

10. Resolving vertically, $2T \cos \theta = mg$

and $\sin \theta = \frac{1}{x}$ so $\cos \theta = \frac{\sqrt{x^2-1}}{x}$

length of stretched string $= \pi + 2 \arcsin\left(\frac{1}{x}\right) + 2\sqrt{x^2-1}$

so by Hooke's law, $T = \frac{\lambda(2\theta + 2\sqrt{x^2-1} - \pi)}{2\pi}$

i.e. $\frac{mg}{2 \cos \theta} = \frac{\lambda}{\pi} \left(\arcsin\left(\frac{1}{x}\right) + 2\sqrt{x^2-1} - \pi \right)$

$x = 2 \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$

so $\frac{mg}{\sqrt{3}} = \frac{\lambda}{\pi} \left(\frac{\pi}{6} + 2\sqrt{3} - \pi \right) = \frac{\lambda}{\pi} \left(2\sqrt{3} - \frac{5\pi}{6} \right)$

so $\lambda = \frac{\pi mg}{\sqrt{3} \left(2\sqrt{3} - \frac{5\pi}{6} \right)} = \frac{6\pi mg}{36-5\pi}$

