The data below give the concentrations $(\mathrm{mM})$ of thiol in the lysate of the blood of two groups of volunteers, the first group being normal and the second group being sufferers from rheumatoid arthritis.

Thiol concentration (mM)

| Normal | Rheumatoid |
| :--- | :--- |
| 1.84 | 1.81 |
| 2.32 | 3.06 |
| 1.94 | 2.62 |
| 1.92 | 3.66 |
| 1.65 | 3.07 |
| 1.91 | 2.76 |
| 2.07 | 1.58 |

The data were analysed using a t-test in Minitab. The output obtained appears in the box below.
Two Sample T-Test and Confidence Interval

Two sample $T$ for Normal vs Rheuma

|  | N | Mean | StDev | SE Mean |
| :--- | :--- | ---: | ---: | ---: |
| Normal | 7 | 1.950 | 0.207 | 0.078 |
| Rheuma | 7 | 2.651 | 0.733 | 0.28 |

95\% CI for $m u$ Normal - mu Rheuma: ( $-1.329,-0.07$ )
T -Test mu Normal $=\mathrm{mu}$ Rheuma (vs not $=$ ) $: \mathrm{T}=-2.44 \mathrm{P}=0.031 \mathrm{DF}=12$
Both use Pooled StDev $=0.539$
(a) Which type of t-test was used? Explain why this was chosen and the assumptions underlying the test.
(b) Write a brief interpretation of the parts of the output in italics/bold type.
(c) What conclusion can be drawn from the output about whether or not the mean thiol concentration of the two groups differ?

Two methods of determining the percentage fibre content of the raw materials used in an oat-based breakfast cereal were compared. Nine samples of the cereal were used. Each was divided into two parts, one part analysed by the old method and one by the new. The results were as follows:

| \% fibre | Sample |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| New Method | 12.7 | 13.6 | 11.6 | 12.7 | 14.2 | 14 | 11.8 | 12.5 | 13.2 |
| Old method | 12.2 | 12.7 | 11.6 | 12.3 | 13.9 | 13.8 | 11.9 | 12.0 | 13.4 |

A paired t-test and an unpaired t-test have been carried out using MINITAB and the analyses are given below.
(i) Which analysis is correct for the above data? Explain your reasoning.
(ii) What assumptions have to be made about the data for your chosen test to be valid?
(iii) Explain what null and alternative hypotheses have been tested.
(iv) What conclusion can be drawn from the test? Your answer should be one sentence suitable for a scientific report.

## Paired T- Test and Confidence Interval

Paired T for new - old

|  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | :---: |
| new | 9 | 12.922 | 0.909 | 0.303 |
| old | 9 | 12.644 | 0.856 | 0.285 |
| Difference | 9 | 0.278 | 0.346 | 0.115 |

95\% CI for mean difference: (0.012, 0.543)
T-Test of mean difference $=0$ (vs not $=0$ ) : T-Value $=2.41$ P-Value $=0.042$

Two Sample T-Test and Confidence Interval
Two sample $T$ for new vs old

|  | N | Mean | StDev | SE Mean |
| ---: | ---: | ---: | ---: | ---: |
| new | 9 | 12.922 | 0.909 | 0.30 |
| old | 9 | 12.644 | 0.856 | 0.29 |

95\% CI for mu new - mu old: ( -0.61, 1.17)
T-Test mu new $=$ mu old (vs not $=$ ) : $T=0.67 \quad \mathrm{P}=0.51 \quad \mathrm{DF}=15$

Two assay methods for measuring the level of vitamin B12 in healthy red blood cells were compared. Ideally they should give the same results. Blood was taken from 10 healthy adults and the level of B12 was found to be as follows:

| Subject | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Method 1 | 224 | 238 | 209 | 277 | 197 | 227 | 207 | 205 | 131 | 282 |
| Method 2 | 204 | 238 | 198 | 253 | 180 | 209 | 217 | 204 | 137 | 250 |

The methods have been compared using both an unpaired $t$-test and a paired $t$-test. The MINITAB analysis is given below.
(i) Explain which is the correct analysis giving your reasoning.
(ii) State the null and alternative hypotheses for the correct test.
(iii) What statistical conclusion can be drawn?
(iv) Explain the implications of this in the context of the question.

