13. maximum height if no ceiling $= \frac{v^2 \sin^2\alpha}{2g}$

Particle will hit the ceiling if $h < \frac{v^2 \sin^2\alpha}{2g} \Rightarrow \sin^2\alpha > \frac{2gh}{v^2} = \frac{1}{4} \Rightarrow \sin\alpha > \frac{1}{2} \Rightarrow \alpha > \frac{\pi}{6}$

Since impact with ceiling is perfectly elastic the path of the particle will be symmetrical about the vertical through the point of impact.

Let velocity of projection be v and time to rach ceiling be t then

$h = vt\sin\alpha - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - (v\sin\alpha)t + h = 0 \Rightarrow t = \frac{1}{g}\left[v\sin\alpha - \sqrt{v^2\sin^2\alpha - 2gh} \right]$

i.e. $t = \frac{1}{g}\left[v\sin\alpha - \sqrt{v^2\sin^2\alpha - \frac{v^2}{4}} \right]$ since $v^2 = 8gh$

so $t = \frac{v}{g}\left[2\sin\alpha - \sqrt{4\sin^2\alpha - 1} \right]$

and range on floor $= vt\cos\alpha = \frac{v^2}{g}\cos\alpha\left[2\sin\alpha - \sqrt{4\sin^2\alpha - 1} \right] = 8h\cos\alpha\left[2\sin\alpha - \sqrt{4\sin^2\alpha - 1} \right]$

as required.