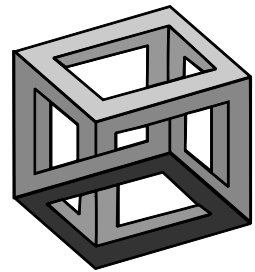


Name.....



Study Leave Revision

GCSE

Mathematics

10-4-10

Ten minutes for ten days

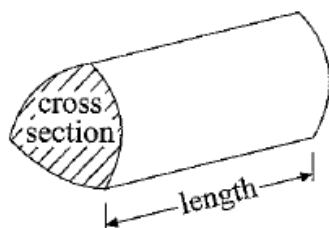
Higher Level A and A* Topics

GCSE Mathematics 1387/8

Formulae: Higher Tier

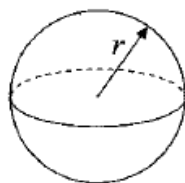
You must not write on this formulae page.
Anything you write on this formulae page will gain NO credit.

Volume of a prism = area of cross section \times length



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

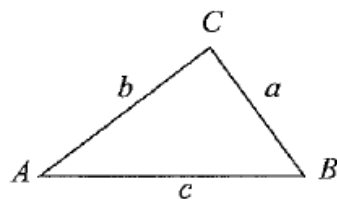


Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



In any triangle ABC



Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A grade A* student can ...



Number	Simplify surds, such as $4(3 + \sqrt{3})$ and $(2 - \sqrt{3})(4 + \sqrt{3})$ in the form $a + b\sqrt{3}$			
	Use index notation and index laws for fractional powers such as $16^{\frac{3}{4}}$			
	Find the upper and lower bounds of more difficult calculations with quantities given to a various degrees of accuracy			
	Simplify harder rational expressions			
	Solve fractional linear equations with the unknown in the denominator such as $\frac{4}{x+2} + \frac{3}{2x-1} = 2$			
	Solve harder quadratic equations ($a \neq 1$) such as $5x^2 - 3x + 5 = 0$ by using the quadratic formula			
	Simplify quadratic expressions by completing the square			
	Use completing the square to solve quadratic equations			
Algebra	Use completing the square to find maximum and minimum values			
	Solve a pair of simultaneous equations where one is linear and one is non-linear such as $x + 4y = 15$ and $x^2 + y^2 = 9$			
	Solve cubic equations by drawing appropriate lines on graphs			
	Plot and sketch graphs of exponential functions			
	Recognise the shapes of graphs of functions			
	Transform the graphs of $y = f(x)$, such as linear, quadratic, cubic, sine and cosine functions, using the transformations $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$ and $y = af(x)$			
	Solve simultaneous equations graphically, such as $y = 2x - 1$ and $x^2 + y^2 = 13$			
	Use trigonometry to find sides and angles in three dimensions			
Shape	Find the angle between a line and a plane			
	Understand the graphs of trigonometric functions for angles of any size			
	Find the volume of the frustum of a truncated cone			
	Solve more difficult vector geometry problems			
hd	Draw tree diagrams and use them to find probabilities of successive dependent events			
	Derive harder algebraic proofs using reasoning and logic			

A grade A student can ...



Number	Rationalise the denominator of a surd such as $\frac{2}{\sqrt{5}}$			
	Use index notation and index laws for simple fractional powers such as $16^{\frac{1}{4}}$			
	Use index notation and index laws for simple negative powers such as 2^{-3}			
	Factorise harder quadratic expressions (a ≠ 1)			
	Solve direct and inverse proportion problems			
	Interpret the graphs of direct and inverse proportion relationships			
	Rearrange formulae where the variable appears twice			
Algebra	Explore the gradients of perpendicular straight-line graphs			
	Use the points of intersection of a quadratic graph such as $y = x^2 - 2x - 4$ with lines such as $y = 2x + 1$ to solve equations like $x^2 - 2x - 4 = 2x + 1$ and simplify this to $x^2 - 4x - 5 = 0$			
	Solve quadratic equations (a=1) such as $x^2 - 3x + 5 = 0$ by using the quadratic formula			
	Solve a pair of simultaneous equations where one is linear and one is non-linear such as $y = 3x - 5$ and $y = x^2$			
	Construct the graphs of loci, including the circle $x^2 + y^2 = r^2$			
	Solve simultaneous equations graphically, such as $y = x - 1$ and $x^2 + y^2 = 9$			
	Calculate the lengths of circular arcs			
	Calculate the areas of sectors			
	Calculate the surface areas of cylinders, cones and spheres			
	Calculate the volumes of cylinders, cones and spheres			
Shape and Space	Prove the angle properties of a circle			
	Prove the tangent / chord properties of a circle			
	Use and prove the alternate segment theorem			
	Enlarge a shape by a negative scale factor			
	Compare areas and volumes of enlarged shapes			
	Find the upper and lower bounds of more difficult calculations with quantities given to a various degrees of accuracy			
	Add, subtract and multiply vectors to solve vector geometry problems			

Data Handling	Understand the relationship between parallel and perpendicular vectors			
	<i>Find the area of a 2-D shape, given the area of a similar shape and the ratio</i>			
	<i>Find the volume of a 3-D solid, given the volume of a similar solid and the ratio</i>			
	Prove that two triangles are congruent			
	Prove the construction theorems			
	<i>Use Pythagoras' theorem in 3-D problems</i>			
	<i>Sketch and draw trigonometric graphs</i>			
	Use the sine rule to find the missing sides and missing angles in any triangle			
	Use the cosine rule to find the missing sides and missing angles in any triangle			
	Use the formula for the area of a non right-angled triangle			
	<i>Use the conditions for congruent triangles in formal geometric proofs</i>			
	<i>Construct and interpret a histogram including unequal class intervals</i>			
	<i>Use stratified sampling methods</i>			
	<i>Understand dependent and independent outcomes</i>			
	<i>Understand probabilities associated with mutually exclusive events</i>			
	Use tree diagrams to find probabilities of successive independent events			
	Derive simple algebraic proofs using reasoning			

Number/Algebra Non-Calculator Day 1

1. $x = 0.\dot{3}7\dot{8}$ (Read as $x = 0.378378378 \dots$)

(a) Write down the value of $1000x$.

.....

(1)

(b) Hence express x as a fraction in its simplest form.

.....

.....

.....

(2)

Number/Algebra Calculator Day 1

2. In a series of experiments, all measurements are taken correct to 3 significant figures. In one particular experiment, the results recorded are

$$v = 24.5, \quad u = 19.2 \quad \text{and} \quad s = 115.$$

Using $f = \frac{v^2 - u^2}{2s}$, what could have been the maximum value for f ?

.....

.....

.....

.....

.....

(4)

Number/Algebra Non-Calculator Day 2

3. (a) Simplify fully the following expression, leaving your answer in surd form.

$$\sqrt{75} - \sqrt{12}$$

.....

.....

.....

- (b) Given that $135 = 3^3 \times 5$,
simplify the expression

$$\frac{\sqrt{135}}{\sqrt{75} - \sqrt{12}}$$

Give your answer in surd form.

.....

.....

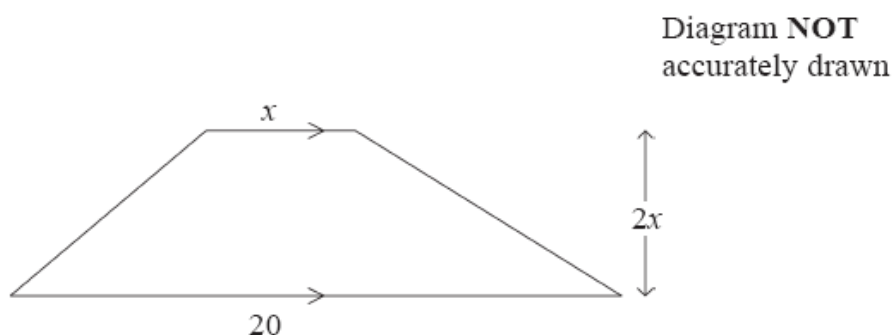
.....

Answer

(3)
(Total 5 marks)

Number/Algebra Calculator Day 2

4.



The diagram shows a trapezium

The measurements on the diagram are in centimetres

The lengths of the parallel sides are x cm and 20 cm

The height of the trapezium is $2x$ cm

The area of the trapezium is 200cm^2

(a) Show that $x^2 + 20x = 200$

(b) Find the value of x (2)

Give your answer correct to 3 decimal places

Number/Algebra Non-Calculator Day 3

5. (a) Find the exact value of $64^{\frac{1}{3}} \times 196^{-\frac{1}{2}}$.

Give your answer as a fraction in its simplest form.

.....

.....

.....

(2)

- (b) Make x the subject of the formula

$$y = \frac{4x}{x+2}$$

.....

.....

.....

.....

.....

.....

(3)

- (c) You are given that $\frac{3}{x+2} - \frac{2}{2x-5} = 1$.

Show that $2x^2 - 5x + 9 = 0$.

.....

.....

.....

.....

.....

.....

.....