## Two Sample T-Test and Confidence Interval

Two sample $T$ for Method 1 vs Method 2

|  |  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Method 1 | 10 | 219.7 | 42.7 | 14 |  |
| Method 2 | 10 | 209.0 | 34.5 | 11 |  |

95\% CI for mu Method 1 - mu Method 2: ( -26 , 47)
T-Test mu Method $1=\mathrm{mu}$ Method 2 (vs not $=$ ) : $\mathrm{T}=0.62 \mathrm{P}=0.55 \mathrm{DF}=17$
Paired T-Test and Confidence Interval
Paired $T$ for Method 1 - Method 2

|  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | ---: |
| Method 1 | 10 | 219.7 | 42.7 | 13.5 |
| Method 2 | 10 | 209.0 | 34.5 | 10.9 |
| Difference | 10 | 10.70 | 13.85 | 4.38 |

95\% CI for mean difference: (0.79, 20.61)
$T$-Test of mean difference $=0$ (vs not $=0$ ) : T-Value $=2.44 \quad \mathrm{P}-\mathrm{Value}=0.037$

Two media are to be compared to determine which one is the better for detecting the presence of Salmonella. Medium 2 is the current standard; medium 1 is a new medium which is said to be more effective in detecting Salmonella. Eight variants of Salmonella were chosen, labelled A to H here. (It is important the medium chosen is effective over the whole spectrum of Salmonella variants.) Each variant was grown on both media. The logarithms of the counts were as follows: Log (count)
strain Medium 1 Medium 2
$\begin{array}{ll}\text { A } & 2.598 \quad 2.365\end{array}$
$\begin{array}{lll}\text { B } & 3.120 \quad 3.320\end{array}$

| C | 4.325 | 4.120 |
| :--- | :--- | :--- |

D $\quad 2.890 \quad 2.780$
E $\quad 3.640 \quad 2.904$
F $2.630 \quad 2.156$
G $\quad 4.450 \quad 4.320$
$\begin{array}{lll}\mathrm{H} & 3.780 & 3.652\end{array}$
The data have been analysed using both a paired t-test and an unpaired t-test. The MINITAB printout is given below.
a) Which is the correct analysis? Explain your choice clearly.
b) For your chosen test state the null and alternative hypotheses which have been used in the MINITAB printout.
c) State the conclusion from the test in statistical terms and in a manner suitable for a scientific report.

## Paired T-Test and CI: Medium 1, Medium 2

```
Paired T for Medium 1 - Medium 2
Log Count
\begin{tabular}{lrrrr} 
& N & Mean & StDev & SE Mean \\
Medium 1 & 8 & 3.429 & 0.730 & 0.258 \\
Medium 2 & 8 & 3.202 & 0.790 & 0.279 \\
Difference & 8 & 0.2270 & 0.2767 & 0.0978
\end{tabular}
```

```
95% lower bound for mean difference: 0.0417
T-Test of mean difference = 0 (vs > 0): T-Value = 2.32 P-Value = 0.027
```

Two-Sample T-Test and CI: Medium 1, Medium 2
Log (count)
Two-sample $T$ for Medium 1 vs Medium 2

|  |  | N | Mean | StDev |
| :--- | :--- | ---: | ---: | ---: |
| Medium | 1 | 8 | 3.429 | 0.730 |
| Medium 2 | 8 | 3.202 | 0.790 | 0.26 |
|  |  | 0.28 |  |  |

Difference = mu Medium 1 - mu Medium 2
Estimate for difference: 0.227
95\% lower bound for difference: -0.443
T-Test of difference $=0$ (vs >): T-Value $=0.60 \quad$ P-Value $=0.280 \quad \mathrm{DF}=14$
Both use Pooled StDev $=0.761$

A botanist hopes to demonstrate that the plants of the species Thlaspi alpestre found in Derbyshire are a different subspecies from the plants found in North Yorkshire. Both are virtually confined to lead mining spoil heaps. The claimed morphological difference is in the height of the plants, those in Derbyshire being taller. The heights (cm) of 8 flowering stems from each area were measured and were as follows:

| Plant | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | ---: | ---: | ---: |
| N. Yorks | 14.6 | 15.2 | 17.1 | 10.4 | 11.6 | 13.8 | 14.2 | 16.1 |
| Derbyshire | 17.3 | 12.4 | 23.6 | 18.0 | 19.6 | 16.2 | 17.3 | 13.4 |

Using Minitab, the data has been analysed to compare heights as both a paired and an unpaired design and the analyses are given below.
a) Explain whether these data should be analysed as a paired or unpaired experiment
b) What are the conditions required for
i. an unpaired t-test
ii. a paired t-test?
c) State, in words, the null and alternative hypotheses for the appropriate analysis.
d) What conclusion should be drawn from the analysis?

