

AQA Core 1 2016 Unofficial Mark scheme (Dan O'F.)

Disclaimer: The marks for each question are accurate but as to where they are allocated exactly is only an estimate based on the responses to previous papers. FT/ECF (Using a wrong answer from previous question part that will still allow you to earn method marks) has not been included as their use is difficult to gauge. Please bear in mind that FT/ECF could allow you to gain additional marks. A grade range estimate is shown below, based on previous years and the difficulty of the paper:

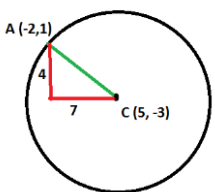
A: 60-64	D: 43-46
B: 55-58	E: 37-40
C: 50-52	

Q1	Question	Solution	Mark	Total
(a)	Find 'm' from $5x + 3y + 3 = 0$	$3y = -5x - 3$ $\therefore y = -\frac{5}{3}x - 1$ $\therefore m = -\frac{5}{3}$	M1 A1	2
(b)	AB & another line intersect at B, Find co-ordinates of B.	$5x + 3y + 3 = 0$ & other equation identified as simultaneous equation. $X = -3$ Or $y = 4$ $B = (-3, 4)$	M1 A1 A1	3
(c)	Co-ordinates involving K [e.g. $(2k-3, k+4)$ or whatever it was] on the line, Find K.	When $x = (\text{e.g.}) (2k-3)$, $y = (\text{e.g.}) (k+4)$ $\Rightarrow 5(2k-3) + 3(k+4) + 3 = 0$ $\Rightarrow K = -30$	M1 A1	2
		Total		7

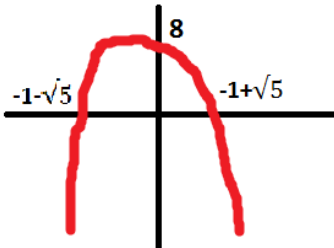
Q2	Question	Solution	Mark	Total	Comments
(a)	Simplify $(3\sqrt{5})^2$	$\therefore = (3 \times 3) \times (\sqrt{5} \times \sqrt{5})$ $\therefore = 9 \times 5$ $\therefore = 45$	B1	1	
(b)	Simplify $\frac{(3\sqrt{5})^2 + \sqrt{5}}{7 + 3\sqrt{5}}$ with your answer in the form $m + n\sqrt{5}$	$\frac{(3\sqrt{5})^2 + \sqrt{5}}{7 + 3\sqrt{5}} \times \frac{7 - 3\sqrt{5}}{7 - 3\sqrt{5}}$ Numerator = $300 - 128\sqrt{5}$ Denominator = 4 $\therefore = 75 - 32\sqrt{5}$	M1 M1 A1	4	45 correctly 'subbed in' from part a)
		Total		5	

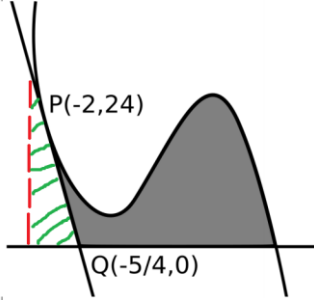
Q3	Question	Solution	Mark	Total	Comments
(a)(i)	Write $x^2 - 7x + 2$ in the form $(x-p)^2 + q$	$(x - \frac{7}{2})^2$ $(x - \frac{7}{2})^2 - \frac{41}{4}$	M1 A1	2	Simplest form but decimals accepted, e.g -3.5 & 20.25
(ii)	Write down the minimum value of the curve.	$-\frac{41}{4}$	B1	1	
(b)	Describe the geometrical transformation that maps the graph of $y = x^2 - 7x + 2$ onto $y = (x-4)^2$	Translation By vector $\begin{pmatrix} \frac{1}{2} \\ \frac{41}{4} \end{pmatrix}$	B1 A1 A1	3	'Translation' = 1 mark Must have fully correct vector for next 2 marks. Word 'vector' can be stated / implied.
		Total		6	

