4. $ze^{i\theta}$ is a rotation by θ anticlockwise about the origin of the point represented by z
the required condition is that $(\beta-\gamma)=(\alpha-\gamma)e^{\frac{i\pi}{3}}$
similarly, $(\alpha-\gamma)=(\beta-\gamma)e^{\frac{i\pi}{3}}$ is the condition for a clockwise triangle.
so for an equilateral triange (in any order) we must have $\left((\beta-\gamma)-(\alpha-\gamma)e^{\frac{i\pi}{3}}\right)\left((\beta-\gamma)-(\alpha-\gamma)e^{-\frac{i\pi}{3}}\right)=0$
$\Rightarrow (\beta-\gamma)^2-(\alpha-\gamma)(\beta-\gamma)\left(e^{\frac{i\pi}{3}}+e^{-\frac{i\pi}{3}}\right)+(\alpha-\gamma)^2e^0=0$
$\Leftrightarrow (\beta-\gamma)^2-(\alpha-\gamma)(\beta-\gamma)+(\alpha-\gamma)^2 += 0$ since $e^{\frac{i\pi}{3}}+e^{-\frac{i\pi}{3}}=1$
$\Leftrightarrow \alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha=0$

Let α,β,γ be the roots of $z^3+az^2b\bar{z}c=0$ then $(\alpha+\beta+\gamma)=-a$ and $\alpha\beta+\beta\gamma+\gamma\alpha=b$
$\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha=0\Rightarrow a^2-3b=0$ which is thus the required condition.
