8. (i) $(\mathbf{a}-\mathbf{b}).\mathbf{r}=\frac{1}{2}\left(|\mathbf{a}|^2-|\mathbf{b}|^2\right)=\frac{1}{2}\left(a^2-b^2\right)=\frac{1}{2}(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b})$
i.e. $(\mathbf{a}-\mathbf{b}).\left(\mathbf{r}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right)=0$ which is the equation of a plane perpendicular to $(\mathbf{a}-\mathbf{b})$ through the point $\frac{1}{2}(\mathbf{a}+\mathbf{b})$.

(ii) $(\mathbf{a}-\mathbf{r}).(\mathbf{b}-\mathbf{r})=0 \Rightarrow r^2-(\mathbf{a}+\mathbf{b}).\mathbf{r}+\mathbf{a}.\mathbf{b}=0 \Rightarrow \left|\mathbf{r}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right|^2=\frac{1}{4}|\mathbf{a}-\mathbf{b}|^2$ which is the equation of a sphere of radius $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ and centre $\frac{1}{2}(\mathbf{a}+\mathbf{b})$

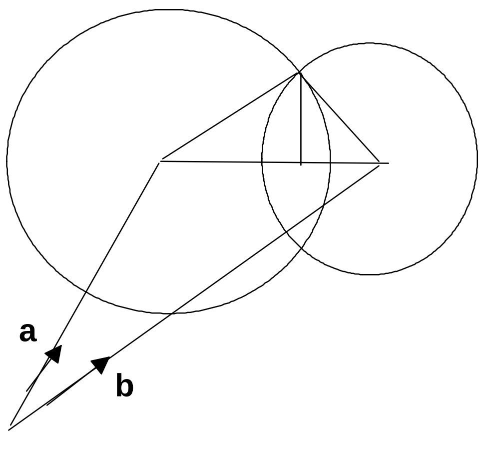
(iii) and (iv) describe spheres, each with radius $\frac{1}{\sqrt{2}}|\mathbf{a}-\mathbf{b}|$ and centres **a** and **b** respectively.

(i) and (ii) $\Rightarrow$ (iv) :
expanding (i) $\mathbf{a}.\mathbf{r}=\mathbf{b}.\mathbf{r}+\frac{1}{2}\left(a^2-b^2\right)$
expanding (ii) $\mathbf{a}.\mathbf{r}=\mathbf{a}.\mathbf{b}+r^2-\mathbf{b}.\mathbf{r}$
hence, $\mathbf{a}.b+r^2-\mathbf{b}.\mathbf{r}=\mathbf{b}.\mathbf{r}+\frac{1}{2}\left(a^2-b^2\right) \Rightarrow r^2-2\mathbf{b}.\mathbf{r}-\frac{1}{2}\left(a^2-b^2\right)+\mathbf{a}.\mathbf{b}=0$ which is the expansion of (iv)
eliminating **b.r** instead of **a.r** gives (iii)

(iii) and (iv) $\Rightarrow$ (i):
subtracting (iv) from (iii) $|\mathbf{r}-\mathbf{a}|^2-|\mathbf{r}-\mathbf{b}|^2=0 \Rightarrow a^2-2\mathbf{a}.\mathbf{r}-b^2+2\mathbf{b}.\mathbf{r}=0 \Rightarrow 2(\mathbf{b}-\mathbf{a}).\mathbf{r}=b^2-a^2$
which is equivalent to (i) and finally

(iii) and (iv) $\Rightarrow$ (ii)
adding (iii) and (iv) $|\mathbf{r}-\mathbf{a}|^2+|\mathbf{r}-\mathbf{b}|^2=|\mathbf{a}-\mathbf{b}|^2 \Rightarrow 2r^2-2\mathbf{a}.\mathbf{r}-2\mathbf{b}.\mathbf{r}=-2\mathbf{a}.\mathbf{b}$
$\Rightarrow (\mathbf{a}-\mathbf{r}).(\mathbf{b}-\mathbf{r})=0$ which is (ii)

Geometrically, the points sastisfying (iii) and (iv) lie on the intersectionof two spheres, i.e. a circle centre $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ whose plane is perpendicular to the line joining their centres i.e. a–b with a radius (see diagram)
$\sqrt{\frac{1}{2}|\mathbf{a}-\mathbf{b}|^2-\frac{1}{4}|\mathbf{a}-\mathbf{b}|^2}=\frac{1}{2}|\mathbf{a}-\mathbf{b}|$



points satisfying (i) and (ii) form the intersection of the plane perpendicular to $(\mathbf{a}-\mathbf{b})$ through the point $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ with the sphere of radius $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ and centre $\frac{1}{2}(\mathbf{a}+\mathbf{b})$i.e. The same circle as above.

So the geometric meaning of the equivalence is that it is the two ways of defining a circle as the intersection of two sheres or the intersection of a sphere and a plane.