THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Summer 2015 examination

MA301

Game Theory I
Half Unit

Suitable for all candidates

Instructions to candidates

This paper contains **5** questions. You may attempt as many questions as you wish, but only your **BEST 4** answers will count towards the final mark. All questions carry equal numbers of marks.

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Answers should be justified by showing work.

Time Allowed -	**Reading Time:**	*None*
	Writing Time:	*2 hours*
You are supplied with:		*Mathematics Answer Booklet*
Calculators:		*Calculators are not allowed in this examination*

1

Recall that in the game Nim, the two players start with some heaps of chips and alternately remove some chips from one of these heaps. Player I moves first. The player to remove the last chip wins. In the following questions, you may use the known way of determining a winning strategy in Nim.

(a) Consider a Nim game with four heaps of sizes 2, 4, 8, and 15. Determine all winning moves from this position, if any.

(b) Let n be a positive integer and consider a Nim game with $n+1$ heaps of chips of sizes 2^1, 2^2, ..., 2^n, and $2^{n+1}-1$. (So the game given in part (a) is the case $n=3$.) Determine all winning moves from this position, if any. Justify your answer.

(c) Consider a winning position in some impartial game where each possible move of the player is a winning move. Show that the Nim value of this position is 1.

(d) The game Take-2-or-3 is a game for two players that is played with heaps of chips as in Nim, but where the player can only remove *two* or *three* chips from one heap. Call $T(n)$ the game Take-2-or-3 that has a single heap with n chips. For $n = 0,1,2,\ldots,9$, list the options of $T(n)$, their Nim values, and the Nim value of $T(n)$ itself.

(e) With the game Take-2-or-3 described in part (d), what are the winning moves, if any, in a game with three heaps of sizes 4, 5 and 7, that is, in the game sum of $T(4)$, $T(5)$, and $T(7)$?

(f) Using the list of Nim values found in part (d), determine the Nim value of $T(10)$, of $T(73)$, and of $T(209)$. Justify your answer.
[*Hint: You should not compute $T(73)$ or $T(209)$, but observe a pattern and explain it.*]

2

Consider the following bimatrix game:

I \ II	a	b	c	d
T	2, 5	4, 4	1, 2	6, 0
B	5, 0	3, 2	3, 4	1, 5

(a) Find all pure and mixed Nash equilibria of the game, and their payoffs to the two players.

(b) Write down the game tree of the commitment game derived from the above game, in which player I moves first and then player II is informed about the move of player I and reacts separately with any of her choices.

(c) Find all subgame perfect equilibria of the game in part (b), and their payoffs to the two players.

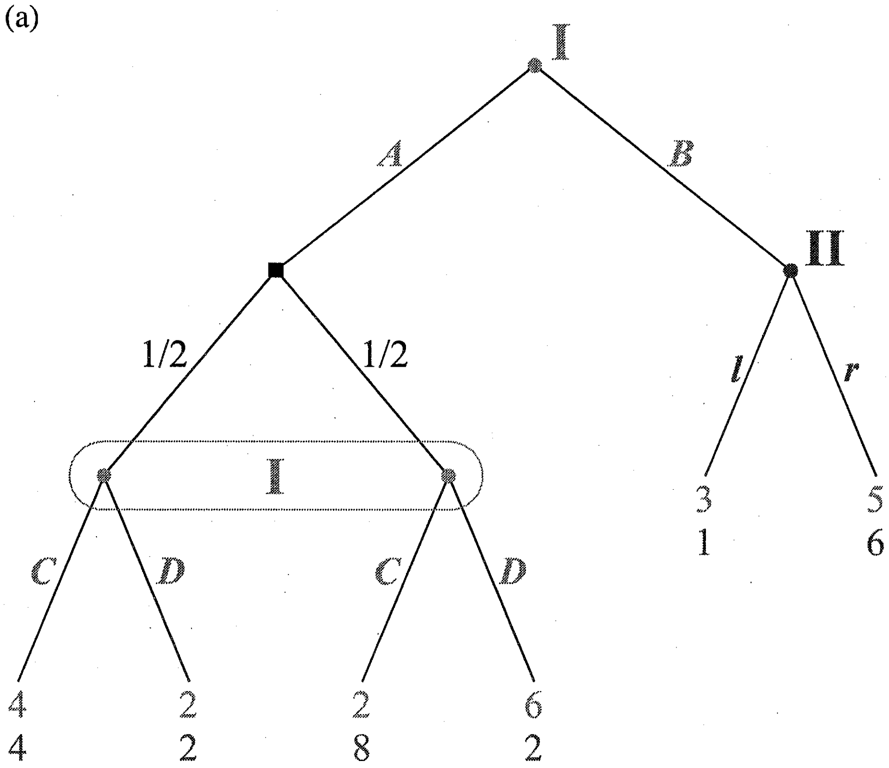
(d) Describe *all* pure Nash equilibria of the game in part (b) (that is, not just those that are subgame perfect). Explain how to find these Nash equilibria by reasoning directly with the game tree, where the Nash equilibrium property implies certain conditions for the moves of each player. How many pure Nash equilibria are there?

