

# Core Mathematics C3 Advanced Level

## For AQA

### Paper I

**Time: 1 hour 30 minutes**

#### *Instructions and Information*

- Full marks may be obtained for answers to ALL questions.
- The formulae booklet, available from AQA, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 75.

#### *Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.

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1. The function  $f$  is defined by

$$f: x \mapsto \frac{1-2x}{2-x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(a) Prove that  $f^{-1}(x) = f(x)$  for all  $x \in \mathbb{R}, x \neq 2$ . (3 marks)

(b) Hence find, in terms of  $k$ ,  $ff(k^2)$ , where  $k^2 \neq 2$ . (3 marks)

2. The function  $g$  is given by

$$g: x \mapsto \ln |4x - 12|, \quad x \in \mathbb{R}, \quad x \neq 3. \quad (3 \text{ marks})$$

(a) Sketch the graph of  $y = g(x)$ .

(b) Find the exact coordinates of all the points at which the curve  $y = g(x)$  meets the coordinate axes. (3 marks)

3.  $f(x) = x - \frac{1}{x-2} + \frac{5}{x^2+x-6}, \quad x \in \mathbb{R}, \quad x > 2.$

(a) Show that  $f(x) = \frac{x^2 + 3x - 1}{x + 3}$ . (5 marks)

(b) Solve the equation

$$f'(x) = \frac{26}{25}. \quad (5 \text{ marks})$$

4. (a) Given  $y = \frac{e^{5x}}{x}$ , find  $\frac{dy}{dx}$  and the value of  $x$  for which  $\frac{dy}{dx} = 0$ . (4 marks)

(b) (i) Given  $x = \sin^2 3y$ , find  $\frac{dx}{dy}$  in terms of  $y$ . (3 marks)

(ii) Evaluate  $\frac{dy}{dx}$  for  $y = \frac{\pi}{12}$ . (3 marks)

5.

Figure 1

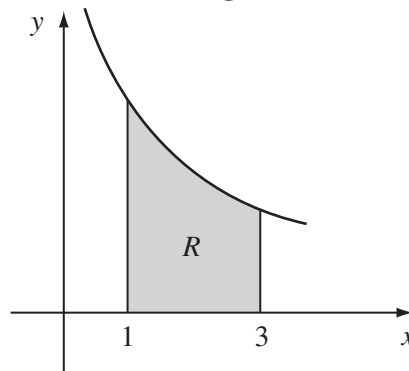


Figure 1 shows a sketch of the curve  $C$  with equation  $y = \frac{2x + 1}{x}$ ,  $x \neq 0$ .

The shaded region  $R$  is bounded by  $C$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 3$ .

(a) Find the area of the region  $R$ . (3 marks)

The region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis to form a solid shape  $S$ .

(b) Show that the volume of  $S$  is  $\pi \left( \frac{26}{3} + 4 \ln 3 \right)$ . (6 marks)

6. The curve  $C$  has the equation  $y = f(x)$  where

$$f(x) = \frac{1}{2} \ln x + \frac{1}{x^2}, \quad x > 0.$$

$P$  is a stationary point on  $C$ .

(a) Calculate the  $x$ -coordinate of  $P$ . (4 marks)

(b) Show that the  $y$ -coordinate of  $P$  can be expressed in the form  $k^{-1} \ln k + k^{-2}$ , where  $k$  is a constant to be found. (2 marks)

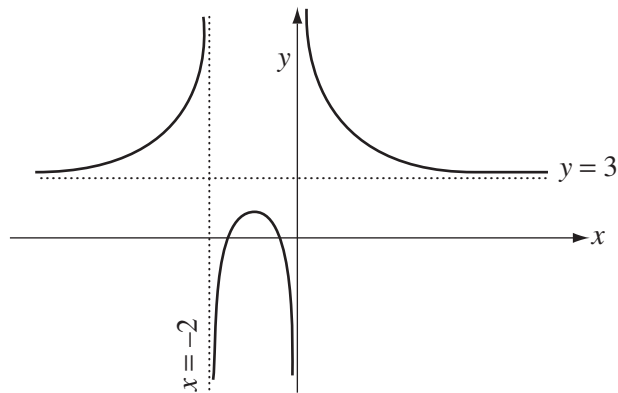
The point  $Q$  on  $C$  has  $x$ -coordinate 1.

(c) Show that the equation to the normal at  $Q$  can be written as

$$Ay + Bx + C = 0$$

where  $A$ ,  $B$  and  $C$  are integers to be found. (3 marks)

7.



The diagram shows the sketch of part of the curve with equation  $y = f(x)$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$ ,  $x \neq -2$ .

The curve has a maximum at  $(-1, 2)$ .

The lines  $y = 3$ ,  $x = -2$  and the  $y$ -axis are asymptotes to the curve as shown.

On separate diagrams sketch the graphs of

- (a)  $y = |f(x)|$  (3 marks)
- (b)  $y = f(2x)$  (3 marks)
- (c)  $y = f(x - 1) - 2$  (4 marks)

In each case state the equations of the new asymptotes and the coordinates of the turning points.

- (d) Solve the equation  $f(x - 1) - 2 = 0$  (2 marks)

8. A cup of tea, initially at boiling point, cools according to Newton's law of cooling so that after  $t$  minutes its temperature,  $T^\circ\text{C}$ , is given by

$$T = 15 + 85e^{-\frac{t}{8}}.$$

- (a) Sketch the graph of  $T$  against  $t$ . (3 marks)
- (b) What is the temperature of the tea after 4 minutes? (2 marks)
- (c) How long does it take the tea to cool to  $40^\circ\text{C}$ ? (3 marks)
- (d) Find  $\frac{dT}{dt}$  and hence find the value of  $T$  at which the temperature is decreasing at the rate of  $1.7^\circ\text{C}$  per minute. (4 marks)
- (e) However long the cup of tea is left to cool down, it never falls below a certain temperature. What temperature is that? (1 mark)