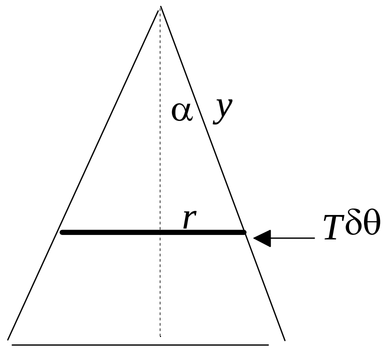
14. Let y be distance of element of band from vertex of cone as shown



component of force on element along slant face of cone is

$T\delta\theta.\sin\alpha$ and $T = \dfrac{\lambda(x-l)}{l}$ so force $= \dfrac{\lambda(x-l)\sin\alpha}{l}\delta\theta$

Hence, ignoring gravitational effects equation of motion is

$$-\frac{m\delta\theta}{2\pi}\frac{d^2y}{dt^2} = \frac{\lambda(x-l)\sin\alpha}{l}\delta\theta$$

$$y = \frac{r}{\sin\alpha} = \frac{x}{2\pi\sin\alpha} \text{ so } \frac{d^2y}{dt^2} = \frac{1}{2\pi\sin\alpha}\frac{d^2x}{dt^2}$$

Hence, $-\dfrac{m}{4\pi^2\sin\alpha}\dfrac{d^2x}{dt^2} = \dfrac{\lambda(x-l)\sin\alpha}{l} \Rightarrow \dfrac{d^2x}{dt^2} + \dfrac{4\pi^2\lambda(x-l)\sin^2\alpha}{ml} = 0$

Writing this as $\dfrac{d^2x}{dt^2} + \dfrac{4\pi^2\lambda x\sin^2\alpha}{ml} = \dfrac{4\pi^2\lambda l\sin^2\alpha}{ml}$ we see that it is a standard differential equation with C.F. $A\cos kt + B\sin kt$ where $k = 2\pi\sin\alpha\sqrt{\dfrac{\lambda}{ml}}$

$x = l$ is an obvious P.I. so solution is $x = A\cos kt + B\sin kt + l$

$\dfrac{d^2x}{dt^2} = 0$ when $x = l \Rightarrow -Ak^2\cos kt - Bk^2 t\sin kt = 0 \Rightarrow A = 0$ so $x = B\sin kt + l$

band will become slack when $x = l \Rightarrow \sqrt{A^2+B^2}\,\cos\left(kt - \frac{\pi}{4}\right) = 0 \Rightarrow t_0 = \dfrac{\pi}{4k} = \dfrac{1}{8\sin\alpha}\sqrt{\dfrac{ml}{\lambda}}$