## Paired T-Test and CI: NYorks, Derbys

Paired $T$ for NYorks - Derbys

|  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | ---: |
| NYorks | 8 | 14.1250 | 2.2199 | 0.7848 |
| Derbys | 8 | 17.2250 | 3.5021 | 1.2382 |
| Difference | 8 | -3.10000 | 4.22442 | 1.49356 |
|  |  |  |  |  |
| 95\% CI for mean difference: | $(-6.63170,0.43170)$ |  |  |  |
| T-Test of mean difference $=0$ | $(\mathrm{Vs} \mathrm{not}=0): \mathrm{T}-$ Value $=-2.08 \quad \mathrm{P}-$ |  |  |  |
| Value $=0.077$ |  |  |  |  |

Two-Sample T-Test and CI: NYorks, Derbys

Two-sample $T$ for NYorks vs Derbys

|  | N | Mean | StDev | SE Mean |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| NYorks | 8 | 14.13 | 2.22 | 0.78 |  |
| Derbys | 8 | 17.23 | 3.50 |  | 1.2 |

```
Difference = mu (NYorks) - mu (Derbys)
```

Estimate for difference: -3.10000
95\% upper bound for difference: -0.51795
T -Test of difference $=0$ (vs <) : T-Value $=-2.11 \quad \mathrm{P}-\mathrm{Value}=0.026$
$\mathrm{DF}=14$
Both use Pooled StDev $=2.9320$

It is claimed that giving children high levels of omega 3 fish oils aids concentration. A sample of 10 children was tested to measure the length of time they could concentrate on doing a jigsaw puzzle. They were then given a fish oil supplement for two months and were tested again. Their concentration times were as follows:

| Time (min) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Child | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |
| Before | 10 | 5 | 6 | 11 | 8 | 2 | 1 | 8 | 5 | 3 |  |  |  |  |
| After | 13 | 4 | 8 | 12 | 10 | 3 | 2 | 9 | 5 | 3 |  |  |  |  |

MINITAB output is given below for an analysis of this experiment as a paired t-test and as an unpaired t-test.
a) Which is the correct analysis? Explain your reasoning.
b) For your chosen test state the null and alternative hypotheses and any necessary assumptions about the data.
c) State your conclusion in one sentence in a form suitable for a report.

```
Paired T-Test and Cl: before, after
Paired T for before - after
\begin{tabular}{lrrrr} 
& \(N\) & Mean & StDev & SE Mean \\
before & 10 & 5.90000 & 3.34830 & 1.05883 \\
after & 10 & 6.90000 & 4.01248 & 1.26886 \\
Difference & 10 & -1.00000 & 1.15470 & 0.36515
\end{tabular}
95% upper bound for mean difference: -0.33064
T-Test of mean difference = 0 (vs < 0): T-Value = -2.74 P-Value =
0.011
Two-Sample T-Test and Cl : before, after
Two-sample T for before vs after
\begin{tabular}{lrrrr} 
& N & Mean & StDev & SE Mean \\
before & 10 & 5.90 & 3.35 & 1.1 \\
after & 10 & 6.90 & 4.01 & 1.3
\end{tabular}
Difference = mu (before) - mu (after)
Estimate for difference: -1.000000
95% upper bound for difference: 1.865726
T-Test of difference = 0 (vs <): T-Value = -0.61 P-Value = 0.276
DF = 18
Both use Pooled StDev = 3.6953
```

An experiment was conducted to study the effect of tamoxifen on patients with cervical cancer. One of the measurements made, both before and after tamoxifen was given, was microvessel density (MVD). MVD, number of vessels per $\mathrm{mm}^{2}$ is a measurement that relates to the formation of blood vessels that feed a tumour and allow it to grow and spread. Thus, small values of MVD are better than large values.
Data for 18 patients are shown.

| Patient | MVD before | MVD after | Patient | MVD before | MVD after |
| ---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 98 | 75 | $\mathbf{1 0}$ | 70 | 60 |
| $\mathbf{2}$ | 100 | 60 | $\mathbf{1 1}$ | 60 | 65 |
| $\mathbf{3}$ | 82 | 25 | $\mathbf{1 2}$ | 88 | 45 |
| $\mathbf{4}$ | 100 | 55 | $\mathbf{1 3}$ | 45 | 36 |
| $\mathbf{5}$ | 93 | 78 | $\mathbf{1 4}$ | 159 | 144 |
| $\mathbf{6}$ | 119 | 102 | $\mathbf{1 5}$ | 65 | 27 |
| $\mathbf{7}$ | 70 | 58 | $\mathbf{1 6}$ | 98 | 90 |
| $\mathbf{8}$ | 78 | 70 | $\mathbf{1 7}$ | 66 | 16 |
| $\mathbf{9}$ | 104 | 90 | $\mathbf{1 8}$ | 67 | 53 |

