## Time: 1 hour 30 minutes

Materials required for examination<br>Mathematical Formulae (Pink)

Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 58 | 51 | 44 | 37 | 29 | 22 |

1. A particle $P$ is moving with constant velocity $(-3 \mathbf{i}+2 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. At time $t=6 \mathrm{~s}, P$ is at the point with position vector $(-4 \mathbf{i}-7 \mathbf{j}) \mathrm{m}$. Find the distance of $P$ from the origin at time $t=2 \mathrm{~s}$.
2. Two particles $A$ and $B$ are moving on a smooth horizontal plane. The mass of $A$ is $k m$, where $2<k<3$, and the mass of $B$ is $m$. The particles are moving along the same straight line, but in opposite directions, and they collide directly. Immediately before they collide the speed of $A$ is $2 u$ and the speed of $B$ is $4 u$. As a result of the collision the speed of $A$ is halved and its direction of motion is reversed.
(a) Find, in terms of $k$ and $u$, the speed of $B$ immediately after the collision.
(b) State whether the direction of motion of $B$ changes as a result of the collision, explaining your answer.

Given that $k=\frac{7}{3}$,
(c) find, in terms of $m$ and $u$, the magnitude of the impulse that $A$ exerts on $B$ in the collision.
3.


Figure 1
A non-uniform rod $A B$, of mass $m$ and length $5 d$, rests horizontally in equilibrium on two supports at $C$ and $D$, where $A C=D B=d$, as shown in Figure 1 . The centre of mass of the rod is at the point $G$. A particle of mass $\frac{5}{2} m$ is placed on the rod at $B$ and the rod is on the point of tipping about $D$.
(a) Show that $G D=\frac{5}{2} d$.

The particle is moved from $B$ to the mid-point of the rod and the rod remains in equilibrium.
(b) Find the magnitude of the normal reaction between the support at $D$ and the rod.
4. A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is $22 \mathrm{~m} \mathrm{~s}^{-1}$. The car maintains this speed for $T$ seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.
(a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights.
(b) Find the value of $T$.

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, $a \mathrm{~m} \mathrm{~s}^{-2}$, and passes the car at the point $A$ which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $22 \mathrm{~m} \mathrm{~s}^{-1}$.
(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point $A$.
(d) Find the value of $a$.
5. A beam $A B$ has length 15 m . The beam rests horizontally in equilibrium on two smooth supports at the points $P$ and $Q$, where $A P=2 \mathrm{~m}$ and $Q B=3 \mathrm{~m}$. When a child of mass 50 kg stands on the beam at $A$, the beam remains in equilibrium and is on the point of tilting about $P$. When the same child of mass 50 kg stands on the beam at $B$, the beam remains in equilibrium and is on the point of tilting about $Q$. The child is modelled as a particle and the beam is modelled as a non-uniform rod.
(a) (i) Find the mass of the beam.
(ii) Find the distance of the centre of mass of the beam from $A$.

When the child stands at the point $X$ on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,
(b) find $A X$.
6.


Figure 2
A beam $A B$ is supported by two vertical ropes, which are attached to the beam at points $P$ and $Q$, where $A P=0.3 \mathrm{~m}$ and $B Q=0.3 \mathrm{~m}$. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg . The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between $P$ and $Q$. The gymnast is modelled as a particle attached to the beam at the point $X$, where $P X=x \mathrm{~m}, 0<x<1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.
(a) Show that the tension in the rope attached to the beam at $P$ is $(588-350 x) \mathrm{N}$.
(b) Find, in terms of $x$, the tension in the rope attached to the beam at $Q$.
(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

Given that the tension in the rope attached at $Q$ is three times the tension in the rope attached at $P$,
(d) find the value of $x$.


Two particles $A$ and $B$, of mass $m$ and $2 m$ respectively, are attached to the ends of a light inextensible string. The particle $A$ lies on a rough horizontal table. The string passes over a small smooth pulley $P$ fixed on the edge of the table. The particle $B$ hangs freely below the pulley, as shown in Figure 3. The coefficient of friction between $A$ and the table is $\mu$. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of $A$ and $B$ is $\frac{4}{9} g$. By writing down separate equations of motion for $A$ and $B$,
(a) find the tension in the string immediately after the particles begin to move,
(b) show that $\mu=\frac{2}{3}$.

When $B$ has fallen a distance $h$, it hits the ground and does not rebound. Particle $A$ is then a distance $\frac{1}{3} h$ from $P$.
(c) Find the speed of $A$ as it reaches $P$.
(d) State how you have used the information that the string is light.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| Q1 | $\begin{aligned} (-4 \mathbf{i}-7 \mathbf{j}) & =\mathbf{r}+4(-3 \mathbf{i}+2 \mathbf{j}) \\ \mathbf{r} & =(8 \mathbf{i}-15 \mathbf{j}) \\ \|\mathbf{r}\| & =\sqrt{8^{2}+(-15)^{2}}=17 \mathrm{~m} \end{aligned}$ | M1 A1 <br> A1 <br> M1 A1 ft <br> [5] |
| 2(a) <br> (b) <br> (c) | $\begin{array}{rlr} 2 u \rightarrow & \leftarrow 4 u & k m 2 u-4 m u=-k m u+m v \\ k m & m & u(3 k-4)=v \\ u \leftarrow & \rightarrow v & \end{array}$ <br> $k>2 \Rightarrow v>0 \Rightarrow \operatorname{dir}^{n}$ of motion reversed <br> For B, $\begin{aligned} & m(u(3 k-4)--4 u) \\ & =7 m u \end{aligned}$ | M1 A1 <br> A1 <br> (3) <br> M1A1A1 <br> cso <br> (3) <br> M1 A1 f.t. <br> A1 (3) <br> [9] |