Number/Algebra Calculator Day 3

6. y is proportional to x^3

- (a) When $x = 4$, $y = 80$.
Find the value of y when $x = 8$.

.....

.....

.....

.....

.....

.....

(3)

Also, x is inversely proportional to the square root of z .

- (b) When $y = 10$, $z = 16$.
Find the value of z when $x = 4$.

.....

.....

.....

.....

.....

(4)

Number/Algebra Non-Calculator Day 4

7. (a) Factorise $x^2 + 6x + 8$

.....

.....

(2)

- (b) Write $x^2 + 6x + 8$ in the form

$$(x + a)^2 + b$$

.....

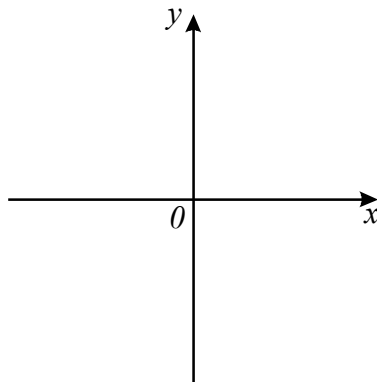
.....

.....

(2)

- (c) Sketch the graph of $y = x^2 + 6x + 8$.

Show how your answers to part (a) and (b) are related to your sketch.



.....

.....

.....

.....

.....

(3)

8.

$$x + \frac{1}{x} = 3$$

- (a) Show that this equation can be re-arranged as

$$x^2 - 3x + 1 = 0.$$

.....

.....

.....

.....

(2)

- (b) Solve this equation to find the values of x correct to 2 decimal places.

.....

.....

.....

.....

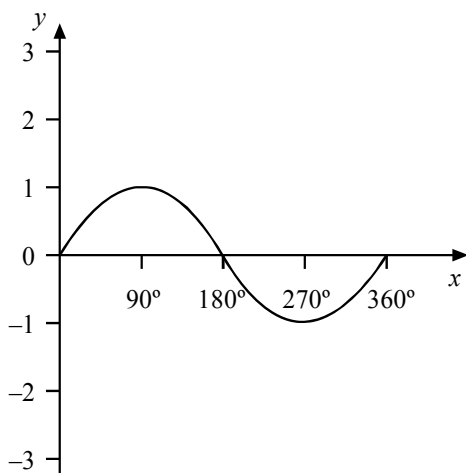
.....

.....

(3)

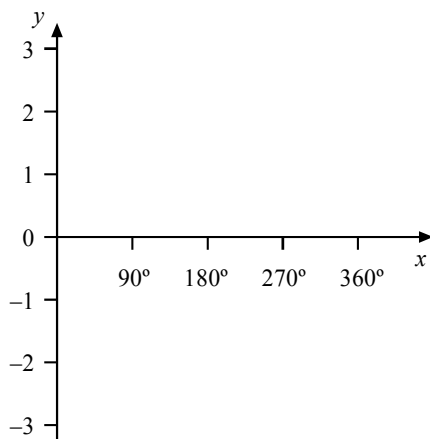
Number/Algebra Non-Calculator Day 5

9. (a) The diagram shows the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$.



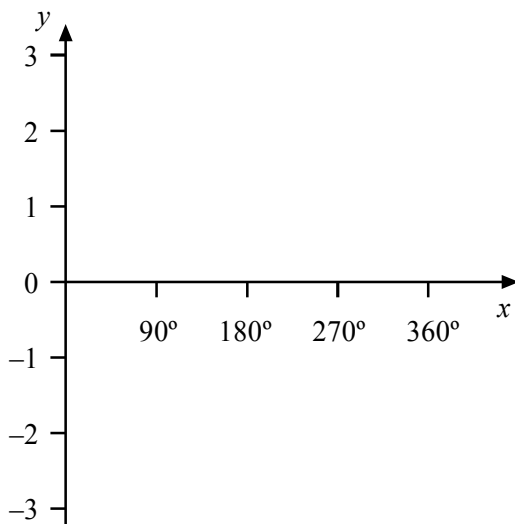
On the axes below sketch the graph of

- (i) $y = 2 \sin x$ for $0^\circ \leq x \leq 360^\circ$



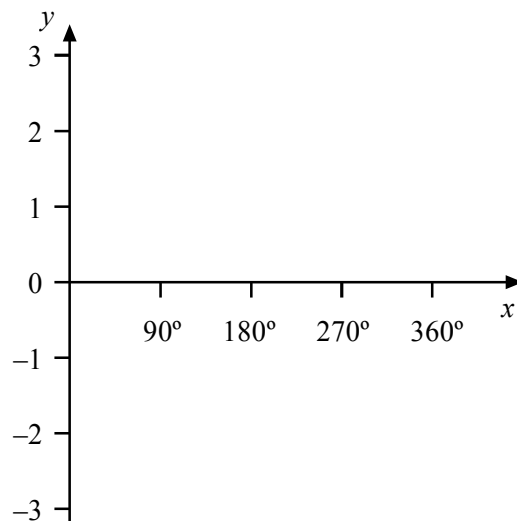
(1)

- (ii) $y = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$



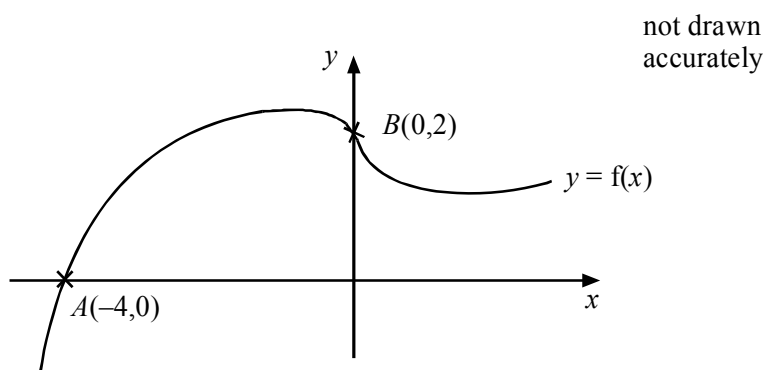
(1)

(iii) $y = \sin x + 2$ for $0^\circ \leq x \leq 360^\circ$



(1)

- (b) The graph of function $y = f(x)$ passes through the points $A(-4, 0)$ and $B(0, 2)$.



The function $y = f(x)$ is transformed to

$$y = 3f(x - 2)$$

The points A and B are transformed to the points A' and B' by this transformation.

Give the coordinates of

- (i) A' ,

.....

(1)

- (ii) B' .

.....

(1)

Number/Algebra Calculator Day 5

10. Given $x=2^p$ $y=2^q$

(a) Express in terms of x and/or y ,

(i) 2^{p+q}

(ii) 2^{2q}

(iii) 2^{p-l}

(3)

$$xy = 32$$

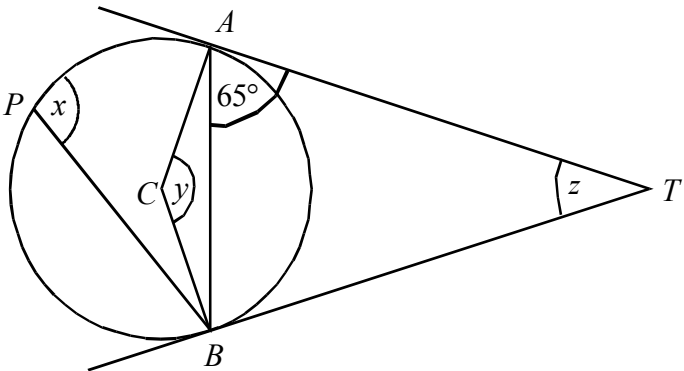
and $2xy^2 = 32$

(b) Find the value of p and the value of q

(2)

Shape, Space and Measures Non-Calculator Day 6

11. AT and BT are tangents to the circle, centre C .
 P is a point on the circumference as shown.
Angle $BAT = 65^\circ$



Calculate the size of

- (a) x ,

.....
.....

(2)

- (b) y ,

.....
.....

(2)

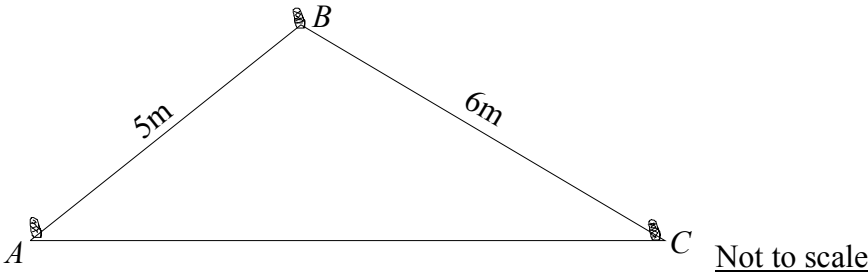
- (c) z .

.....
.....

(2)

Shape, Space and Measures Calculator Day 6

12. A gardener pegs out a rope, 19 metres long, to form a flower bed.



Calculate

- (a) the size of the angle BAC ;

.....

.....

.....

.....

