If each vertex is shared between n m-gons, and if there are F m-gons in the whole polyhedron, then mF counts each vertex n times, hence,

$$mF = nV \implies V = \frac{m}{n}F \tag{1}$$

Likewise, each edge is shared between 2 m-gons, hence,

$$mF = 2E \implies E = \frac{m}{2}F \tag{2}$$

Euler's formula states

$$V - E + F = 2 \tag{3}$$

Hence,

$$\begin{aligned}
\frac{m}{n}F - \frac{m}{2}F + F &= 2 \\
\left(\frac{m}{n} - \frac{m}{2} + 1\right) F &= 2 \\
(2m - mn + 2n)\, F &= 4n \\
(4 - (4 - 2m - 2n + mn))\, F &= 4n \\
(4 - (n-2)(m-2))\, F &= 4n \\
hF = 4n \implies F &= \frac{4n}{h}
\end{aligned}$$

All polyhedra have at least 3 faces meeting at each vertex, and each face has at least 3 edges, hence, $n \geq 3$ and $m \geq 3$. F is a positive integer, hence h must also be positive, therefore $(n-2)(m-2) < 4$. Thus we obtain upper bounds $n < 6$ (if $m = 3$) and $m < 6$ (if $m = 3$). Considering the values of h for $3 \leq n \leq 5$ and $3 \leq m \leq 5$, we find that the only values of (m, n) which give positive h are $(3,3), (3,4), (3,5), (4,3), (5,3)$. These are the five regular polyhedra:

name	m	n	F	V	E
tetrahedron	3	3	4	4	4
octahedron	3	4	8	6	12
icosahedron	3	5	20	12	30
cube	4	3	6	8	12
dodecahedron	5	3	12	20	30