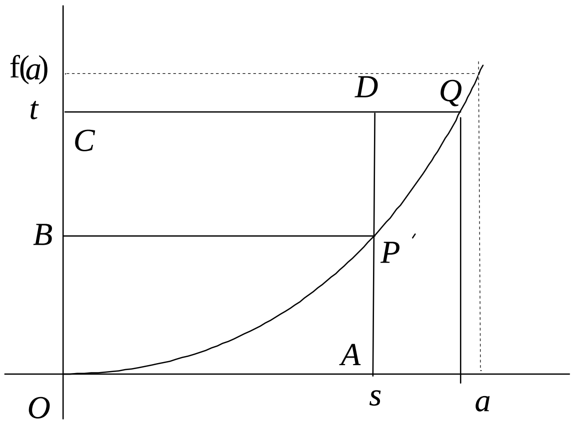
6. $F(s)=\int_0^s f(x)dx$ and $G(t)=\int_0^t g(y)dy$ where $g(y)=x$ and $y=f(x)$

If A is the point $(s,0)$ and C is $(0,t)$
then $F(s)=$ area OAP and $G(t)=$ area OCQ
Clearly the sum of these areas is equal to or greater than $OADC=st$
Hence, $F(s)+G(t)\ge st$



Equality will occur if $t=f(s)$

Now take $f(x)=\sin x,\ g(y)=\sin^{-1}y$
then $G(t)=st-F(s)$ and $s=\sin^{-1}t$
i.e. $\int_0^t \sin^{-1}y\,dy=t\sin^{-1}t-\int_0^s \sin x\,dx=t\sin^{-1}t-[-\cos x]_0^s$

$=t\sin^{-1}t-(-\cos s+1)$ and $\cos s=\cos\left(\sin^{-1}t\right)$ so $\int_0^t \sin^{-1}y\,dy=t\sin^{-1}t-\left(1-\cos\left(\sin^{-1}t\right)\right)$

differentiating both sides w.r.t. t we have, derivative of left hand side $=\sin^{-1}t$
derivative of right hand side $=\frac{t}{\sqrt{1-t^2}}+\sin^{-1}t+\sin\left(\sin^{-1}t\right)\times\frac{1}{\sqrt{1-t^2}}=\sin^{-1}t$

So result checks.