(2)

- (b) the area of the triangular flower bed.

.....

.....

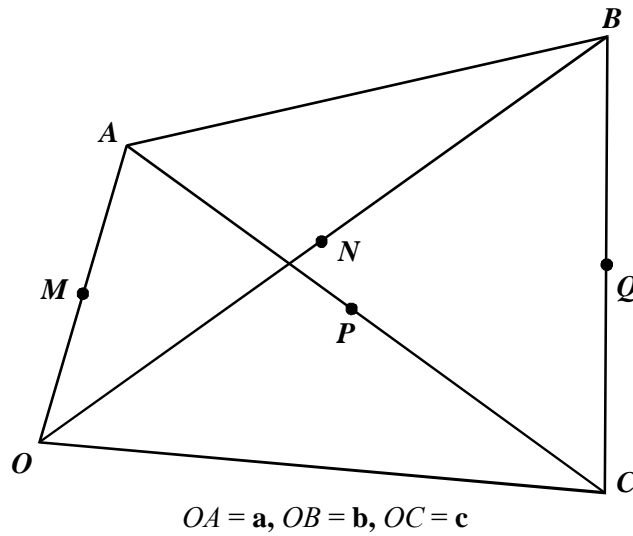
.....

.....

(4)

Shape, Space and Measures Non-Calculator Day 7

13. $OABC$ is a quadrilateral.
 M , N , P and Q are the mid-points of OA , OB , AC and BC .



- (a) Find, in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} expressions for

(i) \overrightarrow{BC}

.....

(1)

(ii) \overrightarrow{NQ}

.....

(1)

(iii) \overrightarrow{MP}

.....

(2)

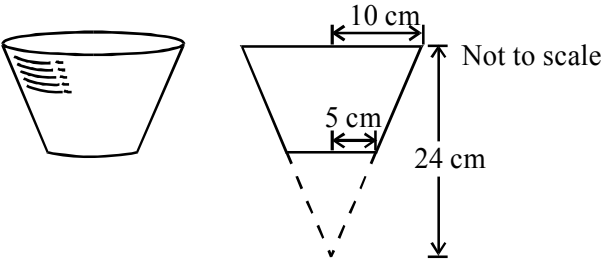
- (b) What can you deduce about the quadrilateral $MNPQ$?

.....

(1)

Shape, Space and Measures Calculator Day 7

- 14.** The sloping sides of a flower bowl are part of a cone as shown.
The radius of the top of the bowl is 10 cm and the radius of the bottom of the bowl is 5 cm.
The height of the full cone is 24 cm.



- (a) Calculate the volume of the full cone.

.....

.....

.....

(2)

- (b) **By using similar figures**, calculate the volume of the flower bowl.

.....

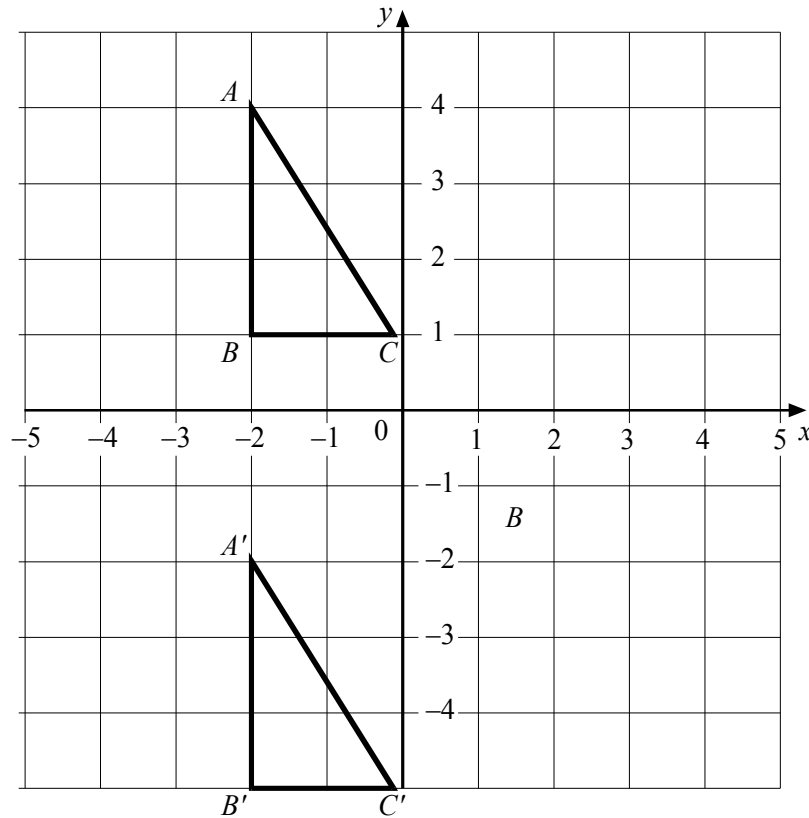
.....

.....

(4)

Shape, Space and Measures Non-Calculator Day 8

15. The grid below shows a triangle ABC and a triangle $A'B'C'$.



- (a) Draw the triangle $A''B''C''$ which is an enlargement of ABC with a scale factor $-\frac{1}{2}$ with centre $(2, 1)$.

(2)

- (b) Describe fully the transformation that takes triangle $A''B''C''$ to triangle $A'B'C'$.

.....

.....

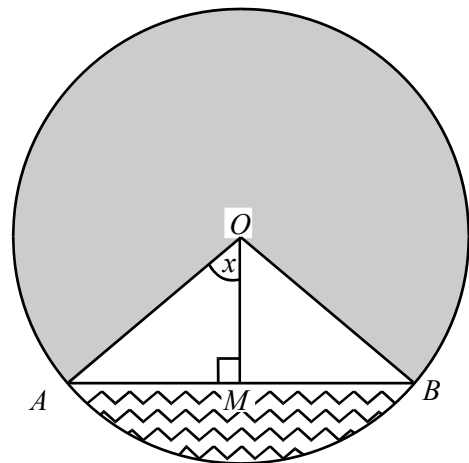
.....

.....

.....

(2)

16.



AB is a chord of the circle, centre O , radius 10 cm. $AB = 16$ cm.


- (a) Calculate the size of angle x .

.....

.....

.....

(2)

- (b) Calculate the area shaded  in the diagram.

.....


.....

.....

.....

.....

(3)

- (c) Calculate the area shaded  in the diagram.

.....

.....

.....

.....

.....

.....

(4)

Handling Data Non-Calculator Day 9

17. Alan, Bob, Charles, David, Evelyn and Fay are asked their ages.
Alan says “23”, Bob says “61”, Charles says “57” and David says “21”.
Evelyn and Fay refuse to give their ages.
It is known that two of the men are older than Evelyn and Fay and the other two are younger.
It is known that the median value of all the ages is 42.

- (a) How do you know that the sum of the ages of Evelyn and Fay is 84?

.....

.....

.....

.....

(2)

- (b) Fay then says that she is twice as old as Evelyn.
How old is Evelyn?

.....

.....

.....

.....

(3)

Handling Data Calculator Day 9

18. A football team has to play two games.

The first game is played away. The second game is played at home.

The probability that the team will win the away game is 0.3

If the team wins the away game, then the probability that it will win the home game is 0.6

If the team does **not** win the away game, then the probability that it will win the home game is 0.45

Calculate the probability that the team

- (a) wins both games,

.....
.....

(2)

- (b) wins only one game.

.....
.....
.....
.....
.....

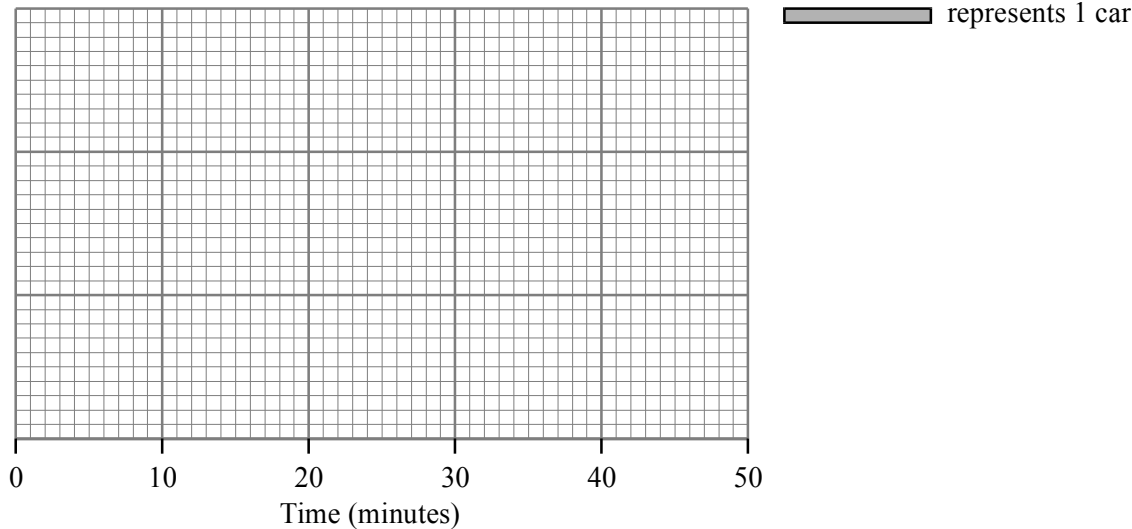
(5)

Handling Data Non-Calculator Day 10

19. Jonathan counted the number of cars arriving at his school in the 50 minutes just before school started.

Time (min)	$0 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$30 \leq t < 50$
Number of cars	2	6	10	48

- (a) Draw a histogram to show this information.