4.
(a)


| Shape | B1 |
| ---: | ---: |
| Figures | B1 |

(2)
(b) $\quad \frac{(120+T) 22}{2}=2145$

Figures B1
5)

9 marks)

6.
(a)

$\mathrm{M}(P), \quad 50 g \times 2=M g \times(x-2)$
$\mathrm{M}(Q), \quad 50 g \times 3=M g \times(12-x)$
M1 A1

DM1 A1
(i) $\quad M=25(\mathrm{~kg})$

DM1 A1
(8)
(b)

$(\uparrow) R+R=25 g+50 g$
$\mathrm{M}(A), \quad 2 R+12 R=25 g \times 6+50 g \times A X$
M1 A1 ft
M1 A1 ft
$A X=7.5$ (m)


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 8(a) | $\begin{aligned} B: \quad 2 m g-T & =2 m \times 4 g / 9 \\ \Rightarrow T & =\underline{10 m g / 9} \end{aligned}$ | M1 A1 <br> A1 <br> (3) |
| (b) | $A: \quad T-\mu \underline{m g}=m \times 4 g / 9$ <br> Sub for $T$ and solve: $\quad \mu=2 / 3^{*}$ | M1 B1 A1 <br> DM1 A1 (5) |
|  | When $B$ hits: $\quad v^{2}=2 \times 4 g / 9 \times h$ <br> Deceleration of $A$ after $B$ hits: $m a=\mu m g \Rightarrow a=2 g / 3$ $\begin{gather*} \text { Speed of } A \text { at } P: \quad V^{2}=8 g h / 9-2 \times 2 g / 3 \times h / 3 \\ \Rightarrow V=\frac{2}{3} \sqrt{ }(g h) \tag{6} \end{gather*}$ | M1 A1 <br> M1 A1 f.t. <br> DM1 <br> A1 |
| (d) | Same tension on $A$ and $B$ | B1 <br> (1) <br> 15 |

## Examiner reports

## Question 1

This proved to be a tricky opening question for many of the candidates. The most popular approach was to find the starting position and then use it to find the position vector at $t=2$. Errors in sign were fairly common at some stage of the working. A significant minority did not use a valid method at all, some just multiplying the given velocity vector by 2 or using a time of 6 only, and others becoming confused with constant acceleration formulae. A number of candidates failed to find the magnitude of their position vector to obtain the distance as required; there were follow through marks available for this even if the vector had been determined incorrectly. A few found the distance from the starting point rather than from the origin. Nevertheless, there were a fair number of entirely correct solutions.

## Question 2

Almost all candidates attempted to use a conservation of momentum equation in part (a) but there were many who either did not draw a diagram at all or else drew one which did not show the directions of motion of each particle after the collision. This lead to problems in all three parts of the question. Few realised the significance of the question asking for the speed of $B$, and gave a negative answer $u(4-3 k)$. There were also sign errors in the momentum equation and general problems dealing with the algebra. The second part required the significance of the range of values of $k$ to be explicitly referred to in the identification of direction and there were a number of fully correct and often well-expressed solutions. However, many did not mention $k$ at all and scored little. In part (c), many knew the relevant impulse-momentum equation and attempted to apply it to one of the particles but there was often confusion over direction and substitution of values and some gave a negative answer, losing the final mark.

## Question 3

Part (a) was answered well by the majority, with most taking moments about $D$. Consistent omission of $g$ 's was allowed since they cancelled out. A few candidates failed to mention $G D$ at all, using an unknown $x$ as the length required, and these were penalised. A few got themselves in a mess by failing to realise that the reaction at $C$ was zero, and although it was still possible to solve the problem, few were able to write down two correct equations and then eliminate the reaction at $C$ successfully to obtain $G D$. The second part was attempted by almost all candidates with the most common error being the omission of $g$ from one or more terms of their moment's equation. Lengths were generally correct for most of those who attempted this question and it was pleasing to see that nearly all the candidates realised that the rod was non-uniform.

