Paper Reference(s)

6678/01

GCE Mathematics

Mechanics M2

Practice Paper 1

Advanced/Advanced Subsidiary

Wednesday 25th May 2016

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Lilac or Green)

Items included with question papers
Nil

Calculators can be used in this examination.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body, your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678/01), your surname, other name and signature.
Check that you have the correct question paper.

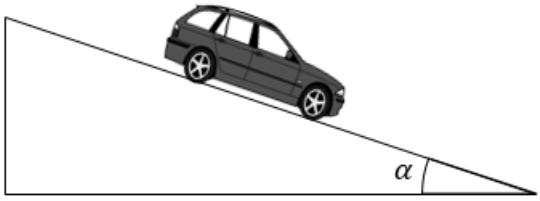
Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

1. An RC car of mass m travels down a rough hill, inclined at an angle α to the horizontal, at a constant velocity. The RC car's engine works at a constant rate of $mg\sin\alpha$ and the car experiences a constant resistive force of magnitude mgR newtons.

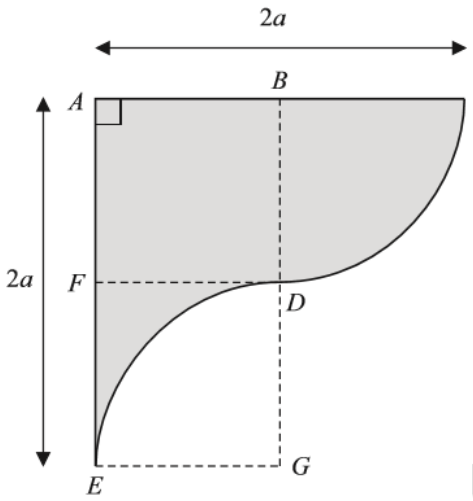


Show that speed of the car is given by $\frac{\sin\alpha}{R-\sin\alpha}$.

(4)

Q1 (4 marks)

2. A rudder of a ship can be constructed by removing a quadrant of a circle of radius a from a rectangle and then attaching it to a side of the rectangle. A diagram of the rudder is shown below.



(a) Show the centre of mass of a quadrant of a circle of radius a is $\frac{4a}{3\pi}$ from either of its straight edges.

(2)

(b) Find the centre of mass of the lamina from AE.

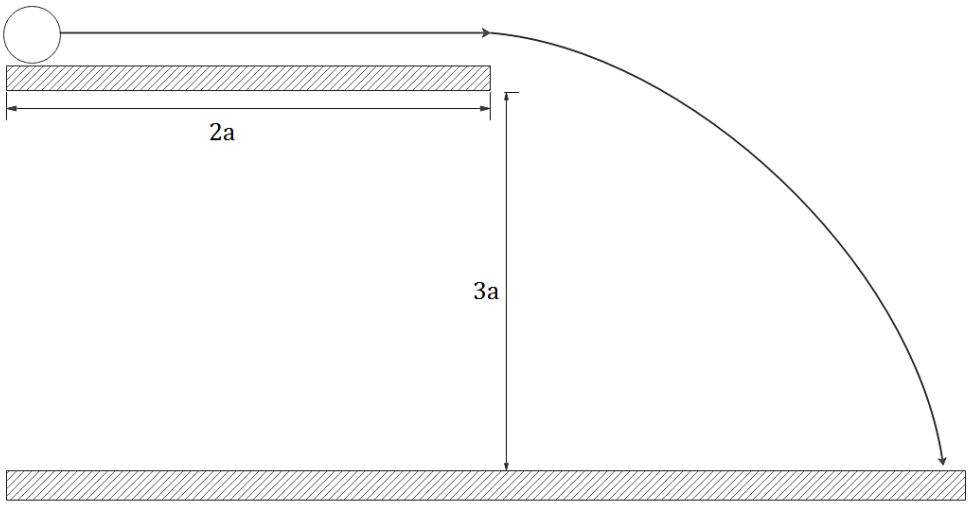
(5)

(c) The centre of mass the lamina is $\frac{a}{2}$ from AB. The lamina is now suspended from A. Find the angle between AB and the downwards vertical.

(3)

Q2 (10 marks)

3. A particle travels across a rough horizontal bench with an initial speed of $2\sqrt{ag}$. After travelling a distance of $2a$, the particle reaches the edge of the bench and has a speed of $\sqrt{2ag}$.



(*a*) Find the value of the coefficient of friction between the particle and the bench.

(3)

(*b*) Given that the bench is at a height $3a$ above the ground, find the angle the particle makes with the horizontal as it collides with the ground.

(4)

Q3 (7 marks)

4. A particle P of mass 0.5kg travels along the positive x-axis and experiences a force at time t seconds of $(6t - t^2)$ newtons. When $t = 0$, P is at O and has a speed of 4ms^{-1}.

(*a*) Find the greatest speed attained by P in the first 10 seconds.

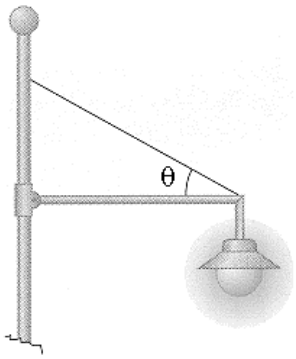
(3)

(*b*) Find the distance of P from the origin when it once again has a velocity of 4ms^{-1}

(5)

Question 4 (8 marks)

5. A street lamp uses a cable to support a uniform horizontal beam that is freely hinged to the lamp post. A lamp of mass m is attached to the end of the beam and the beam is of mass $2m$.