(3)

- (b) Using your histogram to estimate the time by which half of the cars had arrived. Show your working.

.....

.....

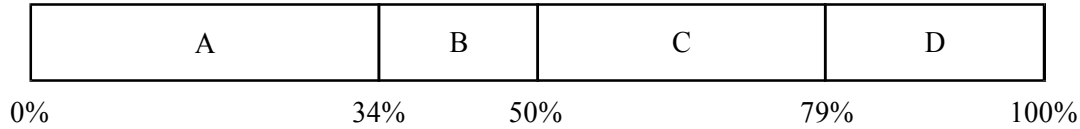
.....

(2)

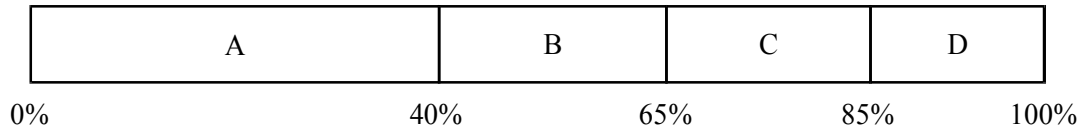
Handling Data Calculator Day 10

- 20.** A company employs 200 men and 320 women.
Each person's job is given a grade (A, B, C or D).
The diagrams show the proportions by grade for men and for women.

Men (200 employees)



Women (320 employees)



The company wishes to survey a 10% sample of the employees, stratified by sex and by grade.
Complete the table to show how many of each group should be sampled.

	A	B	C	D
Men				
Women				

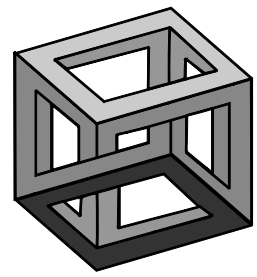
.....

.....

.....

.....

(4)



GCSE Mathematics

Study Leave Revision



Number/Algebra Non-Calculator Answers Day 1

1. $x = 0.\dot{3}78$ (Read as $x = 0.378378378 \dots$)

- (a) Write down the value of $1000x$.

$$\underline{378.378(378\dots)}$$

(NB If $x = 0.4545454545$ you would find the value of $100x$)

(1)

- (b) Hence express x as a fraction in its simplest form.

$$1000x = 378.378(378\dots)$$

$$x = 0.378378378\dots \text{ (If you subtract } x \text{ from } 1000x \text{ you will eliminate the decimals)}$$

$$999x = 378 \text{ M1}$$

$$378/999 \text{ gets M1}$$

(378 & 999 are divisible by 3! $3+7+8=18$; $9+9+9=27$; both 18 and 27 are divisible by 3)

$$x = \frac{14}{37}$$

A1

must be in simplest form

(2)

Number/Algebra Calculator Answers Day 1

2. In a series of experiments, all measurements are taken correct to 3 significant figures. In one particular experiment, the results recorded are

$$v = 24.5, \quad u = 19.2 \quad \text{and} \quad s = 115.$$

Using $f = \frac{v^2 - u^2}{2s}$, what could have been the maximum value for f ?

NB. All measurements are correct to 3 significant figures. Variables v and u are also correct to 1 decimal place (0.1) and s is correct to the nearest whole number (1). To find the maximum and minimum values for these variables

$$\text{Max } v = 24.5 + (0.1 \div 2) = 24.5 + 0.05 = 24.55; \text{ Min } v = 24.5 - (0.1 \div 2) = 24.5 - 0.05 = 24.45$$

$$\text{Max } u = 19.2 + (0.1 \div 2) = 19.2 + 0.05 = 19.25; \text{ Min } u = 19.2 - (0.1 \div 2) = 19.2 - 0.05 = 19.15$$

$$\text{Max } s = 115 + (1 \div 2) = 115 + 0.5 = 115.5; \quad \text{Min } s = 115 - (1 \div 2) = 115 - 0.5 = 114.5$$

To find the maximum value for f , we would need the following values

Max v , Min u and Min s

$$\text{Max } v = 24.55$$

$$\text{M1 (Not 24.54; Round 24.549999.. up to 24.55)}$$

$$\text{Min } u = 19.15$$

$$\text{M1 (Not 19.14; Round 19.149999.. up to 19.15)}$$

$$\text{Min } s = 114.5$$

$$\text{M1 (Not 114.4; Round 114.4999.. up to 114.5)}$$

$$f_{\text{max}} = \frac{24.55^2 - 19.15^2}{2 \times 114.5} = 1.030480349 \dots = \underline{1.03 \text{ (3sf)}}$$

M1

(4)

Number/Algebra Non-Calculator Answers Day 2

3. (a) Simplify fully the following expression, leaving your answer in surd form.

$$\sqrt{75} - \sqrt{12}$$

Hint look for square numbers

$$75 = 25 \times 3 \quad (25 = 5 \times 5)$$

$$12 = 4 \times 3 \quad (4 = 2 \times 2)$$

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$5\sqrt{3} - 2\sqrt{3}$$

B1

Either $5\sqrt{3}$ or $2\sqrt{3}$

$$= 3\sqrt{3}$$

B1

(2)

- (b) Given that $135 = 3^3 \times 5$,
simplify the expression

$$\frac{\sqrt{135}}{\sqrt{75} - \sqrt{12}}$$

$$\sqrt{135} = \sqrt{9 \times 15} = 3\sqrt{15} \text{ or } 3\sqrt{3 \times 5} \quad \text{B2}$$

Give your answer in surd form.

$$\frac{3\sqrt{15}}{3\sqrt{3}}$$

B2

$$= \sqrt{5}$$

B1

(3)

Number/Algebra Calculator Answers Day 2

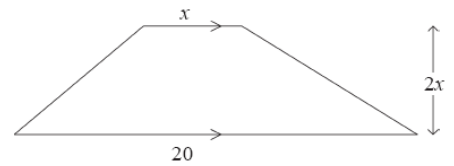
4. The diagram shows a trapezium

The measurements on the diagram are in centimetres

The lengths of the parallel sides are x cm and 20 cm

The height of the trapezium is $2x$ cm

The area of the trapezium is 200cm^2



(a) Show that $x^2 + 20x = 200$

You will need to know this formula!!

$$\text{Area of trapezium} = \frac{(a+b)}{2} x h$$

$$200 = \frac{(20+x)}{2} x 2x$$

$$200 = \frac{(40x + 2x^2)}{2}$$

$$400 = 40x + 2x^2$$

$$200 = 20x + x^2 \text{ or } x^2 + 20x = 200$$

Divide throughout by 2

(c) Find the value of x

Give your answer correct to 3 decimal places

Methods: completing the square / using the quadratic formula to solve $x^2 + 20x - 200 = 0$

Completing the square (see question 8b notes on how to solve using the quadratic formula)

$$(x + 10)^2 = (x + 10)(x + 10) = x^2 + 20x + 100$$

You need

$$x^2 + 20x - 200$$

You now need to subtract 300 from this expression to make

$$\text{So } (x + 10)^2 - 300 = 0$$

$$(x + 10)^2 = 300$$

$$x + 10 = \pm\sqrt{300}$$

$$x = -10 - \sqrt{300} \quad \text{or} \quad x = -10 + \sqrt{300}$$

$$x = -27.3205080 \dots \text{ or } x = 7.3205080 \dots$$

$$x = -27.321 \text{ cm or } 7.321 \text{ cm (3 decimal places)}$$

Take the square root of both sides of the equation to remove the squared term

$$x^2 + 20x - 200$$

Check these solutions are correct by substituting back into the original formula

$$x^2 + 20x = 200$$

(3)

Number/Algebra Non-Calculator Answers Day 3

5. (a) Find the exact value of $64^{\frac{1}{3}} \times 196^{-\frac{1}{2}}$. Give your answer as a fraction in its simplest form.

$$64^{\frac{1}{3}} = \sqrt[3]{64} = 4; \quad 196^{-\frac{1}{2}} = \frac{1}{\sqrt{196}} = \frac{1}{14}$$

$$4 \times \frac{1}{14} = \frac{4}{14} = \frac{2}{7}$$

(2)

