

10. $a_j \circ a_k = a_m$ where a_m is the greater of a_j and a_k

since a_j and a_k are elements of S and $a_m = a_j$ or a_k then the operation is closed

the smallest element of S is an identity element, for suppose it is a_1 then for any k , $a_1 \circ a_k = a_k$ but no element other than the identity has an inverse since $a_j \circ a_k \neq a_1$ for any $j, k (\neq 1)$

$$(a_j \circ a_k) \circ a_l = (\text{larger of } a_j, a_k) \circ a_l = \text{largest of } a_j, a_k, a_l$$

$$a_j \circ (a_k \circ a_l) = a_j \circ (\text{larger of } a_k, a_l) = \text{largest of } a_j, a_k, a_l \text{ so associative.}$$

$$a_j * a_k = a_n \text{ where } n = |j - k| + 1$$

if a_j and a_k are elements of S then $0 \leq |j - k| \leq N - 1$ so $1 \leq |j - k| + 1 \leq N$ so $a_n \in S$ closed

a_1 is an identity element since $a_1 * a_j = a_n \Rightarrow n = |j - 1| + 1 = j$

$a_j * a_k = a_1$ iff $|j - k| + 1 = 1$ so every element is self inverse

$$\text{Let } (a_j * a_k) * a_l = a_n \text{ and } a_j * (a_k * a_l) = a_m$$

If $j = 2, k = 3, l = 4$ then $n = 3$ and $m = 1$ so not associative.