MINITAB output is given in the table below for an analysis of this experiment as a paired t-test and as an unpaired t-test.

```
Paired T-Test and CI: MVD before, MVD after
Paired T for MVD before - MVD after
\begin{tabular}{lrrrr} 
& N & Mean & StDev & SE Mean \\
MVD before & 18 & 86.78 & 26.14 & 6.16 \\
MVD after & 18 & 63.83 & 30.83 & 7.27 \\
Difference & 18 & 22.94 & 17.71 & 4.17
\end{tabular}
95% lower bound for mean difference: 15.68
T-Test of mean difference = 0 (vs > 0): T-Value = 5.50 P-Value = 0.000
Two-Sample T-Test and CI: MVD before, MVD after
```

```
Two-sample T for MVD before vs MVD after
```

Two-sample T for MVD before vs MVD after

|  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | ---: |
| MVD before | 18 | 86.8 | 26.1 | 6.2 |
| MVD after | 18 | 63.8 | 30.8 | 7.3 |

Difference = mu (MVD before) - mu (MVD after)
Estimate for difference: 22.94
95% lower bound for difference: 6.84
T-Test of difference = 0 (vs >): T-Value = 2.41 P-Value = 0.011 DF = 34
Both use Pooled StDev = 28.5810

```
a) Which is the correct analysis? Explain your reasoning.
b) For your chosen test state the null and alternative hypotheses and any necessary assumptions about the data.
c) State your conclusion in one sentence in a form suitable for a report.

A UWE student measured the biomass (milligrams) produced by bacterium \(A\) and bacterium \(B\) in shake flasks containing glucose as substrate.
\begin{tabular}{lllllllll}
\hline Bacterium A & 410 & 447 & 448 & 449 & 454 & 467 & 482 & 507 \\
\hline Bacterium B & 511 & 480 & 466 & 460 & 459 & 455 & 435 & 416 \\
\hline
\end{tabular}
a) Explain which of the two tests in the Minitab output below is using the correct analysis approach.
b) State the hypotheses which are being tested in your chosen analysis.
c) What conclusion can be drawn from the data about her hypothesis?
d) State the assumptions needed for the test to be valid.

\section*{Paired T-Test and Cl : BacteriumA, BacteriumB}
\begin{tabular}{lrrrr} 
Paired T for BacteriumA - BacteriumB \\
& & & & \\
& M & Mean & StDev & SE Mean \\
BacteriumA & 8 & 458.0 & 28.5 & 10.1 \\
BacteriumB & 8 & 460.3 & 28.3 & 10.0 \\
Difference & 8 & -2.3 & 56.5 & 20.0
\end{tabular}
```

95% CI for mean difference: (-49.5, 45.0)
T-Test of mean difference = 0 (vs not = 0): T-Value = -0.11
P-Value = 0.913

```

Two-Sample T-Test and CI: BacteriumA, BacteriumB
Two-sample \(T\) for BacteriumA vs BacteriumB
\begin{tabular}{lrrrr} 
& & & & SE \\
& N & Mean & StDev & Mean \\
BacteriumA & 8 & 458.0 & 28.5 & 10 \\
BacteriumB & 8 & 460.3 & 28.3 & 10
\end{tabular}
```

Difference = mu (BacteriumA) - mu (BacteriumB)

```
Estimate for difference: -2.2
95\% CI for difference: (-32.7, 28.2)
T-Test of difference \(=0\) (vs not \(=\) ) : T-Value \(=-0.16\)
P-Value \(=0.876 \mathrm{DF}=14\)
Both use Pooled StDev \(=28.4147\)

An overdose of paracetamol can cause fatal hepatic necrosis. Rapid and accurate determination of plasma paracetamol is necessary for the administration of antidotal treatment.
To measure paracetamol concentrations, a standard High Performance Liquid Chromatograph (HPLC) method can be used in the laboratory. An alternative commercial method (PCP) has been proposed for use in Accident and Emergency departments. To measure the efficacy of PCP, samples of blood were taken from 10 suspected overdose patients. For those 10 samples the plasma concentrations ( \(\mathrm{mg} / \mathrm{l}\) ) were measured using HPLC and PCP. The measurements and two analyses of these data using MINITAB are shown below below.
\begin{tabular}{rc}
\multicolumn{2}{c}{ Conc. } \\
\begin{tabular}{c} 
HPLC \\
\hline 0
\end{tabular} & \begin{tabular}{c} 
PCP \\
\hline 0
\end{tabular} \\
48 & 53 \\
36 & 46 \\
3 & 7 \\
3 & 3 \\
5 & 6 \\
12 & 14 \\
53 & 50 \\
65 & 61 \\
98 & 107
\end{tabular}
```