- (b) Make x the subject of the formula

$$y = \frac{4x}{x+2}$$

First you need to remove $(x+2)$ from the denominator. To do this you need to multiply the equation throughout by $(x+2)$

$$y(x+2) = 4x$$

Now expand the bracket and collect together the x 's on one side of the equation

$$yx + 2y = 4x \quad M1$$

$$2y = 4x - yx$$

Collect x 's on the RHS to keep $2y$ positive

$$\text{or } yx - 4x = -2y$$

$$2y = x(4 - y) \quad \text{Factorise} \quad M1$$

$$\text{or } x(y - 4) = -2y$$

$$\frac{2y}{(4 - y)} = x \quad (\text{divide both sides by } (4 - y))$$

$$x = \frac{2y}{4 - y} \quad \text{or} \quad x = \frac{-2y}{y - 4}$$

A1

cao

(3)

- (b) You are given that $\frac{3}{x+2} - \frac{2}{2x-5} = 1$.

$$\text{Show that } 2x^2 - 5x + 9 = 0.$$

First find the lowest common multiple of the fraction denominators. This is simply the denominators multiplied together

$$(x+2)(2x-5) \quad \text{Next multiply throughout by } (x+2)(2x-5)$$

$$3(2x-5) - 2(x+2) = (x+2)(2x-5)$$

Now expand the brackets

B2

$$6x - 15 - 2x - 4 = 2x^2 - 5x + 4x - 10$$

Collect together like terms

$$4x - 19 = 2x^2 - x - 10$$

Collect all terms on the RHS

$$0 = 2x^2 - 5x + 9$$

$$\text{OR } 2x^2 - 5x + 9 = 0$$

A2

(4)

Number/Algebra Calculator Answers Day 3

6. y is proportional to x^3

- (a) When $x = 4$, $y = 80$.
Find the value of y when $x = 8$.

First find the constant k by forming an equation and substituting in the given values

$$y = kx^3 \quad \text{M1}$$

$$80 = k \times 4^3; \quad 80 = k \times 64$$

$$\frac{80}{64} = k; \quad k = 1.25 \quad \text{A1}$$

Using this value of $k = 1.25$ and $x = 8$; substitute into the formulae $y = kx^3$

$$y = 1.25 \times 8^3$$

$$\mathbf{y = 640} \quad \text{A1} \quad (3)$$

Also, x is inversely proportional to the square root of z .

- (b) When $y = 10$, $z = 16$.
Find the value of z when $x = 4$.

We have now two equations

$$\text{From part (a)} \quad y = 1.25x^3$$

$$\text{From part (b)} \quad x = \frac{k}{\sqrt{z}};$$

We need to substitute the value of x from part (b) into the equation in part (a) to find the value of the second constant k in part (b)

$$\text{If } y = 1.25x^3 \text{ and } y = 10; \quad \text{then } 10 = 1.25x^3$$

$$\frac{10}{1.25} = x^3; \quad 8 = x^3; \quad \text{Thus } x = 2$$

Now given $x = \frac{k}{\sqrt{z}}$; and $z = 16$, using $x = 2$ we can calculate the value for k

$$2 = \frac{k}{\sqrt{16}}; \quad 2 = \frac{k}{4}; \quad \text{Thus } k = 8$$

Now given $x = 4$ and using $k = 8$ we can find the value of z

$$4 = \frac{8}{\sqrt{z}}; \quad 4\sqrt{z} = 8; \quad \sqrt{z} = 2 \quad \mathbf{Hence } z = 4$$

(4)

Number/Algebra Non-Calculator Answers Day 4

7. (a) Factorise $x^2 + 6x + 8$

$$(x + 2)(x + 4)$$

B1 B1

(2)

(b) Write $x^2 + 6x + 8$ in the form

$$(x + a)^2 + b$$

$$(x + 3)^2$$

Now expand the bracket $(x + 3)^2 = (x + 3)(x + 3)$

$$= x^2 + 6x + 9$$

To find 'a' simply halve the coefficient of x i.e.

$$6x; a = 3$$

We need $x^2 + 6x + 8$ so if we subtract 1 to the above expression this should do it!!

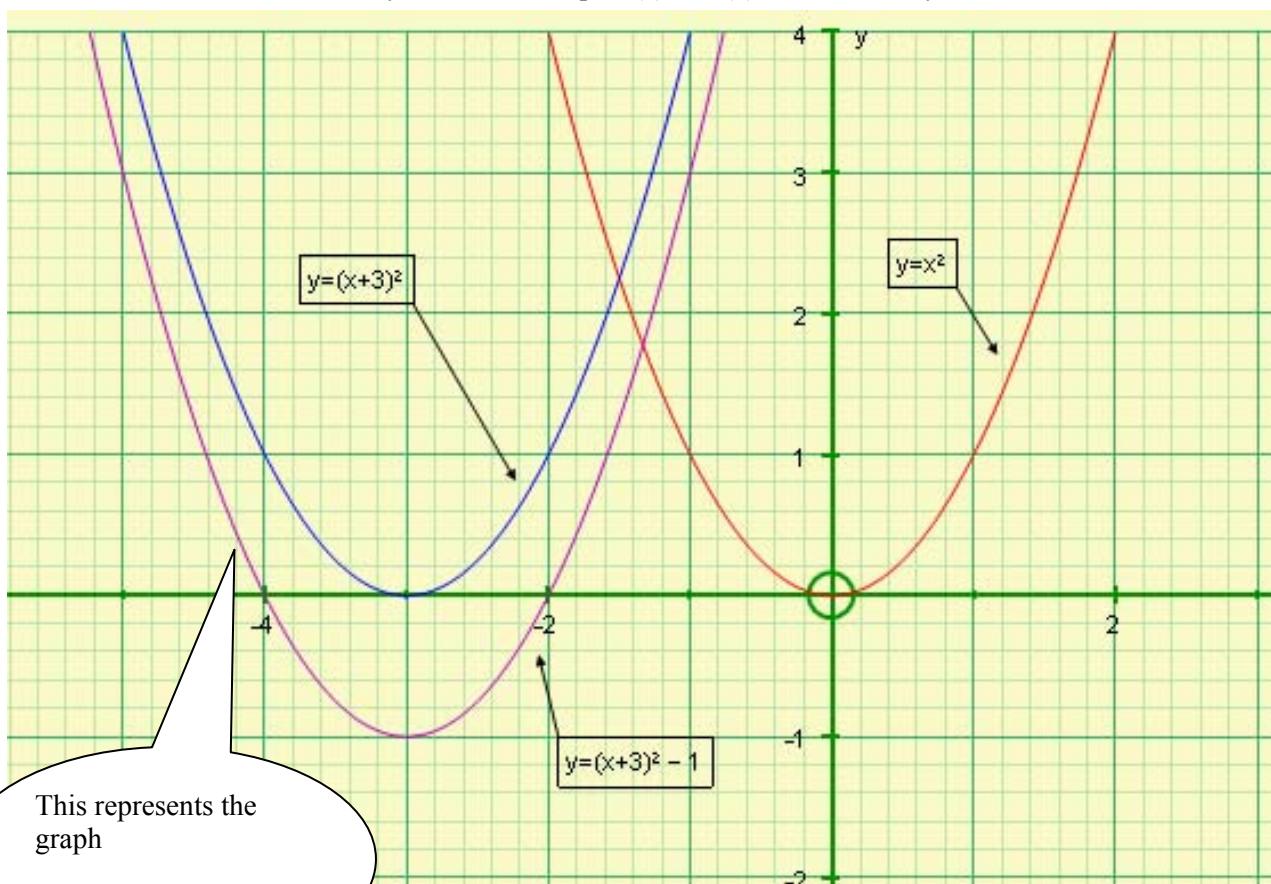
$$(x + 3)^2 - 1 = x^2 + 6x + 8$$

$$(x + 3)^2 - 1$$

(2)

(c) Sketch the graph of $y = x^2 + 6x + 8$.

Show how your answers to part (a) and (b) are related to your sketch.



This represents the graph

$$y = x^2 + 6x + 8$$

(3)

Number/Algebra Calculator Answers Day 4

8. $x + \frac{1}{x} = 3$

- (a) Show that this equation can be re-arranged as

$$x^2 - 3x + 1 = 0.$$

First multiply the equation throughout by x to eliminate the fraction

$$x^2 + 1 = 3x$$

Therefore $x^2 - 3x + 1 = 0$.

(2)

- (b) Solve this equation to find the values of x correct to 2 decimal places.

