$$\begin{aligned}
(x * y) * z &= (x + y + axy) * z \\
&= (x + y + axy) + z + a(x + y + axy)z \\
&= x + y + axy + z + axz + ayz + a^2xyz \\
&= (x + y + z) + a(xy + yz + xz) + a^2xyz
\end{aligned}$$

$$\begin{aligned}
x * (y * z) &= x * (y + z + ayz) \\
&= x + (y + z + ayz) + ax(y + z + ayz) \\
&= x + y + z + ayz + axy + axz + a^2xyz \\
&= (x + y + z) + a(xy + yz + xz) + a^2xyz
\end{aligned}$$

Thus, $(x * y) * z = x * (y * z)$, hence, $*$ is associative.

Since $x * y$ is symmetric, $*$ is also commutative:

$$x * y = x + y + axy = y + x + ayx = y * x$$

We may verify that 0 is the identity under $*$:

$$\begin{aligned}
0 * x = 0 + x + a \cdot 0 \cdot x = x \\
x * 0 = x + 0 + a \cdot x \cdot 0 = x
\end{aligned}$$

Let y be such that $x * y = 0$ for a given x.

$$\begin{aligned}
x * y &= 0 \\
x + y + axy &= 0 \\
y + axy &= -x \\
y(1 + ax) &= -x \\
y &= -\frac{x}{1 + ax}
\end{aligned}$$

Hence, inverses exist for all x except $x = -\frac{1}{a}$.

$x * y \in \mathbb{R}$ for all $x, y \in \mathbb{R}$, hence $*$ is closed, is associative, has an identity, and has inverses in G where $G = \mathbb{R} \setminus \left\{-\frac{1}{a}\right\}$. Therefore $(G, *)$ is a group.

Now, let us consider x such that $x * x = 0$.

$$\begin{aligned}
x * x &= 0 \\
x + x + axx &= 0 \\
2x + ax^2 &= 0 \\
x(2 + ax) &= 0
\end{aligned}$$

Therefore 0 and $-\frac{2}{a}$ are self-inverse under $*$, hence, form a 2-element subgroup of $(G, *)$.