UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **MATH1401**

ASSESSMENT PATTERN : **MATH1401A**

MODULE NAME : **Mathematical Methods 1**

DATE : **11-May-11**

TIME : **14:30**

TIME ALLOWED : **2 Hours 0 Minutes**

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) The equation of a plane is $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. Show that this is equivalent to

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$$

for some $\mathbf{n}$ which you should determine.

(b) Show that the minimum distance between the two lines $\mathbf{r} = \mathbf{a}_1 + \lambda\mathbf{b}_1$ and $\mathbf{r} = \mathbf{a}_2 + \mu\mathbf{b}_2$ is given by

$$\left|\frac{(\mathbf{a}_1 - \mathbf{a}_2) \cdot (\mathbf{b}_1 \wedge \mathbf{b}_2)}{|\mathbf{b}_1 \wedge \mathbf{b}_2|}\right|.$$

(c) Show that the point $(0, 0, 1)$ lies in both of the planes $x + y + z = 1$ and $x - y + 2z = 2$ and that the minimum distance between the x-axis and the line of intersection of these two planes is $1/\sqrt{5}$.

2. (a) Show that the function

$$y(x) = \frac{\exp(ax)}{1 + \exp(x)},$$

has a stationary point at

$$x = \ln\left(\frac{a}{1-a}\right), \quad y = a^a(1-a)^{1-a},$$

for $0 < a < 1$. Sketch the graph of the function for the three cases $a = 1/2$, $a = 1$, $a = 2$ on a single set of axes.

(b) Using De Moivre's theorem show that

$$\sin^5\theta = \frac{1}{16}\left(\sin(5\theta) - 5\sin(3\theta) + 10\sin(\theta)\right).$$

3. Show that for $\alpha \ll 1$

$$\int_0^{\pi} \cos(\alpha \sin x) dx = \pi \left(1 - \frac{\alpha^2}{4} + \frac{\alpha^4}{64} + O(\alpha^5)\right).$$

4. (a) If $I_n(x) = \int_0^x \frac{\sinh^{2n+1} t}{\cosh t} dt$, show that

$$I_{n+1}(x) = \frac{\sinh^{2n+2} x}{2(n+1)} - I_n(x), \qquad n \geqslant 0.$$

Hence, or otherwise, find I_3.

(b) Find

$$\int \frac{dx}{1 - \sin x}.$$

5. Solve the equations

(a) $xy' - 2y = x^3 \ln x, \quad y(1) = -1,$

(b) $x^2 y' = xy + y^2, \quad y(1) = 1,$

(c) $x^2 y'' + xy' - 4y = \ln x, \quad x > 0, \quad y(1) = y'(1) = 1,$

where $y' = dy/dx$ and $y'' = d^2y/dx^2$.

6. Determine the series solution for the following differential equation

$$(x^2 + 1)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$$

about the regular point $x_0 = 0$.