Methods: completing the square(see question 4b)/ using the quadratic formula to solve $x^2 - 3x + 1 = 0$

This approach will use both; first by using the quadratic formula with equations in the form

$$ax^2 + bx + c = 0 \text{ and the formula } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ to solve } x^2 - 3x + 1 = 0.$$

(Remember a is the coefficient of x^2 , b is the coefficient of x and c is the constant)

Substituting $a = 1$, $b = -3$, $c = 1$ into formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3 + \sqrt{5}}{2} \text{ or } x = \frac{3 - \sqrt{5}}{2}$$

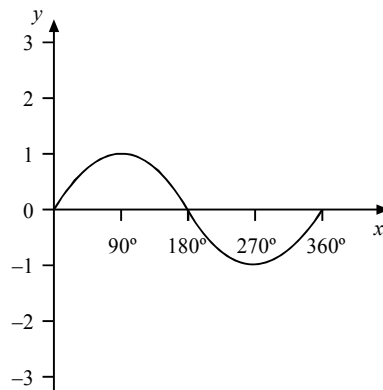
$x = 2.61803... \text{ or } x = 0.381966...$

$x = 2.62 \text{ or } x = 0.38(\text{correct to 2 decimal places})$

(3)

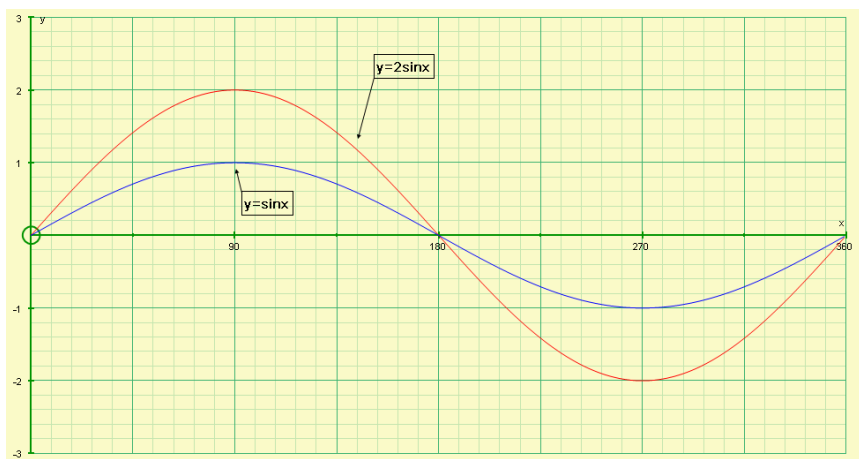
Number/Algebra Non-Calculator Answers Day 5

9. (a) The diagram shows the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$.



On the axes below sketch the graph of

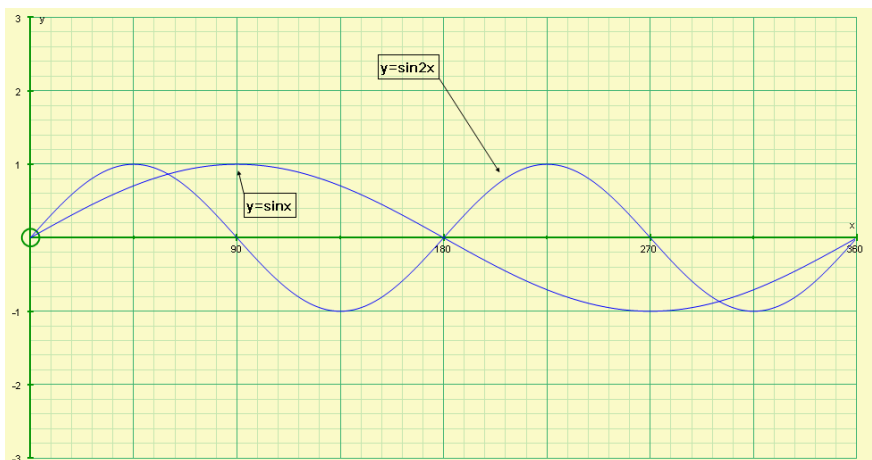
(i) $y = 2 \sin x$ for $0^\circ \leq x \leq 360^\circ$



Graph with max at (90, 2), and min at (270, -2), crossing axis at 0, 180, 360.

(1)

(ii) $y = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$

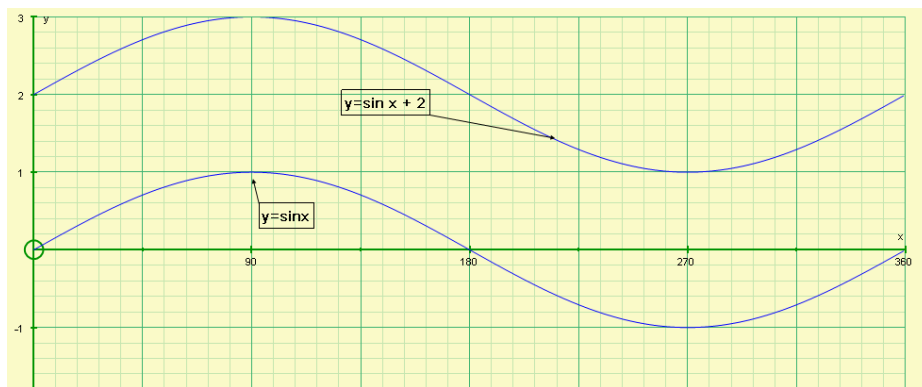


Sine graph with 2 cycles. Max at (45, 1) and (225, 1)

Min at (135, -1) and (315, -1) crossing axis at 0, 90, 180, 270, 360

(1)

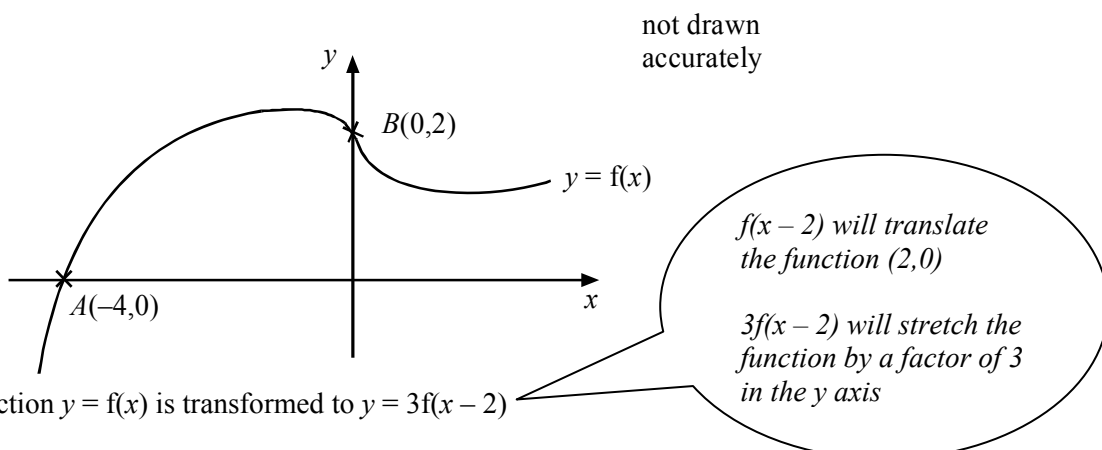
(iii) $y = \sin x + 2$ for $0^\circ \leq x \leq 360^\circ$



Translation of sine graph by (0,2)

(1)

- (b) The graph of function $y = f(x)$ passes through the points $A(-4, 0)$ and $B(0, 2)$.



The function $y = f(x)$ is transformed to $y = 3f(x - 2)$

The points A and B are transformed to the points A' and B' by this transformation.

Give the coordinates of

(i) $A' (-2, 0)$

(ii) $B' (2, 6)$

(2)

Number/Algebra Calculator Answers Day 5

10. Given $x=2^p$ $y=2^q$

(a) Express in terms of x and/or y ,

(i) $2^{p+q} = 2^p \times 2^q = \mathbf{xy}$

(ii) $2^{2q} = (2^q)^2 = \mathbf{y^2}$

(iii) $2^{p-1} = \frac{2^p}{2} = \mathbf{\frac{x}{2}}$

(3)

$$xy = 32$$

and $2xy^2 = 32$

(c) Find the value of p and the value of q

$$xy = 32;$$

$$xy^2 = 16;$$

$$xy \times y = 16$$

$$32 \times y = 16$$

$$\mathbf{y = 0.5}$$

$$xy = 32$$

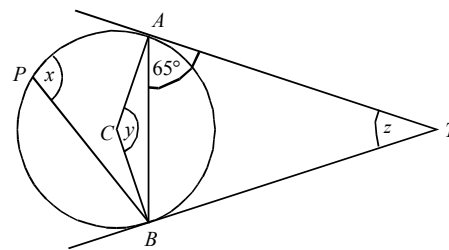
$$0.5x = 32$$

$$\mathbf{x = 64}$$

(2)

Shape, Space and Measures Non-Calculator Answers Day 6

11. AT and BT are tangents to the circle, centre C .
 P is a point on the circumference as shown.
 Angle $BAT = 65^\circ$



Calculate the size of

- (a) x ,

tangent-chord
 65°

*In all parts, Ms can be awarded
 for clear intent or reason.*

M1
 A1 cao

(2)

- (b) y ,

angle at centre = twice angle at circumference
 130°

M1
 A1 cao

(2)

- (c) z .

