

14. Consider the ring B

Resolving vertically and horizontally

$$R \sin \theta + T \cos \theta = mg \text{ and } T \sin \theta - R \cos \theta = m\omega^2 l \sin \theta$$

eliminating R we have

$$T \cos^2 \theta + T \sin^2 \theta = mg \cos \theta + m\omega^2 l \sin^2 \theta$$

$$\text{i.e. } T = \frac{4}{5}mg + \frac{9}{25}ml \cdot \frac{5g}{2a} \text{ since } \theta = \sin^{-1} \frac{3}{5} \text{ and } \omega = \sqrt{\frac{5g}{2a}}$$

$$\text{so } T = \frac{4}{5}mg + \frac{9}{10} \frac{lmg}{a}$$

compression of spring is $a - (2a - l) = l - a$

$$\text{so } T = \frac{kmg(l-a)}{a}$$

$$\text{hence, } \frac{4}{5}mg + \frac{9}{10} \frac{lmg}{a} = \frac{kmg(l-a)}{a}$$

$$\Rightarrow 8a + 9l = 10kl - 10ka \Rightarrow l = \frac{10ka+8a}{10k-9} = \frac{(10k+8)a}{10k-9} \text{ as required}$$

Now consider the particle A and let the reaction of the rod have horizontal and vertical components R_x and R_y

so that $13R_x = 21R_y$

$$\text{Then } R_y - T \cos \theta = mg \text{ and } R_x - T \sin \theta = 2am\omega^2 \sin \theta$$

$$\text{i.e. } \frac{13}{21}R_x - T \cos \theta = mg \text{ and } R_x - T \sin \theta = 5mg \sin \theta$$

$$\text{eliminating } R_x \text{ gives } \frac{13}{21}(5mg \sin \theta + T \sin \theta) - T \cos \theta = mg \Rightarrow \frac{13}{21}\left(3mg + \frac{3}{5}T\right) - \frac{4}{5}T = mg$$

$$\Rightarrow 39mg - 9T = 21mg \Rightarrow T = -\frac{18mg}{9} = 2mg \text{ and } T = \frac{kmg\left(\frac{(10k+8)a}{10k-9} - a\right)}{a} = \frac{17kmg}{10k-9}$$

$$\text{hence, } 20kmg - 18mg = 17kmg \Rightarrow k = \frac{18}{3} = 6$$

$$\text{from resolutions for B we have } \frac{3}{5}R + \frac{8}{5}mg = mg \Rightarrow R = -mg$$

i.e. reaction between rod and ring B is mg in opposite direction to that shown.