Two-Sample T-Test and CI: HPLC, PCP
Two-sample T for HPLC vs PCP
N Mean StDev SE Mean
HPLC 10 32.3 33.3 11
PCP 10 34.9 34.5 11
Difference = mu (HPLC) - mu (PCP)
Estimate for difference: -2.6
95% CI for difference: (-35, 29)
T-Test of difference = 0 (vs not =): T-Value = -0.17 P-Value = 0.866 DF =
17
Paired T-Test and CI: HPLC, PCP
Paired T for HPLC - PCP

|  | N | Mean | StDev | SE Mean |
| :--- | ---: | ---: | ---: | ---: |
| HPLC | 10 | 32.3 | 33.3 | 10.5 |
| PCP | 10 | 34.9 | 34.5 | 10.9 |
| Difference | 10 | -2.60 | 4.58 | 1.45 |

95% CI for mean difference: (-5.87, 0.67)
T-Test of mean difference = 0 (vs not = 0): T-Value = -1.80 P-Value =
0.106

```
a) Explain which of the two tests above is the correct analysis giving your reasoning.
b) Using the appropriate output given above,
(i) State the hypotheses which are being tested in your chosen analysis.
(ii) Give a 95\% confidence interval for the difference between the mean paracetamol concentrations from the two methods and explain what it means. What statistical conclusion can be drawn?
(iii) Explain the implications of this in the context of the question.
(iv) State the assumptions needed for the test to be valid.

\section*{10}

In an investigation of possible brain damage due to alcoholism, an X-ray procedure known as a computerized tomography (CT) scan was used to measure brain densities of 11 chronic alcoholics with a view to determining whether alcoholism reduces brain density.

For each alcoholic, a non-alcoholic control was selected who matched the alcoholic on age, sex, education, and other factors. Their brain densities were as follows:
\begin{tabular}{cc} 
Alcoholics & Control \\
40.1 & 41.3 \\
38.5 & 40.2 \\
36.9 & 37.4 \\
41.4 & 46.1 \\
40.6 & 43.9 \\
42.3 & 41.9 \\
37.2 & 39.9 \\
38.6 & 40.4 \\
38.5 & 38.6 \\
38.4 & 38.1 \\
38.1 & 39.5
\end{tabular}

Two t-tests, paired and unpaired, for the above data using MINITAB are given below.

\section*{Two-Sample T-Test and Confidence Interval:}
```

Two-sample T for Alcoholics vs Control

```
\begin{tabular}{lllrc} 
& N & Mean & StDev & SE Mean \\
Alcoholics & 11 & 39.15 & 1.72 & 0.52 \\
Control & 11 & 40.66 & 2.56 & 0.77
\end{tabular}

Difference \(=\) mu (Alcoholics) - mu (Control)
95\% CI for difference: (-3.460, 0.424)
T-Test of difference \(=0\) (vs <) : T-Value \(=-1.63\) P-Value \(=0.059 \quad \mathrm{DF}=20\)
Both use Pooled StDev \(=2.1833\)
Paired T-Test and Confidence Interval:
Paired T for Alcoholics - Control
\begin{tabular}{lrrrr} 
& N & Mean & StDev & SE Mean \\
Alcoholics & 11 & 39.145 & 1.722 & 0.519 \\
Control & 11 & 40.664 & 2.563 & 0.773 \\
Difference & 11 & -1.518 & 1.580 & 0.476
\end{tabular}

95\% CI for mean difference: (-2.579, -0.457)
T-Test of mean difference \(=0\) (vs \(<0\) ) : T-Value \(=-3.19 \quad \mathrm{P}\)-Value \(=0.005\)
a) Which is the correct analysis? Explain your reasoning. Note that matched-controls are used here. Why?
b) For your chosen test state the null and alternative hypotheses and any necessary assumptions about the data.
c) State your conclusion in one sentence in a form suitable for a report.```

