Core Mathematics C3 For AQA
Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

- Full marks may be obtained for answers to ALL questions.
- The formulae booklet, available from AQA, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 75.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Use integration by parts to find the exact value of $\int_1^e x^2 \ln x \, dx$. *(6 marks)*

2. Given $f: x \mapsto \dfrac{2}{x-3}$, $\quad x \in \mathbb{R}, \quad x \neq 3,$

(a) express f^{-1} in the same form. *(4 marks)*

(b) Evaluate f(4) and $ff^{-1}(7)$. *(2 marks)*

3. You are given $f(x) = \ln(x+2)$, $\quad x \in \mathbb{R}, \quad x > -2.$

(a) On two separate diagrams sketch the graphs of

$$y = f(x) \quad \text{and} \quad y = |f(x)|.$$ *(3 marks)*

(b) Explain how your graph shows that the equation

$$|f(x)| - x = 0 \qquad \ldots(A)$$ *(1 mark)*

has only one solution for x.

(c) Show that the solution to the equation $|f(x)| - x = 0$ lies in the interval $[1, 2]$. *(2 marks)*

(d) Using the iteration

$$x_{n+1} = \ln(x_n + 2) \quad \text{and} \quad x_0 = 1,$$

find the values of x_1, x_2, x_3, x_4, x_5 and hence give the solution to equation (A) to 3 decimal places. *(4 marks)*

4. Differentiate with respect to x,

(a) $x^2 \ln x$ *(4 marks)*

(b) $\cos^2 3x$ *(3 marks)*

(c) $\dfrac{\sin x}{x}$. *(3 marks)*

5. (a) Show that

$$\int_{a}^{a+h} \left(x^2 - a^2\right) \mathrm{d}x = \frac{h^2}{3}(3a + h). \qquad \textit{(4 marks)}$$

(b) Find $\int \tan^2 x \ \mathrm{d}x$. *(2 marks)*

(c) Find $\int_0^{\frac{\pi}{3}} x \sec^2 x \ \mathrm{d}x$. *(5 marks)*

6. (a) Show that the equation

$$\mathrm{e}^x + 6\mathrm{e}^{-x} = 5 \qquad \ldots\text{(A)}$$

can be written in the form

$$(\mathrm{e}^x - 3)(\mathrm{e}^x - 2) = 0 \qquad \textit{(3 marks)}$$

(b) Use this to find the values of x which satisfy equation (A). *(4 marks)*

(c) Hence find the values of x which satisfy the equation

$$\mathrm{e}^{2x+2} - 5\mathrm{e}^{x+1} + 6 = 0. \qquad \textit{(4 marks)}$$

7.

Figure 1

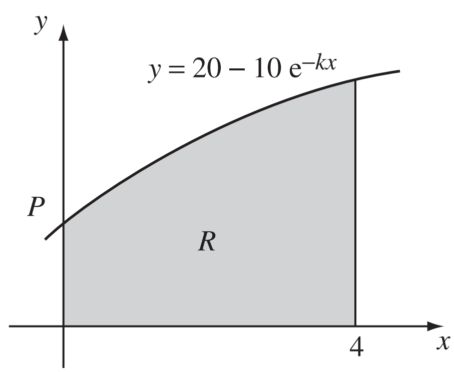

Figure 1 shows part of the curve C with equation

$$y = 20 - 10\,\mathrm{e}^{-kx}.$$

(a) Write down the coordinates of the point P where C crosses the y-axis. *(1 mark)*

(b) The gradient of C at the point P is 5. Show that $k = \frac{1}{2}$. *(3 marks)*

(c) Find the area of the region R which is bounded by C, the positive axes and the line $x = 4$. *(5 marks)*

8. (a) Given $x = \sin y$, find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. *(2 marks)*

The point $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{\pi}{4}\right)$ lies on the curve $y = \arcsin x$.

Using your answer to part (*a*) find,

(b) the gradient of the tangent to the curve at P, *(3 marks)*

(c) the equation of the tangent to the curve at P. *(2 marks)*

The tangent to the curve at P meets the x axis at the point Q.

(d) Show that the coordinates of the point Q are $\left(\dfrac{4-\pi}{4\sqrt{2}}, 0\right)$ *(3 marks)*

(e) Find the exact value of the area of the triangle OPQ. *(2 marks)*