② Let $f(x) = (ax^2 + bx + c)\ln(x + \sqrt{1+x^2}) + (dx+e)\sqrt{1+x^2}$

so $f'(x) = (2ax+b)\ln(x+\sqrt{1+x^2}) + (ax^2+bx+c)\left[\dfrac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\right]$

$+ (dx+e)\cdot\dfrac{x}{\sqrt{1+x^2}} + (dx+e)\sqrt{1+x^2}$

$= (2ax+b)\ln(x+\sqrt{1+x^2}) + \dfrac{1}{\sqrt{1+x^2}}\left(\overset{(a+2d)}{(a+d)}x^2 + (b+e)x + (c+d)\right)$

(i) Choose $a=0,\ b=1,\ d=0,\ e=-1,\ c=0$

so that $\int \ln(x+\sqrt{1+x^2})\,dx = x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + c'$

(ii) Choose $a=0,\ b=0,\ d=\frac{1}{2},\ e=0,\ c=\frac{1}{2}$

so that $\int \sqrt{1+x^2}\,dx = \frac{1}{2}\ln(x+\sqrt{x^2+1}) + \frac{1}{2}x\sqrt{1+x^2} + c''$

(iii) Choose $a=\frac{1}{2},\ b=0,\ c=\frac{1}{4},\ d=-\frac{1}{4},\ e=0$

so that $\int x\ln(x+\sqrt{1+x^2})\,dx = \left(\frac{1}{2}x^2+\frac{1}{4}\right)\ln(x+\sqrt{x^2+1}) - \frac{1}{4}x\sqrt{1+x^2} + c'''$