3.(i) $\left(a+\sqrt{a^2-b}\right)^n = N - r$ and $\left(a-\sqrt{a^2-b}\right)^n = M + s$ with $b < 2a-1$
$a^2 - b > a^2 - 2a + 1 = (a-1)^2$ so $a - \sqrt{a^2-b} < a - (a-1) = 1$ so since M is an integer it must be 0

(ii) $\left(a+\sqrt{a^2-b}\right)^n + \left(a-\sqrt{a^2-b}\right)^n$ contains only even powers of $\sqrt{a^2-b}$ and so must be an integer, hence, $N - r + s$ is an integer $\Rightarrow s - r = 0$ or $r = s$

(iii) $\left(a+\sqrt{a^2-b}\right)^n\left(a-\sqrt{a^2-b}\right)^n = \left(a^2-\left(a^2-b\right)\right)^n = b^n$
So $(N-r)(M+s) = b^n \Rightarrow (N-r)r = b^n$ since $s = r$ and $M = 0 \Rightarrow Nr - r^2 = b^n$ or $r^2 - Nr + b^n = 0$

Taking $a = 8, b = 1$ so that $a^2 - b = 63$ we have $\left(a+\sqrt{a^2-b}\right)^n = (8+3\sqrt{7})^n = N - r$ so we only require the value of $r = s = (8-3\sqrt{7})^n$
$8 - 3\sqrt{7} = 8 - \sqrt{64-1} = 8 - 8\sqrt{1-\frac{1}{64}} \approx 8 - 8\left(1-\frac{1}{128}\right)$ (Binomial approx)
$= \frac{1}{16} = 2^{-4}$ so $r \approx 2^{-4n}$ as required.