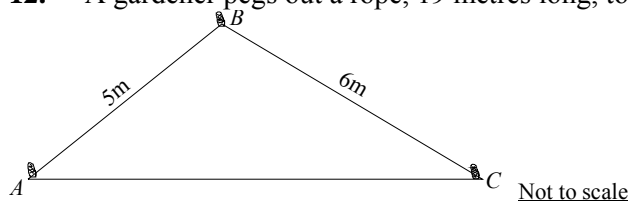
sum of angles in quadrilateral
 50°

M1
 A1 cao

(2)

Shape, Space and Measures Calculator Answers Day 6

12. A gardener pegs out a rope, 19 metres long, to form a flower bed.



Calculate

- (a) *the size of the angle BAC;*

$$6^2 = 5^2 + 19^2 - 2 \times 5 \times 19 \cos BAC$$

$BAC = 48.5^\circ$

M1
 A1

(2)

- (b) *the area of the triangular flower bed.*

$$\text{Area} = \frac{1}{2} \times 5 \times 19 \sin 48.5^\circ$$

$= 14.98 \text{ cm}^2$

M2
 A2

or $h = 5 \sin 48.5^\circ$
 $= 3.745 \text{ cm}$

(M1)
 (A1)
 (M1)

$$\text{area} = \frac{1}{2} \times 19 \times 3.745$$

$= 14.98 \text{ cm}^2$

(A1) (4)

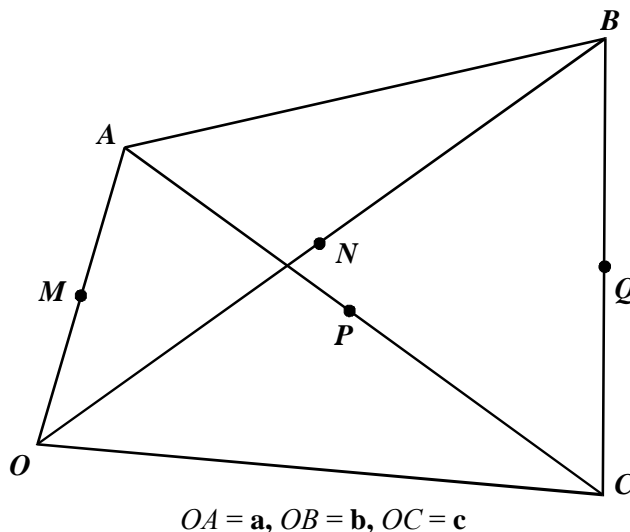
(4)

Shape, Space and Measures Non-Calculator Day 8

2. Triangle ABC has vertices at $A(0, 2)$, $B(2, 5)$, $C(6, 4)$.

Shape, Space and Measures Calculator Answers Day 7

13. $OABC$ is a quadrilateral.
 M, N, P and Q are the mid-points of OA, OB, AC and BC .



- (a) Find, in terms of \mathbf{a}, \mathbf{b} and \mathbf{c} expressions for

Hints

Put information from the question onto the diagram first, put 'a' and an arrow on OA, 'b' and an arrow on OB, 'c' and an arrow on OC. Always follow a path that you know and remember that if you go backwards down an arrow the vector is negative e.g. $AO = -a$

(i) \overrightarrow{BC}

$\mathbf{c} - \mathbf{b}$

(or $-\mathbf{b} + \mathbf{c}$)

(1)

(ii) \overrightarrow{NQ}

You will need to use your answer to BC for this question

Follow the path that you know, to get from N to Q, go from N to O, then O to C and finally C to Q

$NQ = -\frac{1}{2}\mathbf{b} \quad OC = \mathbf{c}$

$BC = \mathbf{c} - \mathbf{b} \quad CB = \mathbf{b} - \mathbf{c} \quad CQ \text{ is half of } CB \quad CQ = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$

So $NQ = -\frac{1}{2}\mathbf{b} + \mathbf{c} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$

$= \frac{1}{2}\mathbf{c}$

(1)

(iii) \overrightarrow{MP}

You will need to work out AC first

$$AC = -a + c$$

$$MP = MA + AP$$

$$MA = \frac{1}{2}a \quad AP = \frac{1}{2}AC$$

$$= \frac{1}{2}(-a + c)$$

$$MP = \frac{1}{2}a + \frac{1}{2}(-a + c)$$

$$= \frac{1}{2}c$$

M1,A1

M1 for attempt to add appropriate vectors.

A1 answer

(2)

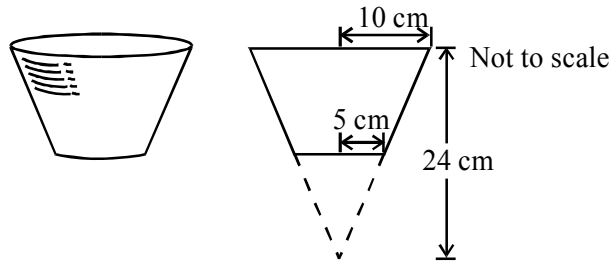
(b) What can you deduce about the quadrilateral $MNQP$?

*Opposite sides have the same vectors, $MP = NQ$ so the lines must be equal in length and parallel so $MNQP$ must be a **Parallelogram*** B1

Or equivalent

Shape, Space and Measures Calculator Answers Day 7

14. The sloping sides of a flower bowl are part of a cone as shown.
The radius of the top of the bowl is 10 cm and the radius of the bottom of the bowl is 5 cm.
The height of the full cone is 24 cm.



- (a) Calculate the volume of the full cone.

Remember to look at the formula page for volume of a cone

$$\text{Volume of a cone} = \frac{1}{3} \times \pi \times r^2 \times h \text{ cm}^3$$

$$= \frac{1}{3} \times \pi \times 10^2 \times 24 \text{ cm}^3$$

$$= 2513.27 \text{ cm}^3$$

M1

A1

(2)

- (b) **By using similar figures**, calculate the volume of the flower bowl.

Radius small cone : radius large cone is
1 : 2

M1 A1

This is the linear scale factor. If we are finding a new volume then we need the volume scale factor

Linear factor is 1 : 2

Area factor is $1^2 : 2^2$ is 1 : 4

Volume factor is $1^3 : 2^3$ is 1 : 8

So the small volume of the small cone is one eighth of the volume of the large cone

$$= 2513.27 \div 8 = 314.16 \text{ cm}^3$$

$$\text{So the volume of the flower bed} = 2513.27 - 314.16$$

$$= 2199.11 \text{ cm}^3$$

$$= 2200 \text{ cm}^3 \text{ A1}$$

M1

[4]

(4)

Shape, Space and Measures Non-Calculator Answers Day 8

15. The grid below shows a triangle ABC and a triangle $A'B'C'$.

Remember that a scale factor of $\frac{1}{2}$ will make the shape smaller (half the size) and half as far from the centre of enlargement.

Remember that a negative scale factor will mean that the image is on the opposite side of the centre of enlargement to the object, it will be inverted (turned upside down)

- (i) **Triangle with coordinates at (3,1), (4, 1) and (4, -1/2)**

M1, A1

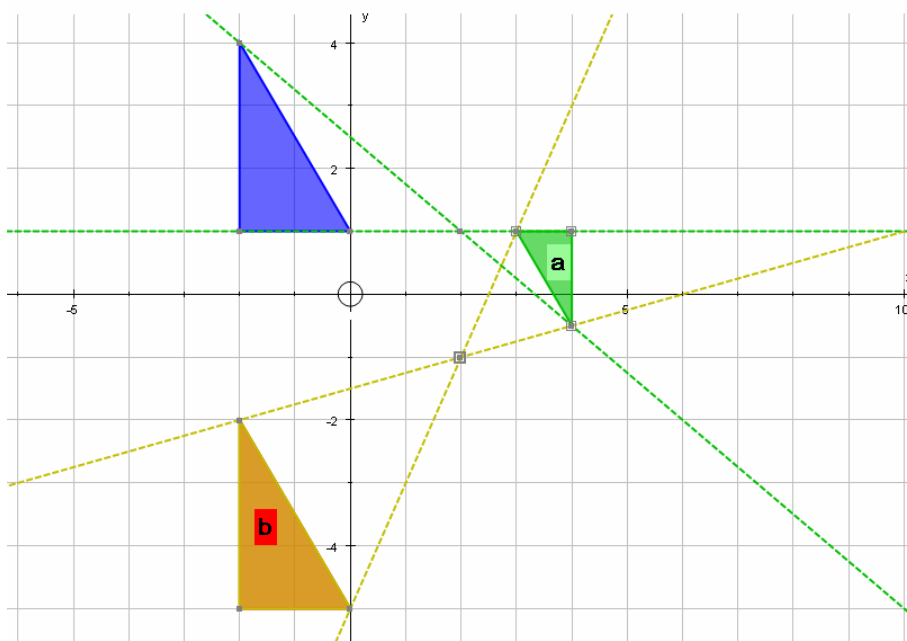
M1 for attempt to enlarge at (2,1), A1 for answer (cao)

When describing a transformation ALL details are needed, usually three details for a rotation (centre, direction, angle) and two for an enlargement (centre, scale factor)

(2)

