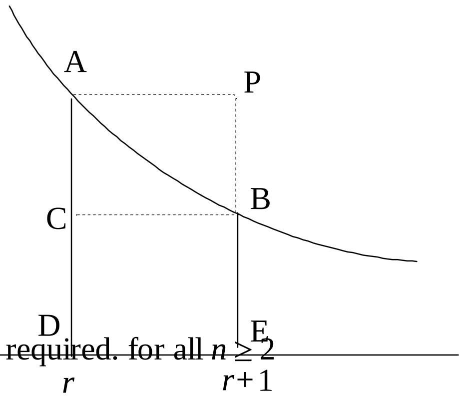
13. P(I find a new card in k^{th} packet) = P(no new card in $k-1$ packets)$\times$ P(new card in k^{th} packet)
$=(\frac{r}{n})^{k-1}\times\frac{n-r}{n}$

E(number of packets required) $=\sum_{k=1}^{\infty}k(\frac{r}{n})^{k-1}\times\frac{n-r}{n}=\frac{n-r}{n}\sum_{k=1}^{\infty}k(\frac{r}{n})^{k-1}=\frac{n-r}{n}\times(1-\frac{r}{n})^{-2}$

$$=\frac{n-r}{n}\times\frac{n^2}{(n-r)^2}=\frac{n}{n-r}\ [0\le r\le n-1]$$

$\int_r^{r+1}\frac{1}{x}dx$ is given by the area $ABED$ (see graph)

Clearly, from graph Area $BCDE\le$ Area $ABDE\le$ Area $APED$



i.e. $(r+1-r)\times\frac{1}{r+1}\le\int_r^{r+1}\frac{1}{x}dx\le(r+1-r)\times\frac{1}{r}$

or $\frac{1}{r+1}\le\int_r^{r+1}\frac{1}{x}dx\le\frac{1}{r}$ as required.

$\ln n=\int_1^n\frac{1}{x}dx=\sum_{r=1}^{n-1}\left(\int_r^{r+1}\frac{1}{x}dx\right)$

so $\sum_{r=1}^{n-1}\frac{1}{r+1}\le\sum_{r=1}^{n-1}\left(\int_r^{r+1}\frac{1}{x}dx\right)\le\sum_{r=1}^{n-1}\frac{1}{r}\Rightarrow\sum_{r=2}^{n}\frac{1}{r}\le\ln n\le\sum_{r=1}^{n-1}\frac{1}{r}$ as required. for all $n\ge2$

If I already have r cards then E(number of packets to get $(r+1)^{th}$ card) $=\frac{n}{n-r}$
E(number of packets to get $(r+2)^{th}$ card) $=\frac{n}{n-(r+1)}$

so E(total number of packets to complete set of cards) $=\sum_{r=0}^{n}\frac{n}{n-r}=n\sum_{r=1}^{n}\frac{1}{r}$

hence, for large n, Expected number of packets is $\le n(1+\ln n)$ but $\ge 1+n\ln n$
i.e. $1+n\ln n\le \mathrm{E}\le n+n\ln n$ so for very large n, he must expect to have to buy $n\ln n$ packets.