Core Mathematics C3 For AQA
Advanced Level

Paper L

Time: 1 hour 30 minutes

Instructions and Information

- Full marks may be obtained for answers to ALL questions.
- The formulae booklet, available from AQA, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 75.

Advice to Candidates
You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

Published by Elmwood Press
80 Attimore Road
Welwyn Garden City
Herts. AL8 6LP

Tel. 01707 333232

1. $\mathrm{f}(x) = x^2 + 2x, \quad x \in \mathbb{R}.$

(a) Write $\mathrm{f}(x)$ in the form $(x + a)^2 + b$. *(2 marks)*

(b) Write down the coordinates of the turning point on the graph of $y = \mathrm{f}(x)$. *(1 mark)*

(c) Sketch the graph of $y = \mathrm{f}(x)$ and $y = |\mathrm{f}(x)|$. *(3 marks)*

(d) Solve the inequality $|\mathrm{f}(x)| > 3$. *(3 marks)*

2. The function f is given by

$$\mathrm{f}: x \mapsto \mathrm{e}^{2x+3}, \qquad x \in \mathbb{R}.$$

(a) Find the exact value of ff(0). *(2 marks)*

(b) Find an expression for $\mathrm{f}^{-1}(x)$. *(3 marks)*

(c) Write down the domain of f^{-1}. *(1 mark)*

3. Given that

$$x = \ln\left(y^2 + 4\right),$$

show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{2} + \dfrac{2}{y}$. *(6 marks)*

4. $\mathrm{f}(x) = \ln x - 3x + 5, \qquad x > 0$

(a) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval [1, 2]. *(2 marks)*

The root α is to be estimated using the iterative formula

$$x_{n+1} = \frac{1}{3}\left(\ln x_n + 5\right), \quad x_0 = 2.$$

(b) Calculate the values of x_1, x_2, x_3 and x_4 giving your answers to 4 significant figures. *(3 marks)*

(c) Prove that α is 1.876, to 4 significant figures. *(2 marks)*

5.

Figure 1

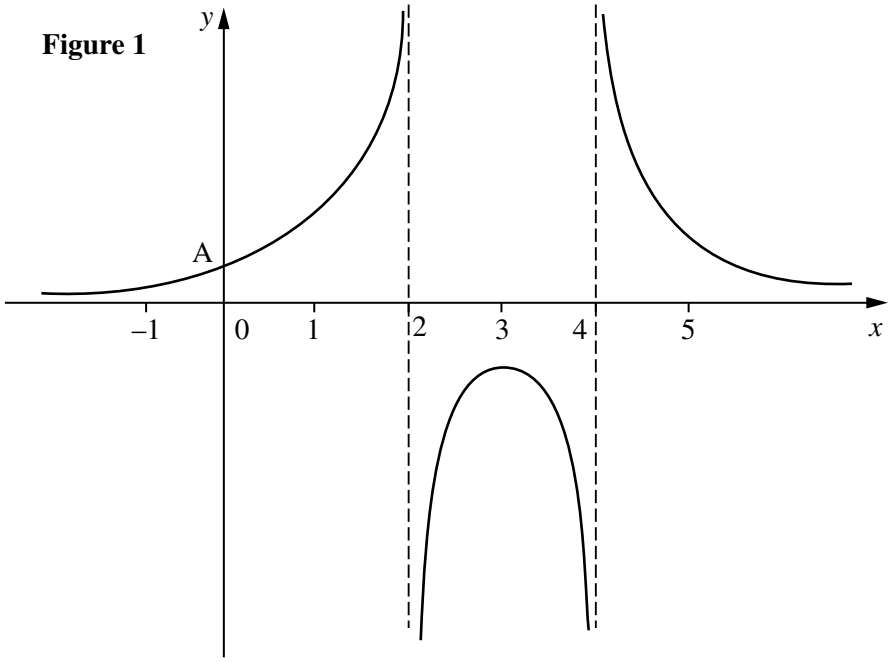


Figure 1 shows a sketch of the curve with the equation $y = \mathrm{f}(x), x \in \mathbb{R}$.

The curve has a maximum point at $(3, -1)$ and meets the y-axis at the point $A\,(0,\ 0.125)$.

The lines $x = 2, x = 4$ and the x-axis are asymptotes to the curve as shown in Fig. 1.

On a separate diagram sketch the graphs of

(a) $y = |4\mathrm{f}(x)|$ *(5 marks)*

(b) $y = \mathrm{f}(x + 3)$ *(4 marks)*

In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

(ii) how the curve approaches the asymptotes of the curve,

(iii) the coordinates of A.

6. (a) Given that $y = \tan x + \sin 2x$, find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \dfrac{\pi}{4}$. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $x = \dfrac{\pi}{4}$. *(3 marks)*

7. (a) (i) Sketch the graph of $y = \sin x \quad \text{for} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. *(2 marks)*

(ii) Sketch the graph of $y = \sin^{-1} x \quad \text{for} \quad -1 \leq x \leq 1$. *(2 marks)*

(iii) Describe the transformation which maps the curve in part (i) onto the curve in part (ii). *(1 mark)*

(b) Solve the equation

$$3\,\text{cosec}^2 2x = 4, \quad \text{for} \quad 0 \leq x \leq 180^\circ \qquad \textit{(5 marks)}$$

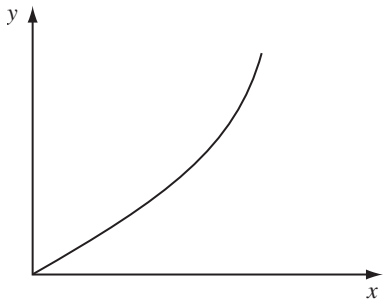
8. (a) On the same pair of axes sketch the graphs of

$y = |x - a|$ and $y = 2a - |x - a|$ where $a > 0$.
Label the graphs clearly. *(5 marks)*

(b) Write down the coordinates of the points of intersection of the two graphs. *(2 marks)*

(c) Find the area of the quadrilateral formed. *(3 marks)*

9.



The diagram shows the graph of $y = x\sqrt{(1 + x^2)}$. The region R is bounded by the curve, the x-axis from the origin to the point $(2, 0)$ and the line $x = 2$.

(a) Use the mid-ordinate with four strips to find an approximation for the area of R, giving your answer to 4 decimal places. *(4 marks)*

(b) Using the calculus find the area of R, giving your answer to 4 decimal places. *(5 marks)*

(c) Find the percentage error in the calculation of the area of R using the method in (a). Give your answer to 2 decimal places. *(2 marks)*