## STEP Mathematics Paper I 1987

14. (b) and (c) could be records of a complete tournament since in eacyh case B has won two games in succession. (a) cannot be since here A, B and C have each won one game.

Sequences in which tournament is still undecided after 5 games are as follows:

ACBAC, BCABC so since each sequence has a probability of  $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$  probability that tournament is undecided after 5 games is  $\frac{1}{16}$ 

A wins in fewer than 5 games with the sequences AA and BCAA so probability  $= \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ B wins in fewer than 5 games with the sequences ACBB and BB so probability  $= \frac{5}{16}$ C wins in fewer than 5 games with the sequences ACC and BCC so probability  $= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ 

A wins eventually with the sequences AA, ACBAA, ACBACBAA etc or BCAA, BCABCAA etc so probability =  $\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^8 + \dots\right] + \left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^7 + \dots\right] = \frac{\left(\frac{1}{2}\right)^2}{1 - \left(\frac{1}{2}\right)^3} + \frac{\left(\frac{1}{2}\right)^4}{1 - \left(\frac{1}{2}\right)^3} = \frac{\frac{5}{16}}{\frac{7}{8}} = \frac{5}{14}$ 

*B* wins eventually with sequences *BB*, *BCABB*, etc or *ACBB*, *ACBACBB* etc so probability =  $\frac{5}{14}$ *C* wins eventually with the sequences *ACC*, *ACBACC* etc or *BCC*, *BCABCC* etc

so probability =  $2 \times \frac{\frac{1}{8}}{1 - \frac{1}{8}} = \frac{4}{14}$