- (ii) **Enlargement s.f. -2 about (2, -1)**

B1, B1

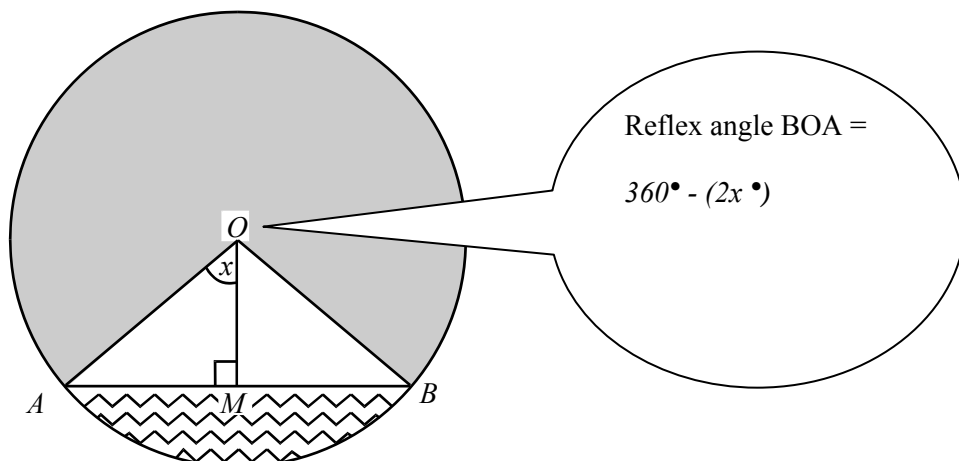


B1 for enlargement s.f. 2, B1 centre
f.t. on their answer if first M1 awarded.

eg enlargement s.f. $\frac{1}{2}$ at (2,1) is Enlargement s.f.2, centre (2,7) with centre (2, 1).

(2)

16.



AB is a chord of the circle, centre O , radius 10 cm. $AB = 16$ cm.

- (a) Calculate the size of angle x .

Label the sides of the triangle with hypotenuse; opposite and adjacent in shorthand form,

Remember this is a right-angled triangle so this is BASIC TRIGONOMETRY

We know the $\sin x = \frac{8}{10}$ *(remember to half AB)*


$$\sin x = 0.8$$

$$x = \sin^{-1} 0.8$$

$$x = 53.1^\circ$$

M1 A1

(2)

- (b) Calculate the area shaded  in the diagram.

We are finding the area of a sector so we need the angle at the centre of the shaded part of the circle (Reflex angle BOA see diagram)

$$360^\circ - (2 \times 53.1^\circ) = 253.8^\circ$$

Area of sector is a fraction of the area of the circle in this case = $\frac{253.8^\circ}{360^\circ} \times \pi r^2$

=


$$= \frac{253.8^\circ}{360^\circ} \times \pi \times 10^2$$

Answer Accepted
221–221.5

$$= 220.96 \text{ cm}^2$$

M1 M1
A1

(3)

- (c) Calculate the area shaded  in the diagram.

Area of segment = area of full circle - area of sector - area of triangle

$$\text{Area of sector} = 220.96 \text{ cm}^2$$

$$\text{Area of triangle} = 0.5 \times 16 \times 6$$

$$= 48 \text{ cm}^2$$

The triangle is right-angled and half of it is a simple 6,8,10 (Pythagorean triple), so the perpendicular height is 6cm

M1 A1

$$\text{Area of circle} = \pi \times 10^2$$

M1

$$\text{So the shaded area} = \pi \times 10^2 - 220.96 - 48$$

$$= 45 \text{ cm}^2$$

accepted answers from 44.7 – 45.2

(4)

Handling Data Non-Calculator Answers Day 9

17. Alan, Bob, Charles, David, Evelyn and Fay are asked their ages.
Alan says “23”, Bob says “61”, Charles says “57” and David says “21”.
Evelyn and Fay refuse to give their ages.
It is known that two of the men are older than Evelyn and Fay and the other two are younger.
It is known that the median value of all the ages is 42.

- (a) How do you know that the sum of the ages of Evelyn and Fay is 84?

Order of ages : 21 23 E F 57 61

M1

$$\text{Median} = \frac{E + F}{2} = 42$$

$$84/2 = 42 \rightarrow 2 \text{ but not } 2 \times 42 = 84$$

A1

(2)

- (b) Fay then says that she is twice as old as Evelyn.
How old is Evelyn?

$$E + F = 84$$

$$F = 2E$$

M1

$$E + 2E = 84$$

M1

$$E = 28$$

A1

(3)

Handling Data Calculator Answers Day 9

18. A football team has to play two games.

The first game is played away. The second game is played at home.

The probability that the team will win the away game is 0.3

If the team wins the away game, then the probability that it will win the home game is 0.6

If the team does **not** win the away game, then the probability that it will win the home game is 0.45

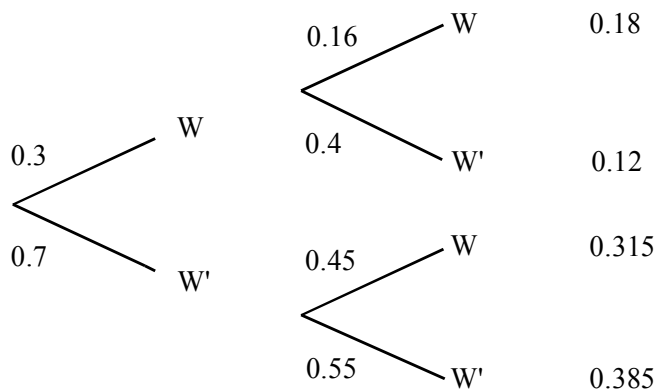
- (a) wins both games,

$$0.3 \times 0.6 = \underline{0.18}$$

M1 A1

(2)

- (b) wins only one game.



Use of $P(W')$ appropriately once
 $P(WW'$ or $W'W)$ or $P(WW$ or $W'W')$

M1

Can be on correct tree diagram

$$= 0.3 \times 0.4 + 0.7 \times 0.45 \text{ or } 0.18 + 0.7 \times 0.55$$

M1, M1

Correct products

$$0.12 + 0.315 \text{ or } 1 - 0.18 - 0.385$$

M1

Probs summed (and subtracted from 1) but **not** halved.

$$= \underline{0.435}$$

A1 cao

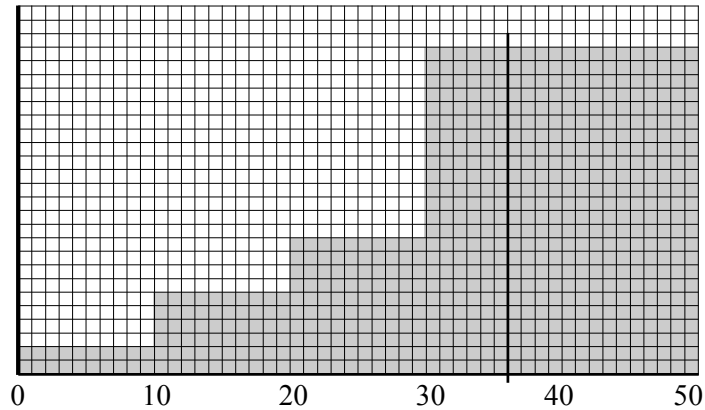
(5)

Handling Data Non-Calculator Answers Day 10

19. Jonathan counted the number of cars arriving at his school in the 50 minutes just before school started.

Time (min)	$0 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$30 \leq t < 50$
Number of cars	2	6	10	48

- (a) Draw a histogram to show this information.



- (a) Using area
correct widths
correct heights
- (b) Using your histogram to estimate the time by which half of the cars had arrived. Show your working.

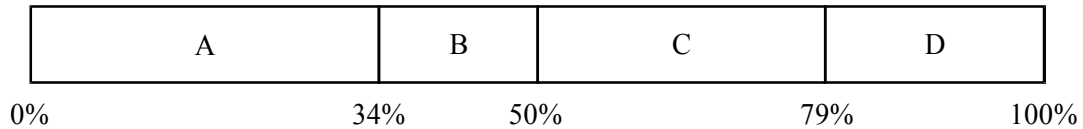
between 36 and 37 minutes

M1 A1

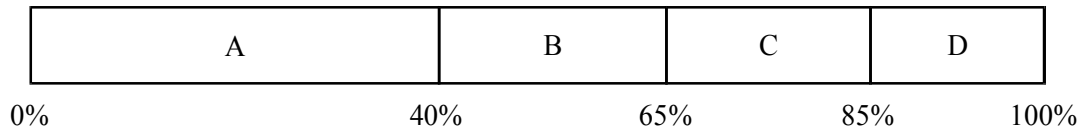
Handling Data Calculator Answers Day 10

20. A company employs 200 men and 320 women.
Each person's job is given a grade (A, B, C or D).
The diagrams show the proportions by grade for men and for women.

Men (200 employees)



Women (320 employees)



The company wishes to survey a 10% sample of the employees, stratified by sex and by grade.
Complete the table to show how many of each group should be sampled.

Actual	A	B	C	D
Men	68	32	58	42
Women	128	80	64	48
10% Stratified (rounded)	A	B	C	D
Men	<u>7</u>	<u>3</u>	<u>6</u>	<u>4</u>
Women	<u>13</u>	<u>8</u>	<u>6</u>	<u>5</u>

(4)

Top 3 grade A topics I need to study further are:



Top 3 grade A* topics I need to study further are:

