

15. distance of P from $(1, 0)$ is $X = 2 \sin \theta$

where θ is uniform in $\left[0, \frac{\pi}{2}\right]$

p.d.f of θ is thus $\frac{2}{\pi}$ for $0 < \theta \leq \frac{\pi}{2}$

$$\text{so } E(X) = \frac{1}{\pi} \int_0^{\pi} 2 \sin \theta d\theta = \frac{2}{\pi} [-\cos \theta]_0^{\pi} = \frac{4}{\pi}$$

$$\begin{aligned} E(X^2) &= \frac{1}{\pi} \int_0^{\pi} 4 \sin^2 \theta d\theta = \frac{2}{\pi} \int_0^{\pi} 1 - \cos 2\theta d\theta = \frac{2}{\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi} \\ &= \frac{2}{\pi} (\pi) = 2 \text{ so } \text{var}(X) = 2 - \left(\frac{4}{\pi}\right)^2 \end{aligned}$$

$$P\left(X > \frac{4}{\pi}\right) = P\left(\sin \theta > \frac{2}{\pi}\right) = 1 - P\left(\theta < \sin^{-1} \frac{2}{\pi}\right) = \frac{2}{\pi} \sin^{-1} \frac{2}{\pi}$$