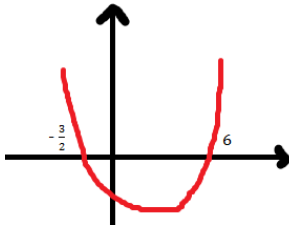
Q4	Question	Solution	Mark	Total	Comments
(a)(i)	Show that $(x+3)$ is a factor of $x^3 - 5x^2 - 8x + 48$	$P(-3) = (-3)^3 - 5(-3)^2 - 8(-3) + 48$	M1	2	P (-3) attempted NOT long division.
		$\therefore = -27 - 45 + 24 + 48 = 0$ $\Rightarrow (x+3)$ is a factor	A1		Must have statement for A1.
			M1	3	Division used to obtain 1 linear and 1 quadratic factor of $p(x)$.
		$\therefore = (x+3)(x^2 - 8x + 16)$	M1		Factorise quadratic for 2 nd M1.
		$\therefore (x^2 - 8x + 16) = (x-4)(x-4)$			
		$\therefore p(x) = (x+3)(x-4)(x-4)$	A1		3 linear factors for A1
(b)(i)	Use the remainder theorem to find the remainder when $p(x)$ is divided by $(x-2)$	$P(2) = 2^3 - 5(2)^2 - 8(2) + 48$	M1	2	'Sub. in' 2 to polynomial.
		$\therefore \text{remainder} = 20$	A1		
				3	Division used to obtain 1 linear and 1 quadratic factor of $p(x)$.
					(r could also be obtained from previous question)
(ii)	Express $x^3 - 5x^2 - 8x + 48$ in the form $(x-2)$	$P(x) = (x-2)(x^2 + bx + c)$ where $b = -3$ And $c = -14$	M1	3	
		$r = 48 - 28 = 20$	M1		
		$P(x) = (x-2)(x^2 - 3x - 14) + 20$	A1		P(x) expressed in correct form.
		Total		10	

Q5	Question	Solution	Mark	Total	Comments
(a)	Centre C (5, -3) given and A (-2, 1). Write the equation of the circle in form $(x-a)^2 + (y-b)^2 = k$	Use co-ordinates of C to obtain $(x-5)^2 + (y+3)^2 = k$	M1	3	Use knowledge that C = (-a, -b) from $(x-a)^2 + (y-b)^2 = r^2$
		Use Pythagoras' theorem to obtain k, as demonstrated below. ($4^2 + 7^2 = r^2 = k$)	M1		Do NOT allow r, or, $k = \sqrt{65}$
		 Obtain $(x-a)^2 + (y-b)^2 = 65$	A1		Fully correct equation in completed square form.
(b)	Points A & B form the diameter. Find the co-ordinates of B.	For B, $x = 5 + 7$ OR $y = (-3) - 4$	M1	2	Use the distance from C to A to find the distance from C to B.
		$\therefore B = (12, -7)$	A1		(Probably!) Doesn't have to be in co-ordinate brackets.
(c)	Calculate the equation of the tangent at A. (In form $px + qy + n = 0$)	Grad of radius (normal) $= \frac{1+3}{-2-5}$ $\therefore = -\frac{4}{7}$	M1 M1	5	Allow $y = mx + c$ method. Must be in this form for A1.
		\therefore gradient of tangent $= \frac{-1}{\frac{-4}{7}}$			
		$= \frac{7}{4}$	M1		
		$\therefore y - (1) = \frac{7}{4}(x + 2)$ $\therefore 7x - 4y + 18 = 0$	M1 A1		

(d)	Given that T is on tangent at A & length of AT = 4, find Length of CT.	Identify that $CT^2 = CA^2 + AT^2$ $\therefore CT^2 = 65 + 4^2$ $\therefore CT^2 = 81$ $\therefore CT = \sqrt{81}$ $= 9$	M1 M1 A1	3	Use of Pythagoras. Must simplify $\sqrt{81}$ to 9 for A1.
		Total		13	

Q6	Question	Solution	Mark	Total	Comments
(a)(i)	Curve $y = 8 - 4x - 2x^2$ is given. Find the x-intercepts of the curve in the form $m \pm \sqrt{n}$	When $y = 0$ $\Rightarrow 8 - 4x - 2x^2 = 0$ $\Rightarrow a = -2, b = -4, c = 8$ $\Rightarrow x = -1 \pm \sqrt{5}$	M1 A1	2	Quadratic formula /similar used (not factorising) Must be in this form for A1.
(ii)	Sketch the graph of $y = 8 - 4x - 2x^2$, showing the y-intercept.		B1 B1	2	-ve (n) shape, max point NOT on y-axis for B1 y intercept & roots stated for 2 nd B1.
(b)(i)	$y = k(x+4)$ meets the curve at a tangent. Hence show that: $2x^2 + (k+4)x + 4(k-2) = 0$	$\therefore k(x+4) = 8 - 4x - 2x^2$ $\therefore kx + 4k = 8 - 4x - 2x^2$ $\therefore 8 - 4x - kx - 4k - 2x^2 = 0$ $\therefore 2x^2 + (k+4)x + 4(k-2) = 0$	A1	1	Must be in this form.
(ii)	Hence find the 2 possible values of k.	Tangent $\Rightarrow b^2 - 4ac = 0$ $\therefore (k+4)^2 - [4 \times 2(4k-8)] = 0$ $\therefore k^2 - 24k + 80 = 0$ $\therefore k = -20, \text{ or, } k = -4$	M1 M1 A1	3	Correct substitution of a, b, c values into discriminant.
		Total		8	