(e) Repeat parts (b), (c), (d) with the roles of the players exchanged, that is, for the game where player II commits by moving first and player I moves second (you do not have to repeat your general explanation from part (d), but you should describe all pure Nash equilibria and state how many there are).

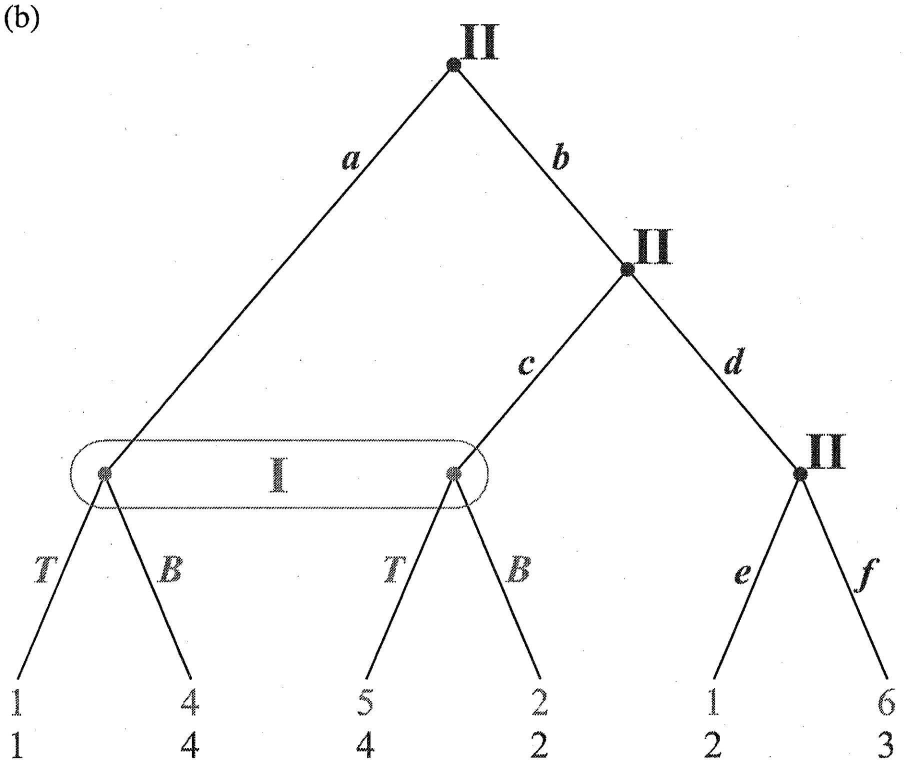
3

For each of the following extensive form games (a)–(d) (continued on the next page), decide if they have perfect recall or not. For those that do, find all their Nash equilibria in mixed or pure strategies, and state which of them are subgame perfect. Justify your answers.

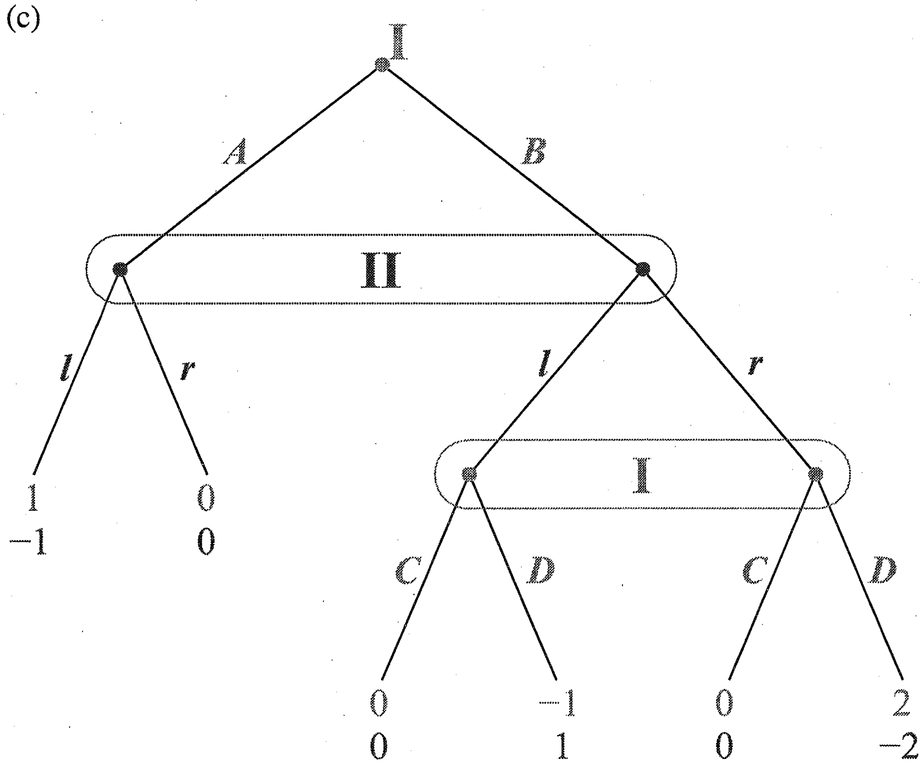
(a)



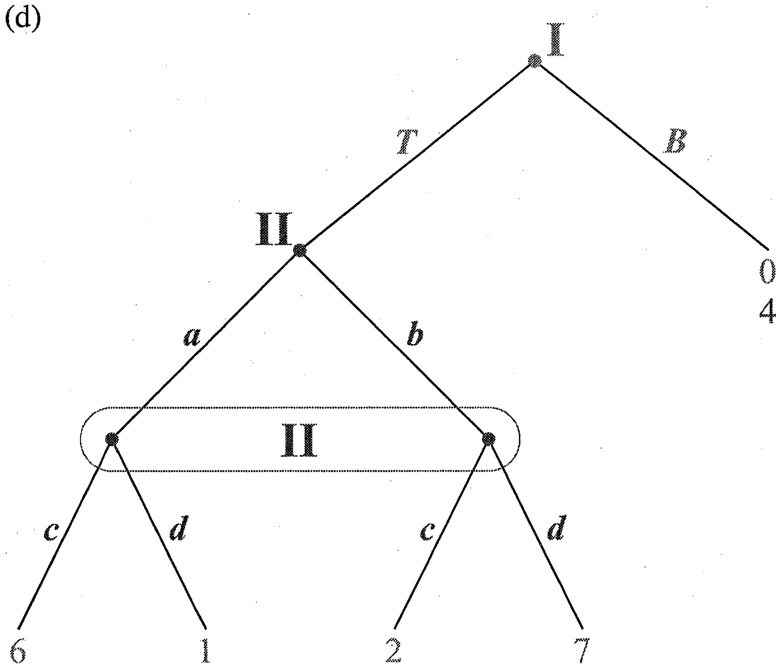
(b)



(c)



(d)



4

Consider the following 3×2 game:

I \ II	l	r
T	6, 0	0, 1
M	0, 3	3, 3
B	0, 0	2, 2

(a) Find all Nash equilibria of this game in pure or mixed strategies.

(b) For player I, find a max-min strategy $\hat{x}$, and the corresponding max-min value u_0. For player II, find a max-min strategy $\hat{y}$, and the corresponding max-min value v_0. In each case, explain why these are the max-min values.

(c) Draw the convex hull of the payoff pairs in the 3×2 game, and the *bargaining set* S resulting from the payoff pairs (u, v) with $u \geq u_0$ and $v \geq v_0$ with the threat point (u_0, v_0) found in (b). What is the Pareto-frontier of the set S?

