

14. Consider the ring B

Resolving vertically and horizontally

$$R \sin \theta + T \cos \theta = mg \text{ and } T \sin \theta - R \cos \theta = m\omega^2 l \sin \theta$$

eliminating R we have

$$T \cos^2 \theta + T \sin^2 \theta = mg \cos \theta + m\omega^2 l \sin^2 \theta$$

$$\text{i.e. } T = \frac{4}{5}mg + \frac{9}{25}ml \cdot \frac{5g}{2a} \text{ since } \theta = \sin^{-1} \frac{3}{5} \text{ and } \omega = \sqrt{\frac{5g}{2a}}$$

$$\text{so } T = \frac{4}{5}mg + \frac{9}{10}lmg$$

compression of spring is $a - (2a - l) = l - a$

$$\text{so } T = \frac{kmg(l-a)}{a}$$

$$\text{hence, } \frac{4}{5}mg + \frac{9}{10}lmg = \frac{kmg(l-a)}{a}$$

$$\Rightarrow 8a + 9la = 10kl - 10ka \Rightarrow l = \frac{10ka+8a}{10k-9} = \frac{(10k+8)a}{10k-9} \text{ as required}$$

