

10 (i) $\begin{array}{ccc} \xrightarrow{u} & & \xrightarrow{0} \\ \textcircled{\lambda m} & & \textcircled{m} \\ \xrightarrow{V_A} & & \xrightarrow{V_B} \end{array}$

Colm: $u\lambda = \lambda V_A + V_B$

NLR: $eu = V_B - V_A$

So $\lambda(V_B - eu) + V_B = u\lambda \Leftrightarrow (\lambda+1)V_B = u\lambda(1+e)$

$\Leftrightarrow V_B = \frac{u\lambda(1+e)}{\lambda+1}$

$\begin{array}{ccc} \xrightarrow{0} & & \xrightarrow{-u} \\ \textcircled{m} & & \textcircled{m} \\ \xrightarrow{V_C} & & \xrightarrow{V_D} \end{array}$

Colm: $-um = mV_C + mV_D$

NLR: $eu = V_D - V_C$

[alternatively, just take $\lambda=1$]

So $-u = V_C + V_D + eu \Leftrightarrow V_C = -\frac{u}{2}(1+e)$ ($\rightarrow +$).

$\begin{array}{ccc} \xrightarrow{\frac{u\lambda(1+e)}{\lambda+1}} & & \xrightarrow{-\frac{u}{2}(1+e)} \\ \textcircled{m} & & \textcircled{m} \\ \xrightarrow{0} & & \xrightarrow{V'_C} \end{array}$

Colm: $\frac{u\lambda(1+e)}{\lambda+1} - \frac{u}{2}(1+e) = V_C$

So NLR: $eu \Leftrightarrow V_C = \frac{eu\lambda(1+e)}{\lambda+1} + \frac{ue}{2}(1+e)$

So $\frac{u\lambda(1+e)}{\lambda+1} - \frac{u}{2}(1+e) = \frac{eu\lambda(1+e)}{\lambda+1} + \frac{ue}{2}(1+e)$

($ue \neq 0, \lambda \neq -1$)

$\Leftrightarrow \frac{\lambda}{\lambda+1} - \frac{1}{2} = \frac{e\lambda}{\lambda+1} + \frac{e}{2} \Leftrightarrow \frac{\lambda-1}{2(\lambda+1)} = \frac{e}{2} \left(\frac{3\lambda+1}{\lambda+1} \right)$

Since $\lambda > 1 \Rightarrow \lambda-1 = e(3\lambda+1) \Rightarrow e = \frac{\lambda-1}{3\lambda+1} = \frac{\lambda - \frac{1}{3} - \frac{2}{3}}{3\lambda+1} = \frac{1}{3} - \frac{2}{3(3\lambda+1)} < \frac{1}{3}$

(ii) $V_D = eu - \frac{u}{2}(1+e) = \frac{u}{2}(2e - e - 1) = \frac{u}{2}(e-1)$

$V_{DC} = \frac{u}{2}(1+e) \left(\frac{\lambda-1}{\lambda+1} \right)$

So since $1-e = 1 - \frac{\lambda-1}{3\lambda+1} = \frac{3\lambda+1 - \lambda+1}{3\lambda+1} = \frac{2\lambda+2}{3\lambda+1}$ and $1+e = \frac{4\lambda}{3\lambda+1}$

So $4\lambda(\lambda-1) = 2(\lambda+1)^2 \Leftrightarrow 2\lambda^2 - 8\lambda - 2 = 0 \Leftrightarrow \lambda^2 - 4\lambda - 1 = 0$

$\Rightarrow \lambda \approx (\lambda-2)^2 = 5 \Rightarrow \lambda = 2 \pm \sqrt{5} \Rightarrow \lambda = 2 + \sqrt{5}$

$e = \frac{1+\sqrt{5}}{7+3\sqrt{5}}$