Pure Mathematics Revision Topics

*Revision for GCSE A**

&

Quick start for GCE AS Core 1

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ACCURACY

You may be asked to give answers correct to so many decimal places or so many significant figures.

Decimal Places

<u>Examples</u>

i. 3.7463 to 2 decimal places

3.74|63

if this number is 5 or over, round up

3.75 i.e., 3.7463 is closer to 3.75 than 3.74

ii. 0.0634 to 3 decimal places

0.063|4

0.063

iii. To 2 decimal places

84.73|9 = 84.74

0.01|99 = 0.02

6.10|4 = 6.10

0.99|9 = 1.00

The zeros at the end MUST be included to show you have corrected to 2 dp

Significant Figures

Start counting the significant figures from the first non-zero digit. The rule for rounding up or not is the same as for decimal places.

<u>Examples</u>

i. 33.762 to 4 significant figures

33.76|2

33.76

ii. 10.076 to 3 significant figures

10.0|76

10.1

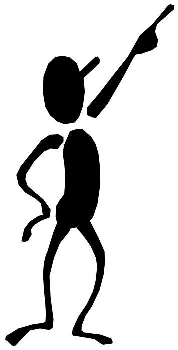
iii. to 3 significant figures:

128|.4 = 128

6.09|3 = 6.09

0.0149|8 = 0.0150

In Pure Mathematics we always give answers to 3 significant figures unless otherwise stated. Obviously, in practical work this may not be the case as the degree of accuracy depends on how accurate your measurements are.



IMPORTANT

If you are to give an answer to 3 significant figures it is very important that any intermediate answers you obtain are given to at least <u>4</u> significant figures.

If you use the Memory on your calculator efficiently this should not be a problem, but if you write intermediate answers down, *take care.*

<u>Exercise 1</u>

1. Write the following numbers correct to 2 decimal places:

i). 8.689
ii). 26.134
iii). 0.094
iv). 0.099
v). 2.019
vi). 2.190
vii). 9.999

2. Write the following numbers correct to 3 significant figures:

i). 86.41
ii). 186.9
iii). 0.09
iv). 1.999
v). 169.86
vi). 3.004
vii). 8694
viii). 9999

3.Do you understand the difference between giving an answer as 2 or 2.00?

FRACTIONS

The following examples should refresh your memory about the rules of working with fractions and remind you of all those phrases and commands we associate with fractions.

Example 1

$\frac{1}{4}+\frac{2}{3}$ find the Lowest Common Denominator

$= \frac{3}{12}+\frac{8}{12}$ write as equivalent fractions

$= \frac{11}{12}$

Example 2

$4\frac{3}{5}-2\frac{5}{6}$ make into improper fractions, i.e., top heavy

$= \frac{23}{5}-\frac{17}{6}$

$= \frac{138}{30}-\frac{85}{30}$

$= \frac{53}{30} = 1\frac{23}{30}$

Example 3

$\frac{4}{9}\times 1\frac{1}{4}$

$= \frac{\cancel{4}}{9}\times\frac{5}{\cancel{4}}$ cancel when appropriate

$= \frac{5}{9}$

Example 4

$3\frac{1}{2}\div 2\frac{1}{4}$

$= \frac{7}{2}\div\frac{9}{4}$

$= \frac{7}{\cancel{2}}\times\frac{\cancel{4}^{2}}{9}$ turn second fraction upside down and multiply

$= \frac{14}{9} = 1\frac{5}{9}$

Example 5

$$\left(\frac{3}{8}+\frac{1}{4}\right)\times 2\frac{1}{2}$$ do the part in brackets first

$$= \left(\frac{3}{8}+\frac{2}{8}\right)\times 2\frac{1}{2}$$

$$= \frac{5}{8}\times\frac{5}{2}$$

$$= \frac{25}{16} = 1\frac{9}{16}$$

Exercise 2

Carry out the following without using a calculator and then check your answers using the fraction key on your calculator.

1. $\frac{3}{5}+\frac{3}{4}$

2. $\frac{1}{3}+\frac{1}{5}$

3. $\frac{2}{5}-\frac{1}{4}$

4. $\frac{3}{8}-\frac{1}{6}$

5. $\frac{2}{5}\times\frac{7}{8}$

6. $\frac{3}{4}\times 1\frac{1}{2}$

7. $\frac{3}{5}\div\frac{2}{5}$

8. $1\frac{1}{2}\div 2\frac{1}{2}$

9. $2\frac{1}{3}+4\frac{1}{2}$

10. $\left(\frac{2}{5}\times\frac{1}{4}\right)\div\frac{1}{2}$

11. $\left(\frac{1}{3}-\frac{1}{4}\right)\times\left(2\frac{3}{4}-1\frac{1}{5}\right)$

12. $3\frac{1}{3}\div\left(\frac{1}{4}+\frac{1}{8}\right)$

Note: It is important you can do fractions both with and without a calculator.

Example

Express as a single fraction $\frac{(x-1)}{2}-\frac{(2x+1)}{6}$

The LCM of 2 and 6 is 6:

$\frac{(x-1)}{2}-\frac{(2x+1)}{6}$

$=\frac{3(x-1)}{6}-\frac{1(2x+1)}{6}$ as equivalent fraction

$=\frac{3(x-1)-(2x+1)}{6}$ as a single fraction

$=\frac{3x-3-2x-1}{6}$ multiplying out brackets

$=\frac{x-4}{6}$ collecting like terms

Exercise 3

Express each of the following as a single fraction and simplify where possible:

1. $\frac{2x}{5}+\frac{x}{5}$
2. $\frac{2a}{3}+\frac{a}{3}$
3. $\frac{x}{2}-\frac{x}{4}$
4. $x+\frac{3x}{4}$
5. $\frac{x}{4}+\frac{x}{3}$
6. $\frac{2x}{5}-\frac{x}{3}$
7. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}$
8. $\frac{3a}{4}+\frac{a}{3}+\frac{5a}{6}$
9. $\frac{y}{6}+\frac{y}{2}-\frac{y}{3}$
10. $\frac{2y}{5}-\frac{y}{3}$
11. $\frac{1}{x}+\frac{2}{x}$
12. $\frac{(5-2x)}{6}+\frac{(4x-1)}{3}$
13. $\frac{4x}{5}-\frac{(x+3)}{2}$
14. $\frac{2}{3y}+\frac{5}{6y}-\frac{1}{y}$
15. $\frac{1}{4a}+\frac{2}{5a}-\frac{1}{2a}$

INDICES

$$3^7$$

BASE NUMBER → 3; INDEX NUMBER (or POWER) → 7

RULES OF INDICES

- $3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5$ so $\boldsymbol{a^m \times a^n = a^{m+n}}$
- $7^6 \div 7^2 = \dfrac{7\times7\times7\times7\times7\times7}{7\times7} = 7^4$ so $\boldsymbol{a^m \div a^n = a^{m-n}}$

  $$\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$$

- $(4^5)^3 = 4^5 \times 4^5 \times 4^5 = 4^{5+5+5} = 4^{15}$ so $\boldsymbol{(a^m)^n = a^{mn}}$
- $\dfrac{2^3}{2^3} = \dfrac{8}{8} = 1$

  also, $\dfrac{2^3}{2^3} = 2^{3-3} = 2^0$

  $\therefore\ 2^0 = 1$ so $\boldsymbol{a^0 = 1}$

Examples

1. $p^3 \times p = p^{3+1} = p^4$ *NB*: $p = p^1$

2. $(2x)^2 \times (3x)^3 = 4x^2 \times 27x^3$
   $= 108x^5$

3. $\left(2\frac{1}{3}\right)^4 = \left(\frac{7}{3}\right)^4 = \frac{7^4}{3^4} = \frac{2401}{81}$

Exercise 4

Simplify:

1.	$x \times x \times x \times x$	2.	$a \times a \times a \times a \times a \times a$
3.	$4 \times a \times a \times a$	4.	$6 \times a \times a \times 5 \times b \times b$
5.	$x^3 \times x^7$	6.	$y^3 \times y^7 \times y^2$
7.	$3y^2 \times 4y^5$	8.	$2y^2 \times 3y^2 \times 4y^4$
9.	$3a^3 \times 4b^2 \times 5b^2 \times a^2$	10.	$6a^3 \times 4b^2 \times 2a^2b^3$
11.	$5p^3 \times 2p^2q^3 \times q^4 \times 3q^2$	12.	$15p^5 \div 3p^2$
13.	$16(p^2)^2 \div 4p^4$	14.	$2a^3 \times 4ab^2 \times 6bc^3 \times c^4$
15.	$12a^{15} \div (a^2)^4$	16.	$3a^2bc^3 \times 4a^2bc \div 2a^2b^4c^2$

EVALUATING EXPRESSIONS WITH INDICES

- A fractional power indicates a root

  e.g a power of $\frac{1}{2}$ means 'square root' so $25^{\frac{1}{2}} = \sqrt{25} = 5$

  e.g a power of $\frac{1}{3}$ means 'cube root' so $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

- A negative power indicates a reciprocal

  e.g 6^{-2} means $\frac{1}{6^2} = \frac{1}{36}$

  e.g 5^{-3} means $\frac{1}{5^3} = \frac{1}{125}$

Examples

1. $100^{\frac{1}{2}} = \sqrt{100} = 10$

2. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

3. $144^{-\frac{1}{2}} = \frac{1}{144^{\frac{1}{2}}} = \frac{1}{12}$

4. $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2^3 = 8$

NB: on C1 Module, you will NOT have a calculator.

Exercise 5

Evaluate:

1.	$1000^{\frac{1}{3}}$	2.	$81^{\frac{1}{2}}$
3.	$32^{\frac{1}{5}}$	4.	12^{-2}
5.	2^{-6}	6.	10^{-4}
7.	$9^{-\frac{1}{2}}$	8.	$1000000^{-\frac{1}{3}}$
9.	$81^{-\frac{1}{4}}$	10.	$25^{\frac{3}{2}}$
11.	$4^{\frac{5}{2}}$	12.	$32^{\frac{3}{5}}$
13.	$27^{\frac{2}{3}}$	14.	$1000^{-\frac{2}{3}}$
15.	$16^{-\frac{7}{4}}$	16.	$64^{-\frac{2}{3}}$

SURDS

A surd is an IRRATIONAL ROOT

e.g., $\sqrt{2}$, $\sqrt{3}$, etc., but not $\sqrt{4}$ because $\sqrt{4} = 2$

SIMPLIFY SURDS

- $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

e.g., $\sqrt{80} = \sqrt{(16 \times 5)} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

e.g., $\sqrt{5\dfrac{4}{9}} = \sqrt{\dfrac{49}{9}} = \dfrac{\sqrt{49}}{\sqrt{9}} = \dfrac{7}{3}$

- $a\sqrt{c} + b\sqrt{c} = (a + b)\sqrt{c}$

e.g., $\sqrt{75} + 2\sqrt{48} - 5\sqrt{12}$	$= \sqrt{(25 \times 3)} + 2\sqrt{(16 \times 3)} - 5\sqrt{(4 \times 3)}$
	$= \sqrt{25} \times \sqrt{3} + 2(\sqrt{16} \times \sqrt{3}) - 5(\sqrt{4} \times \sqrt{3})$
	$= 5\sqrt{3} + 8\sqrt{3} - 10\sqrt{3}$
	$= 3\sqrt{3}$

Exercise 6

Simplify:

1. $\sqrt{27}$
2. $\sqrt{45}$
3. $\sqrt{162}$
4. $\sqrt{48}$
5. $\sqrt{75}$
6. $\sqrt{147}$
7. $\sqrt{567}$
8. $\sqrt{112}$
9. $\dfrac{\sqrt{12}}{2}$
10. $\dfrac{\sqrt{98}}{7}$
11. $\dfrac{\sqrt{18}}{\sqrt{2}}$
12. $\dfrac{\sqrt{27}}{\sqrt{3}}$
13. $\sqrt{12} + 3\sqrt{75}$
14. $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$
15. $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$
16. $5\sqrt{6} - \sqrt{24} + \sqrt{295}$
17. $\sqrt{63} - 2\sqrt{28} + \sqrt{175}$

Rationalising the Denominator

<u>Examples</u>

1. $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ i.e., multiply top and bottom by √3

$= \dfrac{\sqrt{3}}{3}$

2. $\dfrac{1}{4\sqrt{2}} = \dfrac{\sqrt{2}}{4\sqrt{2}\times\sqrt{2}}$

$= \dfrac{\sqrt{2}}{4\times 2}$

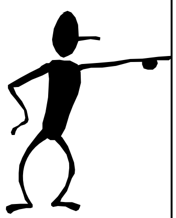
$= \dfrac{\sqrt{2}}{8}$

<u>Exercise 7</u>

Rationalise the denominators:

1. $\dfrac{1}{\sqrt{2}}$
2. $\dfrac{1}{\sqrt{7}}$
3. $\dfrac{7}{\sqrt{5}}$
4. $\dfrac{\sqrt{2}}{3\sqrt{3}}$
5. $\dfrac{\sqrt{8}}{\sqrt{32}}$
6. $\dfrac{\sqrt{5}}{\sqrt{45}}$
7. $\dfrac{\sqrt{3}}{\sqrt{21}}$
8. $\dfrac{\sqrt{11}}{\sqrt{132}}$

SUBSTITUTION



Substitution into Formulae

Don't be tempted to skip this section. Many errors are due to careless substitution or misunderstanding of substitution.

Don't be tempted to just write down an answer – you need to practise first to gain understanding and confidence.

REMEMBER

$2x^2$ means 2 lots of x squared so if $x = 5$, $2x^2$ = 2 lots of 25 = 50

BUT

$(2x)^2$ means the term $2x$ is squared

so if $x = 5$, $(2x)^2 = (2 \times 5)$ all squared $= 10^2 = 100$

Example

Evaluate the following function:

A. $f(x) = x^2 - 5x + 1$ when $x = -2$

$$\begin{aligned} f(-2) &= (-2)^2 - 5x - 2 + 1 \\ &= 4 + 10 + 1 \\ &= 15 \end{aligned}$$

B. $g(x) = \dfrac{1}{2} + \dfrac{x}{3} + \dfrac{x^2}{4}$ when $x = \dfrac{1}{2}$

$$\begin{aligned} g\left(\frac{1}{2}\right) &= \frac{1}{2} + \frac{\frac{1}{2}}{3} + \frac{\left(\frac{1}{2}\right)^2}{4} \\ &= \frac{1}{2} + \frac{1}{6} + \frac{1}{16} \\ &= \frac{24}{48} + \frac{8}{48} + \frac{3}{48} \\ &= \frac{35}{48} \end{aligned}$$

Exercise 8

1. If $x = -5$, $y = 2$ and $z = 3$, evaluate the following:

a) $10 - x^2$ b) $(2x)^2 - 3y^2$ c) xy^2

d) $-7 - 2x$ e) $4xyz - \dfrac{xy}{4z}$ f) $\dfrac{(4y-z)^2}{(4y-z^2)}$

2. The volume of a box is given by $(x + 3)(2x - 1)(x - 4)$.
Find the volume if $x = 7$ cm.

3. One of the equations of motion when acceleration is constant can be written $v^2 = u^2 + 2as$. Find v if u = 2.3, a = 0.8 and s = 28.6.

4. If $C = \dfrac{5}{9}(F - 32)$ is the formula which changes Fahrenheit into Centigrade, find the Centigrade equivalent of 102^{o}F to the nearest degree.

5. When $x = -2$ find the value of the following:

a) $x^2 + 3x$ b) $x^2 - 4x - 2$ c) $3x^2 - (2x)^2$ d) $12 - x^3$

6. $p = -1$ $q = 2$ $r = -3$

a) $5 - (p^2 + 1) =$ b) $2p - 2(q^2 + r^2) =$ c) $(3pq)^2 =$

d) $\dfrac{(r-q)}{p} + \dfrac{(3q-r)}{2p} =$

7. Work out the value of $x^3 + 3x^2 - 2x - 16$ when

i) $x = 0$ ii) $x = 1$ iii) $x = 2$ iv) $x = 3$ v) $x = -1$

8. What value of x makes the expression $x^3 - 2x - 21$ equal to zero?

REMOVING BRACKETS

Example 1
Remove the brackets and simplify:

a) $3(2x - y) - 2(x - 4y)$ b) $a(a + b) - b(a + 2b)$

a) $3(2x - y) - 2(x - 4y) = 6x - 3y - 2x + 8y$
$= 4x + 5y$

b) $a(a + b) - b(a + 2b) = a^2 + ab - ba - 2b^2$
$= a^2 - 2b^2$ (since $ba = ab$)

Example 2
Expand $(3x - 4)(2x - 3)$

$$(3x - 4)(2x - 3) = 6x^2 - 8 - 9x + 12$$

$\therefore$ $(3x - 4)(2x - 3) = 6x^2 - 17x + 12$

Exercise 9

Remove the brackets and simplify:

1. $4(5x + 2y)$
2. $3(2a - 4b)$
3. $-2(3p - 6q)$
4. $5(x - 3y + 2z)$
5. $-(a - b - c)$
6. $x(3x - y + 2z)$
7. $-a(a + b - c)$
8. $x(2x^2 + 3x + 2)$
9. $4(x + y) - 3(x - y) + 2(x - y)$
10. $3xy(x + y) + 2x(xy - y^2)$

Remove the brackets from the following:

11. $(2x + 5)(2x - 5)$
12. $(3x - 4)(3x + 4)$
13. $(x + 2)(x + 2)$
14. $(x - 1)(x - 1)$
15. $(3x + 1)(3x + 1)$
16. $(2x - 3)(2x - 3)$
17. $(2x + 1)(x + 4)$
18. $(3x + 2)(x - 2)$
19. $(2x + 1)(3x + 2)$
20. $(3x - 2)(4x + 3)$

FACTORISING

Common Factors

Example 1

Factorise $6p + 3q + 9r$

The factor which common to each term is 3

$\therefore\ 6p + 3q + 9r = 3(2p + q + 3r)$

Example 2

Factorise $x^2 + xy + 6x$

The factor which is common to each term is x

$\therefore\ x^2 + xy + 6x = x(x + y + 6)$

Example 3

Factorise $2x^2 - 4xy$

This expression has more than one common factor.
Both 2 and x are common factors.

$\therefore\ 2x^2 - 4xy = 2x(x - 2y)$

$2x^2 - 4xy$ therefore has three factors: 2, x and $(x - 2y)$

Exercise 10

Factorise the following expressions:

1. $4x + 12y$
2. $3p - 6q$
3. $5a + 10$
4. $10b - 5$
5. $14m - 21n$
6. $12s + 20t$
7. $xy + xz$
8. $xy + x^2$
9. $y^2 - 2y$
10. $2y^2 - 4y$
11. $6xy + 3x$
12. $5x - 10x^2$
13. $2a + 8b - 4c$
14. $x^2y + xyz + xy^2$
15. $4pqr - 12p^2q$

Difference of Two Squares

Example 1

Factorise $x^2 - 9$

$x^2 - 9 = (x - 3)(x + 3)$

Example 2

Factorise $9x^2 - 16$

$$9x^2 - 16 = (3x)^2 - (4)^2$$
$$= (3x - 4)(3x + 4)$$

Example 3

Factorise $8x^2 - 2$

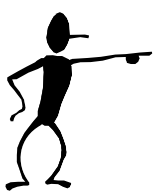
$8x^2$ and 2 are not perfect squares, but 2 is a common factor of the expression.

$8x^2 - 2 = 2(4x^2 - 1)$

$4x^2 - 1 = (2x)^2 - 1^2$ which is the difference of two squares

$\therefore\ 8x^2 - 2 = 2(2x - 1)(2x + 1)$

(*Note*: If a quadratic has a common factor, always take out the common factor before factorising the quadratic into two brackets.)



Exercise 11

Factorise:

1. $x^2 - 1$
2. $x^2 - 16$
3. $x^2 - 25$
4. $4x^2 - 9$
5. $9x^2 - 1$
6. $16x^2 - 25$
7. $49 - x^2$
8. $36 - 25x^2$
9. $25 - 49x^2$
10. $2x^2 - 8$
11. $9x^2 - 36$
12. $12x^2 - 75$

Factorising Quadratic Expressions

When the coefficient of x^2 is unity

<u>Example 1</u>

Factorise $x^2 + 5x + 6$

Reversing the process of multiplying out brackets, we can see that

$x^2 + 5x + 6 = (x + ?)\,(x + ?)$

and 6 must factorise as 6×1 or 3×2.

The key to factorising the quadratic expression is the x term.

$5x$ is the sum of two x terms and the coefficients of these two terms must be the factors of 6.

$3 \times 2 = 6$ and $3x + 2x = 5x$

$\therefore\; x^2 + 5x + 6 = (x + 3)\,(x + 2)$

<u>Example 2</u>

Factorise $x^2 - x - 6$

The factors of -6, when added together, must equal the coefficient of x, which is -1.

The factors of -6 are -6×1, 6×-1, -3×2, 3×-2.
Only $-3 + 2 = -1$

$\therefore\; x^2 - x - 6 = (x - 3)(x + 2)$

<u>Example 1</u>

Factorise $x^2 + 6x$

This is a much simpler quadratic expression to factorise than the standard type because it has a common factor which is x.

