$$
\begin{aligned}
{u_{n+2}}^2 + {u_{n-1}}^2 &= (u_{n+1} + u_n)^2 + (u_{n-1})^2 \\
&= (u_n + u_{n-1} + u_n)^2 + (u_{n-1})^2 = (2u_n + u_{n-1})^2 + (u_{n-1})^2 \\
&= 4{u_n}^2 + 4u_n u_{n-1} + 2{u_{n-1}}^2 = 2\left(2{u_n}^2 + 2u_n u_{n-1} + {u_{n-1}}^2\right) \\
&= 2\left({u_n}^2 + \left({u_n}^2 + 2u_n u_{n-1} + {u_{n-1}}^2\right)\right) \\
&= 2\left({u_n}^2 + (u_n + u_{n-1})^2\right) \\
{u_{n+2}}^2 + {u_{n-1}}^2 &= 2\left({u_{n+1}}^2 + {u_n}^2\right)
\end{aligned}
$$

Suppose $u_{2n} = {u_n}^2 + {u_{n-1}}^2$ and ${u_{2n+1}}^2 = u_{n+1}^2 - u_{n-1}^2$ are true for some $n = k$. Consider the case of $n = k + 1$:

$$
\begin{aligned}
u_{2(k+1)} &= u_{2k+2} \\
&= u_{2k+1} + u_{2k} \\
&= \left({u_{k+1}}^2 - {u_{k-1}}^2\right) + \left({u_k}^2 + {u_{k-1}}^2\right) \\
&= {u_{k+1}}^2 + {u_k}^2 \\
u_{2(k+1)} &= {u_{(k+1)}}^2 + {u_{(k+1)-1}}^2
\end{aligned}
$$

$$
\begin{aligned}
u_{2(k+1)+1} &= u_{2k+3} \\
&= u_{2k+2} + u_{2k+1} \\
&= \left({u_{k+1}}^2 + {u_k}^2\right) + \left({u_{k+1}}^2 - {u_{k-1}}^2\right) \\
&= \left({u_{k+1}}^2 + 2u_{k+1}u_k + {u_k}^2\right) + \left({u_{k+1}}^2 - 2u_{k+1}u_k - {u_{k-1}}^2\right) \\
&= (u_{k+1} + u_k)^2 + \left({u_{k+1}}^2 - 2u_{k+1}u_k + {u_k}^2\right) - {u_k}^2 - {u_{k-1}}^2 \\
&= (u_{k+1} + u_k)^2 + (u_{k+1} - u_k)^2 - {u_k}^2 - {u_{k-1}}^2 \\
&= (u_{k+1} + u_k)^2 + (u_k + u_{k-1} - u_k)^2 - {u_k}^2 - {u_{k-1}}^2 \\
&= {u_{k+2}}^2 + {u_{k-1}}^2 - {u_k}^2 - {u_{k-1}}^2 \\
&= {u_{k+2}}^2 - {u_k}^2 \\
u_{2(k+1)+1} &= {u_{(k+1)+1}}^2 - {u_{(k+1)-1}}^2
\end{aligned}
$$

Hence it is also true for $n = k + 1$.

Consider the case of $n = 1$:

$$
\begin{aligned}
u_2 &= u_1 + u_0 = 1 + 1 = 2 \\
u_2 &= {u_1}^2 + {u_0}^2 = 1^2 + 1^2 = 2
\end{aligned}
$$

$$
\begin{aligned}
u_3 &= u_2 + u_1 = 2 + 1 = 3 \\
u_3 &= {u_2}^2 - {u_0}^2 = 2^2 - 1^2 = 3
\end{aligned}
$$

Therefore, by induction, the result is true for all positive n.