Let $\alpha = e^{i\frac{pi}{3}}$. Then:

$$\alpha = e^{i\frac{1}{3}\pi} = \cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\alpha^2 = e^{i\frac{2}{3}\pi} = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

Hence, $\alpha^2 + 1 = \frac{1}{2} + i\frac{\sqrt{3}}{2} = \alpha$.

(Sketch omitted.)

Let a, b, c, l, m, n be the complex numbers representing A, B, C, L, M, N. The line from N to the midpoint of AC is perpendicular to AC and has the length $AC\sin\frac{1}{3}\pi = \frac{\sqrt{3}}{2}AC$. $-i(a-c)$ is perpendicular to $(a-c)$, therefore the line is a scalar multiple of $-i(a-c)$:

$$-i(a-c) = -ip - ir\alpha = -ip - ir\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}r - i\left(p + \frac{1}{2}r\right)$$

$|-i(a-c)| = |a-c|$, hence:

$$n - \frac{1}{2}(a+c) = \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}r - i\left(p + \frac{1}{2}r\right)\right)$$

$$n = \left(\frac{1}{2}p - \frac{1}{2}r\alpha\right) + \left(\frac{3}{4}r - i\frac{\sqrt{3}}{2}\left(p + \frac{1}{2}r\right)\right)$$

$$n = p\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) - r\left(\frac{1}{2}\alpha - \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$n = p\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) - r\left(\frac{1}{4} + i\frac{\sqrt{3}}{4} - \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$n = -p\alpha^2 - r\alpha^2$$

Since $\alpha^2 = \alpha - 1$,

$$n = (1-\alpha)p - \alpha^2 r = -\alpha^2(p+r) \tag{1}$$

Consider $i(a-b)$, which is perpendicular to AB:

$$i(a-b) = i\left(p - q\alpha^2\right) = i\left(p - q\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right) = ip + i\frac{1}{2}q + \frac{\sqrt{3}}{2}q$$

$$i(a-b) = \frac{\sqrt{3}}{2}q + i\left(p + \frac{1}{2}q\right)$$

As with n to $(a-c)$, $l - \frac{1}{2}(a+b)$ is perpendicular to $(a-b)$ and has the length $\frac{\sqrt{3}}{2}AB$.

$$l - \frac{1}{2}(a+b) = i\frac{\sqrt{3}}{2}(a-b)$$

$$l = \left(\frac{1}{2}p + \frac{1}{2}q\alpha^2\right) + \left(\frac{3}{4}q + i\left(\frac{\sqrt{3}}{2}p + \frac{\sqrt{3}}{4}q\right)\right)$$

$$l = p\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + q\left(\frac{1}{2}\alpha^2 + \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$l = p\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + q\left(-\frac{1}{4} + i\frac{\sqrt{3}}{4} + \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$l = p\alpha + q\alpha = \alpha(p+q) \tag{2}$$

Likewise for m:

$$i(b-c) = i\left(q\alpha^2 + r\alpha\right) = q\left(-i\frac{1}{2} - \frac{\sqrt{3}}{2}\right) + r\left(i\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$i(b-c) = -\frac{\sqrt{3}}{2}(q+r) + i\frac{1}{2}(r-q)$$

$$m - \frac{1}{2}(b+c) = i\frac{\sqrt{3}}{2}(b-c)$$

$$m = \left(\frac{1}{2}q\alpha^2 - \frac{1}{2}r\alpha\right) + \left(-\frac{3}{4}(q+r) + i\frac{\sqrt{3}}{4}(r-q)\right)$$

$$m = q\left(\frac{1}{2}\alpha^2 - \frac{3}{4} - i\frac{\sqrt{3}}{4}\right) + r\left(-\frac{1}{2}\alpha - \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$m = q\left(-\frac{1}{4} + i\frac{\sqrt{3}}{4} - \frac{3}{4} - i\frac{\sqrt{3}}{4}\right) + r\left(-\frac{1}{4} - \frac{\sqrt{3}}{4} - \frac{3}{4} + i\frac{\sqrt{3}}{4}\right)$$

$$m = -q - r = -(q+r) \tag{3}$$

$$\begin{aligned} l &= \alpha(p+q) \\ m &= -(q+r) \\ n &= -\alpha^2(r+p) \end{aligned}$$

Hence, each of OL & OC, OM & OA, and ON & OB are antiparallel, therefore, LC, MA, and NB are all straight line segments through the origin.

$$|l - c| = |\alpha(p + q) + r\alpha| = |\alpha(p + q + r)| = p + q + r$$
$$|m - a| = | - (q + r) - p| = | - (p + q + r)| = p + q + r$$
$$|n - b| = | - \alpha^2(r + p) - q\alpha^2| = | - \alpha^2(p + q + r)| = p + q + r$$

So,

$$|l - c| = |m - a| = |n - b| = p + q + r$$

Therefore LC, MA and NB have the same length.