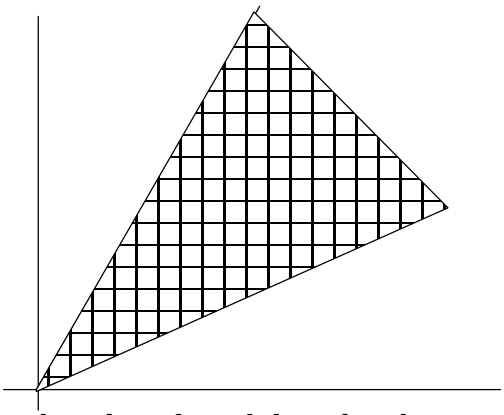
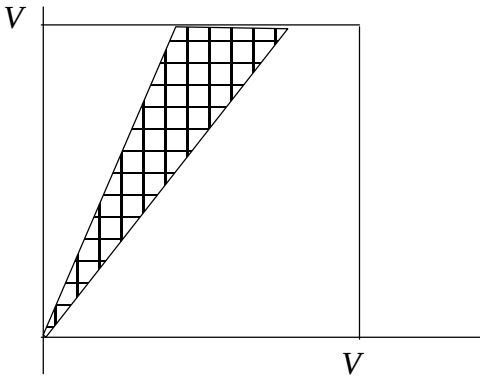
15. Train and lorry will collide if $\frac{d_2}{u_2} < \frac{d_1}{u_1} < \frac{d_2+l_2}{u_2}$ or $\frac{d_1}{u_1} < \frac{d_2}{u_2} < \frac{d_1+l_1}{u_1}$
i.e. if $\frac{d_2}{d_1} < \frac{u_2}{u_1} < \frac{d_2+l_2}{d_1}$ or $\frac{d_2}{d_1+l_1} < \frac{u_2}{u_1} < \frac{d_2}{d_1}$ and hence, $\frac{d_2}{d_1+l_1} < \frac{u_2}{u_1} < \frac{d_2+l_2}{d_1}$

The region between the lines through the origin with gradients $\frac{d_2}{d_1+l_1}$ and $\frac{d_2+l_2}{d_1}$ represent the collisions.



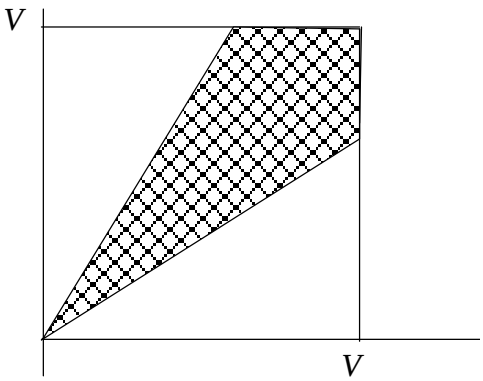
If the back of the train is nearer to crossing than front of lorry then $l_1 + d_1 < d_2$ and $d_1 < d_2 + l_2$ so both gradients are greater than 1 so diagram is as shown



lines exit the square when $u_2 = V$, $u_1 = \frac{d_1 V}{d_2+l_2}$ or $\frac{(d_1+l_1)V}{d_2}$
Probability of a collision is given by the ratio of the area of the shaded triangle to that of the square.
Area of triangle $= \frac{1}{2}\left[\frac{(d_1+l_1)V}{d_2} - \frac{d_1 V}{d_2+l_2}\right]V = \frac{[d_1 l_2 + l_1 d_2 + l_1 l_2]V^2}{2d_2(d_2+l_2)}$
so probability of a collision is $\frac{l_1 l_2 + l_1 d_2 + l_2 d_1}{2d_2(d_2+l_2)}$

If back of lorry is nearer than front of train then both gradients are lews than 1 and the two lines exit the righgt hand side of the square with $u_2 = \frac{d_2}{d_1+l_1}V$ or $\frac{d_2+l_2}{d_1}V$ so probability of a collision is $\frac{1}{2}\left[\frac{d_2+l_2}{d_1} - \frac{d_2}{d_1+l_1}\right] = \frac{l_1 l_2 + l_1 d_2 + l_2 d_1}{2d_1(d_1+l_1)}$

If neither of these conditions apply then we must have $\frac{d_2}{d_1+l_1} < 1$ and $\frac{d_2+l_2}{d_1} > 1$ see diagram on right



Probability is now $1 - \frac{1}{2}\left(\frac{d_2}{d_1+l_1} + \frac{d_1}{d_2+l_2}\right)$
$= \frac{2(d_1+l_1)(d_2+l_2) - d_2(d_2+l_2) - d_1(d_1+l_1)}{2(d_1+l_1)(d_2+l_2)}$
$= \frac{(d_1+l_1)(d_2+l_2-d_1) + (d_2+l_2)(d_1+l_1-d_2)}{2(d_1+l_1)(d_2+l_2)}$