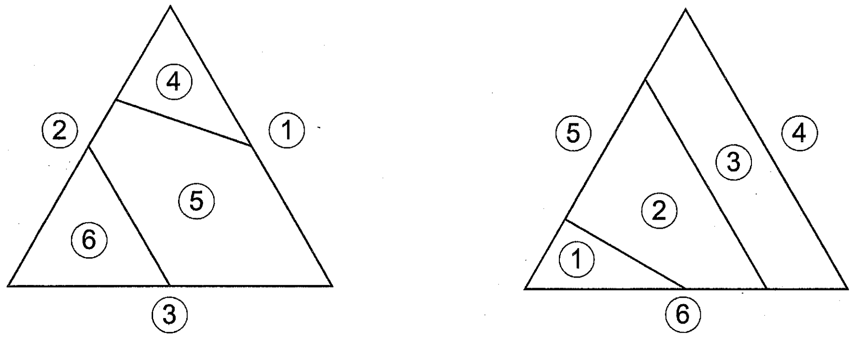
(d) Find the Nash *bargaining solution* for S. Explain your method.

(e) Show how the players can implement the bargaining solution for S with a joint lottery over strategy pairs in the 3×2 game.

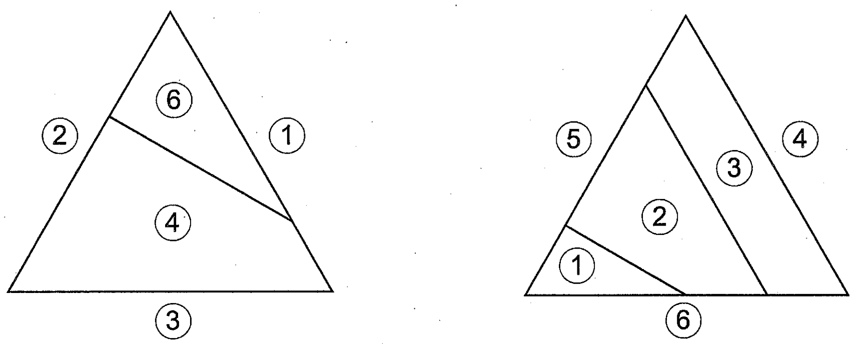
5

The following three pairs of diagrams in (a), (b), (c) show labeled mixed strategy sets X and Y of player I and II, respectively.

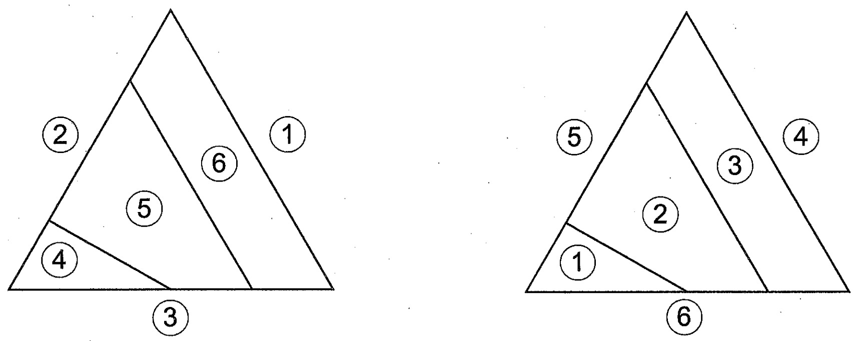
(a)



(b)



(c)



Explain in general what these diagrams mean, which triangle represents X and which represents Y, the role of the circled numbers in these diagrams, and how and precisely *why* they can be used to identify Nash equilibria.

Then, for each of (a), (b), (c), find these Nash equilibria, where you can refer to a point by the triple of labels that identify it, such as 345 for the top corner of the right triangle in (a).

(d) Consider the 3×3 *zero-sum game* with payoff matrix A to the maximizing row player given by

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 4 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

Exactly one of the above diagram pairs (a), (b), or (c) applies to this zero-sum game. Explain which one, and justify your answer. Find the exact mixed strategy pairs (x, y), as pairs of probability vectors, in the Nash equilibria of this game, and the respective payoffs to the players.