Q7	Question	Solution	Mark	Total	Comments
(a)(i)	The curve $y = 4 - x^2 - 3x^3$ is given. Find the equation of the tangent at P (-2,24) giving your answer in the form $y = mx + c$	$\frac{dy}{dx} = -2x - 9x^2$	M1	5	1 term correct
		A1			All correct
		\therefore when $x = -2$, $\frac{dy}{dx} = -2(-2) - 9(-2)^2$	M1		Sub in $x = -2$
		$\therefore y - 24 = -32(x + 2)$	M1		Sub $\frac{dy}{dx}$ and x & y of P into equation.
(ii)	Find the x co-ordinate of Q.	$\therefore y = -32x - 40$	A1	1	Rearrange to get $y = mx + c$
		When $y = 0$ $\Rightarrow -32x - 40 = 0$ $\Rightarrow x = -\frac{5}{4}$	B1		Accept -1.25 and $-1\frac{1}{4}$
(b)(i)	Calculate $\int_{-2}^1 4 - x^2 - 3x^3 \, dx$	$\left[\frac{-3x^4}{4} - \frac{x^3}{3} + 4x \right]_{-2}^1$	M1	5	2 terms correct
		A1			all correct
		$\therefore = \left[4 - \frac{1}{3} - \frac{3}{4} \right] - \left[-8 + \frac{8}{3} - 12 \right]$	M1		Attempt to correctly use limits
		A1			Correct, unsimplified
(ii)	Hence determine the area of the shaded region bounded by the curve between Q & R (1, 0).	$= \frac{81}{4}$	A1	3	Accept 20.25 and $20\frac{1}{4}$
		Area of triangle (see below) = $\frac{24 + \frac{3}{4}}{2} = 9$ (units ²)	B1		
		\therefore shaded area = $\left(\frac{81}{4} - 9 \right)$	M1		
		$= \frac{45}{4}$	A1		OE ; accept 11.25 and $11\frac{1}{4}$
					
		Total		14	

Q8	Question	Solution	Mark	Total	Comments
(a)(i)	The gradient of a curve is given by $\frac{dy}{dx} = 54 + 27x - 6x^2$ Calculate d^2y/dx^2	$d^2y/dx^2 = 27 - 12x$	M1 A1	2	1 term correct all correct
(ii)	The curve passes through the point P $(-1\frac{1}{2}, 4)$. Verify that there is a minimum point at P.	When $x = -1\frac{1}{2}$ $\Rightarrow \frac{dy}{dx} = 54 + 27(-1\frac{1}{2}) - 6(-1\frac{1}{2})^2$ $= 0$ \Rightarrow stationary point	M1 M1	4	Must prove $\frac{dy}{dx} = 0$ for M1 Must have statement.
		When $x = -1\frac{1}{2}$ $\Rightarrow d^2y/dx^2 = 27 - 12(-1\frac{1}{2})$ $= 45 > 0$ \Rightarrow minimum point (at $x = -1\frac{1}{2}$)	M1 A1		Prove $d^2y/dx^2 > 0$ Must have statement for A1
(b)(i)	Hence show that, when y is decreasing, $2x^2 - 9x - 18 > 0$	When y is decreasing $\Rightarrow \frac{dy}{dx} < 0$ $\Rightarrow 54 + 27x - 6x^2 < 0$ $\Rightarrow 6x^2 - 27x - 54 > 0$ $\Rightarrow 2x^2 - 9x - 18 > 0$ as req'd	M1 A1	2	Attempt to show that $\frac{dy}{dx} < 0$ Clear that > 0 . 'Be convinced'
(ii)	Hence solve $2x^2 - 9x - 18 > 0$	$\therefore (k-6)(2k+3) > 0$ \therefore Critical values are $k = 6$ and $-\frac{3}{2}$	M1 A1		Correct factors / use of quadratic formula.
			M1		Use of sign diagram OR graph
		$\therefore k > 6, \quad k < -\frac{3}{2}$	A1		Fraction must be simplified for A1.
		Total		12	