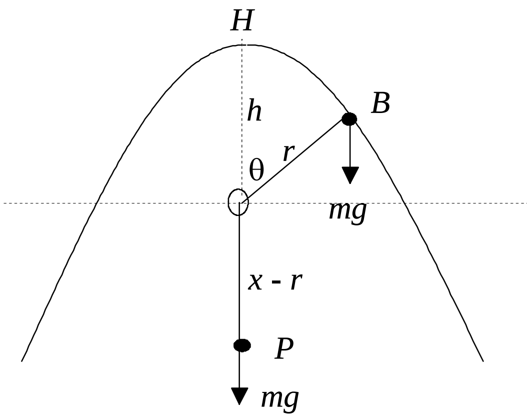

12. . System will be in static equilibrium for all positions of the bead if there is no change in P.E. when moved from one position to another.

Let H be the highest point of the curve and consider when bead is at B Taking a horizontal plane through the ring as . zero level for P.E. we have

P.E. of system when bead is at H is $mgh - mg(x-h)$ where x is

Taking horizontal plane through O as zero level for P.E. we have

P.E. of system when bead is at H is $mgh - mg(x-h)$ where x is the length of the string.

P.E. when bead is at B is $mgr\cos\theta - mg(x-r)$

so there is no change in P.E if $mgh - mg(x-h) = mgr\cos\theta - mg(x-r)$

$\Rightarrow h + h = r\cos\theta\_ + r$ or $r = \dfrac{2h}{1+\cos\theta}$ which is the equation of the curve.

shortest distance of curve from ring is obviously when $1+\cos\theta$ is a maximum, i.e. $\cos\theta = 1$

so minimum value of r is h, i.e. $h = d$ and equation is $r(1+\cos\theta) = 2d$

$\cos\theta = \dfrac{2d}{r} - 1 \Rightarrow -\sin\theta\dot\theta = -\dfrac{2d}{r^2}\dot r$ and $\sin\theta = \sqrt{1-\left(\dfrac{2d}{r}-1\right)^2} = \dfrac{\sqrt{4dr-4d^2}}{r}$

so $\dot\theta = \dfrac{2d}{r^2} \times \dfrac{r\dot r}{\sqrt{4dr-4d^2}} = \dfrac{d\dot r}{r\sqrt{dr-d^2}} \Rightarrow \dot\theta^2 = \dfrac{d^2\dot r^2}{r^2(dr-d^2)} = \dfrac{d\dot r^2}{r^2(r-d)}$

P.E. constant $\Rightarrow$ K.E. constant

speed of P is $\dot r$

at B, bead has a speed of $r\dot\theta$ tangentially and $-\dot r$ radially so by the constancy of K.E.

$(r\dot\theta)^2 + 2\dot r^2 = v^2 \Rightarrow \left(\dfrac{r^2 d}{r^2(r-d)} + 2\right)\dot r^2 = v^2 \Rightarrow \left(\dfrac{2r-d}{r-d}\right)\dot r^2 = v^2 \Rightarrow \dot r^2 = \left(\dfrac{r-d}{2r-d}\right)v^2$

so $\dot\theta^2 = \dfrac{d}{r^2(r-d)} \times \left(\dfrac{r-d}{2r-d}\right)v^2 = \left(\dfrac{d}{r^2(2r-d)}\right)v^2$ and $(r\dot\theta)^2 = \left(\dfrac{d}{2r-d}\right)v^2$

so finally the speed of the bead is $\left((r\dot\theta)^2 + \dot r^2\right)^{\frac12} = \left(\left(\dfrac{d}{2r-d}\right) + \left(\dfrac{r-d}{2r-d}\right)\right)^{\frac12} v = \left(\dfrac{r}{2r-d}\right)^{\frac12} v$ as required