(*a*) Show that the tension in the cable is

$$T = \frac{2mg}{\sin\theta}$$

(3)

(*b*) The reaction force at the end of the horizontal beam makes an angle α with the horizontal. Show that

$$\tan\theta = 2\tan\alpha$$

(6)

Question 5 (9 marks)

6. Three identical spheres P, Q and R of mass m, $2m$ and $3m$ respectively are at rest in a straight line on a smooth horizontal surface. P is projected towards Q with speed $2u$. The coefficient of restitution between all the spheres is $2e$.

(a) Given that the direction of P is reversed by the collision, show that

$$\frac{1}{2} \geq e > \frac{1}{4}$$

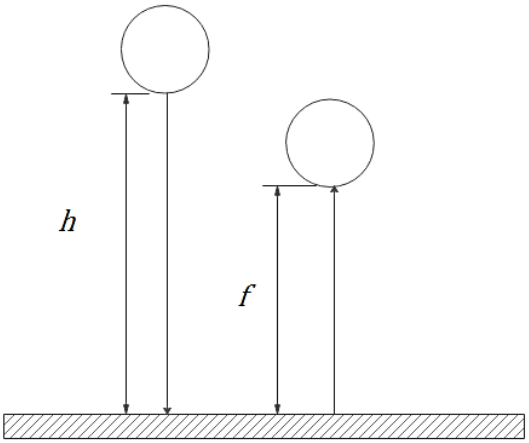
(4)

(*b*) Q then collides directly with R. After the collision, R travels with speed $\frac{49}{60}u$. Find the value of e.

(9)

Question 6 (13 marks)

7. A ball is dropped from a height h metres above a horizontal surface and the ball then bounces back up and reaches a height f metres. The only force acting on the ball is its weight. Given that the coefficient of restitution between the ball and the surface is e and that $e < 1$,



(*a*) Show that $f = e^2h$

(3)

(*b*) Show that the ball can travel no further than $\left(\frac{2h}{1-e^2} - h\right)$ metres before it comes to rest.

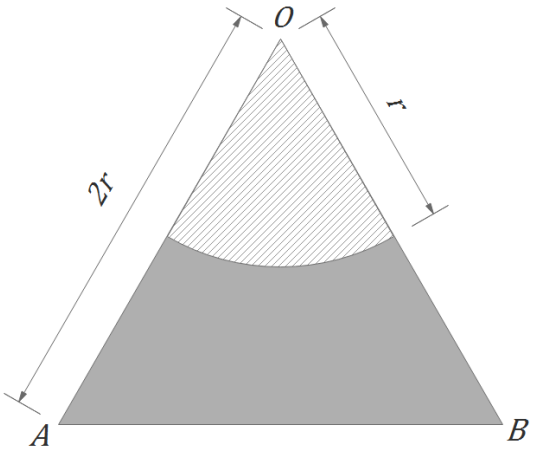
(3)

(*c*) Upon colliding with the surface for a fourth time, the speed of the ball is $\frac{16\sqrt{2gh}}{81}$. Find the value of e.

(4)

Question 7 (10 marks)

8. A uniform lamina is made by removing a sector of radius r from an equilateral triangle OAB with side lengths $2r$.

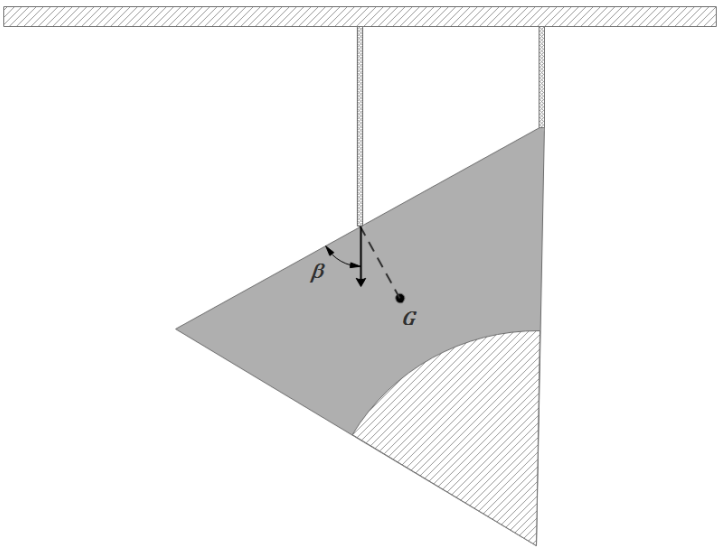


(*a*) Show that the distance of the centre of mass of the lamina is $\frac{10}{6\sqrt{3}-\pi}r$ from O.

(5)

(*b*) Hence show that the distance of the centre of mass of the lamina is $\frac{8-\pi\sqrt{3}}{6\sqrt{3}-\pi}r$ from AB.

(2)



(*c*) The lamina is suspended from two light, inextensible strings. β is the angle between AB and the downwards vertical. Given that the tension in one of the strings is twice the tension in the other string and that both strings are vertical, find the two possible values of β.

(7)

Question 8 (14 marks)

TOTAL FOR PAPER: 75 MARKS

END OF QUESTIONS