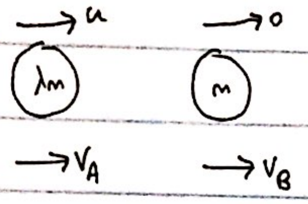
10 (i)

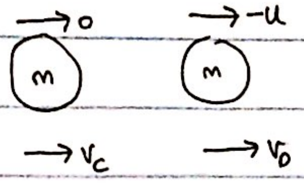


Colm: $u\lambda = \lambda V_A + V_B$

NLR: $eu = V_B - V_A$

So $\lambda(V_B - eu) + V_B = u\lambda \Leftrightarrow (\lambda+1)V_B = u\lambda(1+e)$

$\Leftrightarrow V_B = \dfrac{u\lambda(1+e)}{\lambda+1}$.

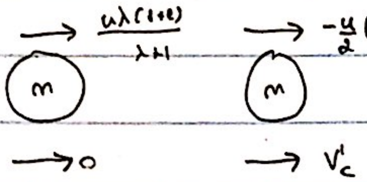


Colm: $-um = mV_C + mV_D$

NLR: $eu = V_D - V_C$

[alternatively, just done λ = 1] (→ +).

So $-u = V_C + V_C + eu \Leftrightarrow V_C = -\dfrac{u}{2}(1+e)$



Colm: $\dfrac{u\lambda(1+e)}{\lambda+1} - \dfrac{u}{2}(1+e) = V_C$.

NLR: $V_C = e\dfrac{u\lambda}{\lambda+1}(1+e) + \dfrac{ue}{2}(1+e)$

So $\dfrac{u\lambda(1+e)}{\lambda+1} - \dfrac{u}{2}(1+e) = \dfrac{eu\lambda}{\lambda+1}(1+e) + \dfrac{ue}{2}(1+e)$

$(u\neq 0,\ e\neq -1)$

$\Leftrightarrow \dfrac{\lambda}{\lambda+1} - \dfrac{1}{2} = \dfrac{e\lambda}{\lambda+1} + \dfrac{e}{2} \Leftrightarrow \dfrac{\lambda-1}{2(\lambda+1)} = \dfrac{e}{2}\left(\dfrac{3\lambda+1}{\lambda+1}\right)$

since $\lambda > 1 \Rightarrow \lambda - 1 = e(3\lambda+1) \Rightarrow e = \dfrac{\lambda-1}{3\lambda+1} = \dfrac{\lambda - \frac{1}{3} - \frac{2}{3}}{3\lambda+1} = \dfrac{1}{3} - \dfrac{2}{3(3\lambda+1)} < \dfrac{1}{3}$

(ii) $V_D = eu - \dfrac{u}{2}(1+e) = \dfrac{u}{2}(2e - e - 1) = \dfrac{u}{2}(e-1)$.

$V_C = \dfrac{u}{2}(1+e)\left(\dfrac{\lambda-1}{\lambda+1}\right)$.

So since $1 - e = 1 - \dfrac{\lambda-1}{3\lambda+1} = \dfrac{3\lambda+1-\lambda+1}{3\lambda+1} = \dfrac{2\lambda+2}{3\lambda+1}$ and $1+e = \dfrac{4\lambda}{3\lambda+1}$

So $4\lambda(\lambda-1) = 2(\lambda+1)^2 \Leftrightarrow 2\lambda^2 - 8\lambda - 2 = 0 \Leftrightarrow \lambda^2 - 4\lambda - 1 = 0$

$\Rightarrow (\lambda-2)^2 = 5 \Rightarrow \lambda = 2 \pm \sqrt{5} \xRightarrow{\lambda>1} \lambda = 2 + \sqrt{5}$ //

$e = \dfrac{1+\sqrt{5}}{7+3\sqrt{5}}$ //