General Certificate of Education
June 2003
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP4**
Unit Pure 4

Friday 13 June 2003 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The cubic equation

$$x^3 + 2px^2 - 8 = 0, \qquad \text{where } p \text{ is real,}$$

has roots α, β and $\alpha + \beta$.

(a) Show that:

(i) $\alpha + \beta = -p$; *(2 marks)*

(ii) $\alpha\beta = -\dfrac{8}{p}$. *(2 marks)*

(b) Show that $p = 2$. *(5 marks)*

2 (a) Show that the equation

$$4\sinh x + e^x = 5$$

can be expressed as

$$3e^{2x} - 5e^x - 2 = 0.$$ *(3 marks)*

(b) Hence solve, for real x,

$$4\sinh x + e^x = 5,$$

giving your answer as a natural logarithm. *(4 marks)*

3 (a) Shade, on an Argand diagram, the region in which

$$|z - 2i| \leqslant 1.$$ *(4 marks)*

(b) Find the greatest and least values of the argument of complex numbers z satisfying

$$|z - 2i| \leqslant 1,$$

giving your answers in terms of π. *(4 marks)*

4 (a) Evaluate:

(i) $\displaystyle\int \cosh^2 x \, \mathrm{d}x\,;$ *(3 marks)*

(ii) $\displaystyle\int x \cosh x \, \mathrm{d}x\,.$ *(3 marks)*

(b) A curve C is given parametrically by the equations

$$x = \cosh t + t, \qquad y = \cosh t - t.$$

Express

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$$

in terms of $\cosh t$. *(5 marks)*

(c) (i) The arc of C from $t = 0$ to $t = 1$ is rotated through 2π radians about the x-axis.

Show that S, the area of the curved surface generated, is given by

$$S = 2\pi\sqrt{2}\int_0^1 (\cosh t - t)\cosh t \, \mathrm{d}t.$$ *(1 mark)*

(ii) Hence find S, leaving your answer in terms of hyperbolic functions. *(4 marks)*

5 (a) Show that

$$\frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}.$$ *(2 marks)*

(b) Hence find

$$\sum_{r=1}^{n} \frac{r}{(r+1)!}.$$ *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

6 It is given that

$$w = \frac{1}{\sqrt{2}}(-1 + \mathrm{i}).$$

(a) (i) Show that $|w| = 1$.

(ii) Express w in the form $\mathrm{e}^{\mathrm{i}\theta}$ where $-\pi < \theta \leqslant \pi$. *(3 marks)*

(b) Solve $z^3 = w$, giving your answers in the form $\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leqslant \pi$. *(4 marks)*

(c) (i) Show that

$$(1 - w)(1 - w^*) = 2 + \sqrt{2},$$

where w^* is the complex conjugate of w. *(3 marks)*

(ii) The sum of the geometric series $\sum_{r=0}^{11} w^r$ is S.

Show that

$$S = \frac{2}{1 - w}$$

and hence express S in the form $1 + p\mathrm{i}$, where p is real. *(5 marks)*

END OF QUESTIONS