dx = \frac{1}{2}(b-a)\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{\frac{1}{2}} \sin\theta\, d\theta$$

Using the previous result:

$$\int_p^q \left(\frac{x-a}{b-x}\right)^{\frac{1}{2}} dx = \frac{1}{2}(b-a)\cdot\frac{2(\pi + 3\sqrt{3} - 6)}{12} = \frac{(b-a)(\pi + 3\sqrt{3} - 6)}{12}$$