## Question 4

In part (a) very few candidates failed to score the first B1, but the second was lost in one of two ways either by omitting a figure (usually the 120), or by labelling the $T+30$ term as $T$. In the second part almost all candidates tried equating the area under the graph to 2145 ; those who used the whole trapezium were almost invariably successful, but the candidates who split it into two triangles and a rectangle often made errors such as writing the last time interval as $(120-T)$ or simplifying $(120-(30+T))$ to obtain $(90+T)$. Candidates were able to score full marks in parts (c) and (d) even if part (b) was wrong. The most common error in part (c) was assuming that when the motorcycle passed the car they had not only covered equal
distances of 990 m but were also both travelling at $22 \mathrm{~m} \mathrm{~s}^{-1}$. Many subtracted the distance travelled in the first part of the motion, 330 m , from 990 m and divided by 22 to obtain the 30 s part of the car's time, but failed to carry out one or both of the remaining steps (adding the other 30 s and subtracting 10 s ). In the final part many scored a method mark for using a wrong time from part (c) correctly, but many scored no marks by persisting with $v=22$ for the motorcycle.

## Question 5

Many candidates struggled to produce a clear strategy for solving part (a) of this problem. Clear separate diagrams of the two situations (child at one end of the beam and then at the other) would have helped. Those who failed to recognise that the implication of 'on the point of tilting' is that one of the reactions is zero, could make no significant progress and this led to considerable wasted effort in trying to solve a variety of equations in too many unknowns. The most direct method of solution was, for each case, to take moments about the pivot with the non-zero reaction, leading to simultaneous equations in the required distance and mass. Often the unknown distance(s) were not made clear and sometimes the same letter was used to represent the distance to the centre of mass from whatever point a moment was taken about, for example, $50 g \times 2=m g x$ followed by $50 g \times 3=m g x$. If these two equations were added together and the fact that the sum of the two distances was 10 was used, the answer for $m$ fell out. Another valid approach was to find the reaction (same in both situations) by vertical resolution and then use this in appropriate moments equations. However, it was not always obvious which points were being used, where the child was standing, or what unknown distances represented; lack of clarity made some work difficult to decipher with candidates writing down too many equations with a variety of unknowns (and often much crossing out). A number of fully correct solutions were seen, although some candidates penalised themselves by giving weight as their answer rather than mass, and/or giving the distance to the centre of mass from the wrong point. Although some candidates gave up before tackling the second part, this was generally attempted with a much greater degree of success. Those who carried forward incorrect answers from part (a) could achieve 5 out of the 6 available marks and many did so. Most candidates drew a diagram for the new situation and then followed the standard approach of resolving vertically to find the two equal reactions and then taking moments about a point (generally $A$ or $P$ ). Those who had no values to carry forward from the first part could still achieve the method marks here.

## Question 6

Most candidates chose to take moments about $Q$ in part (a). Common errors were incorrect distances, missing g's or lack of a distance in the moment of $T_{P}$. Some used the answer for part (a) and resolved vertically to obtain the answer for part (b).

The third part proved to be difficult for many candidates and answers with inequalities in $x$ were offered. Some candidates used $x=0.1$ or 1.39 to calculate the boundaries. A few managed to get the correct boundaries but did not express their answers correctly.

In the final part a significant minority lost marks by using $T_{P}=3 T_{q}$ to obtain their answer.

## Question 7

Most candidates attempted parts (a) and (b) using simultaneous equations, with the most common mistake being to cancel out either $m$ or $g$ when it was not a factor in every term. This resulted in the $m$ term of $T$ being missing. A relatively large number of candidates also lost the final A1 mark for part (b) as they worked through the question using decimals.

The first section of part (c) for calculating the velocity of $A$ after $B$ hits the ground was often calculated correctly although a common mistake was to use $h / 3$. A large number of candidates took this to be the new velocity and finished the question at this point. Some continued to calculate the new acceleration but then struggled to form the final equation and a number used either g as the acceleration or $4 \mathrm{~g} / 9$.

## Statistics for M1 Practice Paper Gold Level G3

| Original paper | Qu | Mean score | Max score | Mean score for students achieving grade: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Mean \% | ALL | A | B | C | D | E | U |
| 1006 | 1 | 2.81 | 5 | 56 | 2.81 | 4.03 | 3.07 | 2.31 | 1.76 | 1.31 | 0.68 |
| 0901 | 2 | 4.88 | 9 | 54 | 4.88 | 6.50 | 4.95 | 3.81 | 3.20 | 2.27 | 1.04 |
| 1201 | 3 | 5.15 | 9 | 57 | 5.15 | 6.96 | 5.08 | 3.86 | 2.68 | 1.71 | 0.77 |
| 1306 | 4 | 6.07 | 11 | 55 | 6.07 | 8.08 | 6.42 | 5.59 | 4.81 | 4.10 | 2.68 |
| 1306 | 5 | 6.03 | 14 | 43 | 6.03 | 10.61 | 6.62 | 4.18 | 2.51 | 1.43 | 0.41 |
| 0906 | 6 | 6.09 | 12 | 51 | 6.09 | 9.39 | 6.91 | 5.07 | 3.25 | 1.93 | 0.58 |
| 0801 | 7 | 6.16 | 15 | 41 | 6.16 | 10.68 | 6.66 | 4.13 | 3.16 | 1.68 | 1.09 |
|  |  | 37.19 | 75 | 50 | 37.19 | 56.25 | 39.71 | 28.95 | 21.37 | 14.43 | 7.25 |

