UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**MATH1301**
ASSESSMENT PATTERN	:	**MATH1301A**
MODULE NAME	:	**Applied Mathematics 1**
DATE	:	**27-May-11**
TIME	:	**10:00**
TIME ALLOWED	:	**2 Hours 0 Minutes**

All questions may be attempted but only marks obtained on the best four *solutions will count.*
The use of an electronic calculator is not *permitted in this examination.*

1. A fair coin is tossed six times.

   (i) Find the probability that exactly three heads appear.

   (ii) Find the probability that exactly three heads appear, one of which appears on the second throw.

   (iii) Find the probability that exactly three heads appear, one of which appears on the second throw and the other two appear after the second throw.

   (iv) Given that exactly three heads appear, find the *conditional* probability that they occur on the second, third and fourth throws.

   (v) Are the events $A = \{\text{exactly three heads are thrown}\}$ and $B = \{\text{the second throw is a head}\}$ independent?

2. (i) A fair die is rolled repeatedly. Let P_r be the probability that the r^{th} roll is the first roll resulting in the number "1" showing. Find a formula for P_r.

   (ii) Show that $\sum_{r=1}^{\infty} P_r = 1$.

   (iii) Show from the definition of mean value that the mean (or expected) number of rolls required to first reveal a "1" is six.

   The identity

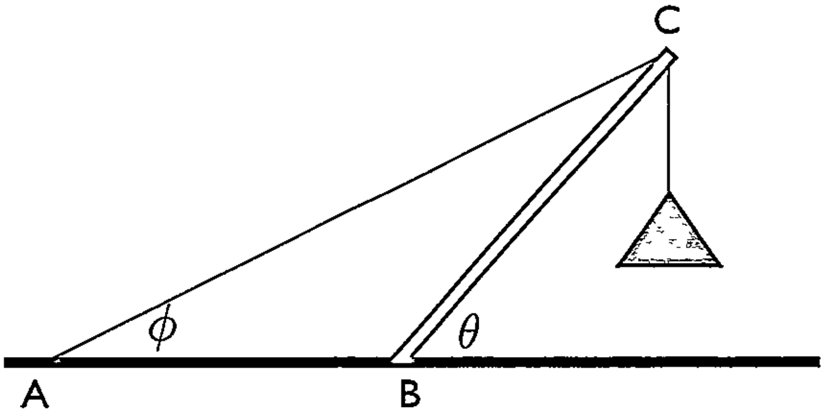
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad |x| < 1,$$

   and its derivative might be useful in the above.

   (iv) A particular office receives on average ten telephone calls each lunch time. Assume that the distribution of calls is Poisson. Find the probability that at least two calls are received during a particular lunch time.

3. (i) State the conditions for a system of forces $\mathbf{F}_1, \ldots, \mathbf{F}_n$ acting at points with position vectors $\mathbf{r}_1, \ldots, \mathbf{r}_n$ respectively to be in equilibrium.

   (ii) A strut (rod) of mass M rests on a rough surface at an angle θ. The top of the strut is supported by a light string which is secured to the ground at an angle $\phi < \theta$ as shown in the diagram. A weight of mass m is suspended from a second light string secured to the top of the strut. The coefficient of static friction between the strut and the ground is μ.



Show that the tension T in the string connecting A and C is

$$T = \frac{(2m + M)g\cos\theta}{2\sin(\theta - \phi)}.$$

Show that for limiting equilibrium, the coefficient of static friction between the strut BC and the ground is

$$\mu = \frac{2m + M}{2(m + M)\tan\theta - M\tan\phi}.$$

4. A particle of mass $m = 1$ moves in the potential

$$V(x) = x(x-1)^2.$$

   (i) Find the force acting on the particle when it is at position x.

   (ii) Locate all equilibrium points and classify them as stable or unstable.

   (iii) Find the (approximate) period of small oscillations near the stable equilibrium point.

   (iv) Suppose that the particle is at the point $x = 0$ at some time and is later found to be at the point $x = 1$. What can you conclude about the possible values of the energy?

5. A small heavy ball B_1 is dropped (with zero initial velocity) off the edge of a high cliff. Air resistance on the ball is negligible. At the same time, a second ball B_2 is thrown vertically downwards from the cliff with an initial speed U. The second ball is significantly larger and lighter than the first. Air resistance on B_2 is kv^2 per unit mass, where k is a positive constant and v is the speed of B_2. Show that if the two balls have the same speed V at some time T after both balls are released, then

$$U = a\frac{bT+1+(bT-1)\,e^{2bT}}{bT+1-(bT-1)\,e^{2bT}}, \quad \text{where } a=\sqrt{g/k}, \quad b=\sqrt{gk}$$

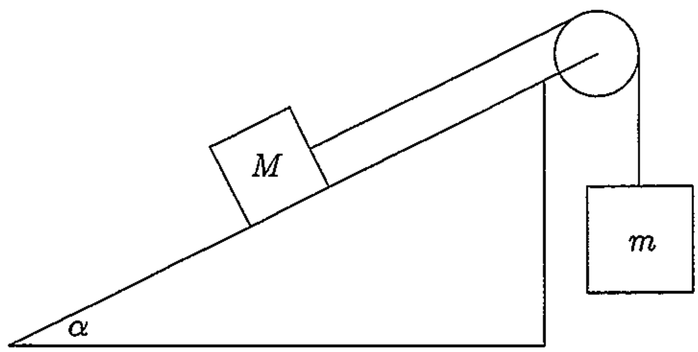
and

$$V = gT.$$

Express the distance h_1 that the ball B_1 has fallen and the distance h_2 that the ball B_2 has fallen in terms of k, g, T and U.

6. (i) A light cable is wrapped around a cylinder and tensions T_1 and T_2 are applied to the ends. The coefficient of friction between the cylinder and the cable is μ. By considering the forces acting on a small arc of the cable, show that if the system is in equilibrium, with the cable poised to slip in the direction of T_2, then $T_2 = T_1 e^{\mu\theta}$, where θ is the angle subtended at the centre of the circle by the arc of the cable which is in contact with the cylinder.

   (ii) Consider two masses connected by a light string as illustrated in the figure. The coefficient of friction between the mass M and the inclined plane is μ_1. The system is in limiting equilibrium with the mass m poised on the point of descending.



If the pulley is massless and rotates freely, show that

$$\frac{m}{M} = \mu_1 \cos\alpha + \sin\alpha.$$

Now suppose that the pulley becomes jammed. The system will be in limiting equilibrium again if m is increased such that the cable is about to slip around the pulley. The coefficient of static friction between the cable and the pulley is μ_2. Show that

$$\frac{m}{M} = e^{(\pi+2\alpha)/2}\left\{\mu_1 \cos\alpha + \sin\alpha\right\}.$$