General Certificate of Education
January 2003
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP4**
Unit Pure 4

Monday 20 January 2003 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Sheets of graph paper are available on request.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

1 The cubic equation

$$x^3 + 2x^2 + 5x + k = 0,$$

where k is real, has roots α, β and γ.

(a) Write down the values of:

(i) $\alpha + \beta + \gamma$; *(1 mark)*

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$. *(1 mark)*

(b) (i) Show that $\alpha^2 + \beta^2 + \gamma^2 = -6$. *(3 marks)*

(ii) Hence explain why the cubic equation must have two non-real roots. *(2 marks)*

(c) Given that one root is $-2 + 3\mathrm{i}$, find the value of k. *(5 marks)*

2 The complex numbers z_1 and z_2 are given by

$$z_1 = 1 + \sqrt{3}\,\mathrm{i} \qquad \text{and} \qquad z_2 = \mathrm{i}z_1.$$

(a) (i) Express z_2 in the form $a + \mathrm{i}b$. *(1 mark)*

(ii) Find the modulus and argument of z_2. *(2 marks)*

(b) Label the points representing z_1 and z_2 on an Argand diagram. *(1 mark)*

(c) On the **same** Argand diagram, sketch the locus of points z satisfying:

(i) $|z - z_1| = |z - z_2|$; *(2 marks)*

(ii) $\arg(z - z_1) = \arg z_2$. *(2 marks)*

3 (a) Express $\dfrac{1}{(r-1)(r+1)}$ in partial fractions. *(2 marks)*

(b) Hence find

$$\sum_{r=2}^{n} \frac{1}{(r^2-1)},$$

giving your answer in the form

$$A + \frac{B}{n} + \frac{C}{n+1}.$$ *(5 marks)*

4 (a) Use the definition $\cosh t = \frac{1}{2}(\mathrm{e}^t + \mathrm{e}^{-t})$ to show that

$$2\cosh^2 t = 1 + \cosh 2t.$$ *(3 marks)*

(b) A curve is given parametrically by the equations

$$x = 2\sinh t, \quad y = \cosh^2 t.$$

(i) Show that $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 4\cosh^4 t.$ *(6 marks)*

(ii) Hence show that the length of arc of the curve from the point where $t = 0$ to the point where $t = \frac{1}{2}$ is

$$\tfrac{1}{2}(1 + \sinh 1).$$ *(4 marks)*

(c) Find the Cartesian equation of the curve. *(2 marks)*

5 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 1, \qquad 3u_{n+1} = 2u_n - 1.$$

Prove by induction that, for all $n \geqslant 1$,

$$u_n = 3\left(\tfrac{2}{3}\right)^n - 1.$$ *(6 marks)*

Turn over ▶

6 (a) (i) Use de Moivre's theorem to show that

$$(\cos\theta + \mathrm{i}\sin\theta)^4 + (\cos\theta - \mathrm{i}\sin\theta)^4 = 2\cos 4\theta.$$ *(2 marks)*

(ii) Deduce that

$$(\cot\theta + \mathrm{i})^4 + (\cot\theta - \mathrm{i})^4 = \frac{2\cos 4\theta}{\sin^4\theta}, \quad \theta \neq r\pi.$$ *(1 mark)*

(b) Verify that $\cot\frac{1}{8}\pi$ is a root of

$$(z + \mathrm{i})^4 + (z - \mathrm{i})^4 = 0$$

and find the **three** other roots of this equation giving each answer in the form $+\cot\alpha$ or $-\cot\alpha$, where $0 < \alpha \leqslant \frac{\pi}{2}$. *(4 marks)*

(c) Express the equation in part (b) in the form

$$z^4 + bz^2 + c = 0,$$

where b and c are real numbers to be determined. *(2 marks)*

(d) Hence, or otherwise, find in surd form the value of $\cot^2\dfrac{\pi}{8}$. *(3 marks)*

END OF QUESTIONS