$x^2 + 6x = x(x + 6)$

<u>Exercise 12</u>

Factorise the following:

1. $x^2 + 8x + 7$
2. $x^2 + 7x + 10$
3. $x^2 + x - 6$
4. $x^2 - 5x + 6$
5. $x^2 - 7x$
6. $x^2 + 2x - 8$
7. $x^2 - 2x - 15$
8. $x^2 - 6x + 9$
9. $2x^2 + 6x$
10. $x^2 - 4x - 12$
11. $x^2 - 7x + 12$
12. $x^2 + 11x - 12$
13. $6x^2 - 3x$
14. $x^2 - 5x - 6$
15. $x^2 - 6x - 16$
16. $x - 2x^2$
17. $x^2 + 12x + 36$
18. $x^2 - 8x + 16$

When the coefficient of x^2 is not unity

Example

Factorise $2x^2 + 11x + 12$

The method previously used must be adapted to take into account the coefficient 2.

As 2 will only factorise as 2 × 1, it can be see that

$2x^2 + 11x + 12 = (2x + a)(x + b)$

As before, $a \times b = 12$, but now we require

$a + 2b = 11$ (not $a + b$ as before)

Factors of 12 are $a \times b$		and $a + 2b$	
	$= 1 \times 12$		$= 1 + 24 = 25$
	2×6		$= 2 + 12 = 14$
	3×4		$= 3 + 8 = 11$
	4×3		$= 4 + 6 = 10$
etc.			

$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$

Alternatively list all possible solutions and select the one which 'works'.

$(2x + 1)(x + 12) = 2x^2 + 25x + 12$
$(2x + 2)(x + 6) = 2x^2 + 14x + 12$
$(2x + 3)(x + 4) = 2x^2 + 11x + 12$

Exercise 13

Factorise:

1. $2x^2 + 5x + 2$
2. $2x^2 + 7x + 6$
3. $2x^2 + 9x - 5$
4. $2x^2 - 13x - 7$
5. $3x^2 + 8x + 5$
6. $3x^2 - 6x - 9$
7. $3x^2 - 8x + 4$
8. $3x^2 - 13x - 10$
9. $2x^2 - 11x + 12$
10. $3x^2 + 20x + 12$
11. $3x^2 - 22x - 16$
12. $2x^2 - 3x - 14$

COMPLETING THE SQUARE

Some expressions can be factorised as $(x+a)^2$ or $(x-a)^2$

e.g. x^2+6x+9 = $(x+3)^2$ - check that you agree!!

$x^2-10x+25$ = $(x-5)^2$

These expressions are called perfect squares.

For expressions which are not perfect squares, we 'complete the square', which means adjusting the constant term:

<u>Example 1</u>

Express $x^2+6x+11$ in the completed square form $(x+a)^2+b$.

Firstly, halve the coefficient of the x term for inside the bracket:

so we have $(x+3)^2$

If we multiply out, this gives x^2+6x+9 so we need to add 2 to obtain $x^2+6x+11$.

So $x^2+6x+11$ = $(x+3)^2+2$

<u>Example 2</u>

Express $x^2-10x+13$ in the completed square form $(x-a)^2+b$.

Again, halve the coefficient of the x term to give $(x-5)^2$

If we multiply out, this gives $x^2-10x+25$ so we need to subtract 12 to obtain $x^2-10x+13$.

So $x^2-10x+13$ = $(x-5)^2-12$

In general terms, the formula for completing the square for x^2+px+q is:

$$\left(x+\frac{1}{2}p\right)^2-\left(\frac{1}{2}p\right)^2+q$$

Exercise 14

Write the following in completed square form:

1. $x^2 + 8x + 7$
2. $x^2 + 6x + 10$
3. $x^2 + 2x - 6$
4. $x^2 - 4x + 6$
5. $x^2 - 8x$
6. $x^2 - 2x - 15$
7. $x^2 - 10x + 9$
8. $x^2 + 6x - 5$
9. $x^2 - 4x - 12$
10. $x^2 - 8x + 11$
11. $x^2 - 3x$
12. $x^2 + 12x + 100$
13. $x^2 - 6x - 16$
14. $x^2 + 5x + 7$
15. $x^2 + 11x + 30$

SOLVING LINEAR EQUATIONS

Example 1

Solve the equation $5x - 4 = 3x + 12$

In this type of equation, the x terms should be collected on one side of the equation and the numerical terms on the other, i.e., $3x$ must be eliminated from the RHS and -4 from the LHS.

$$5x - 4 = 3x + 12$$

Transposing $3x$ and -4 gives:

$$5x - 3x = 12 + 4$$
$$2x = 16$$
$$x = 8$$

Answers to algebraic equation should *always* be checked by substituting back into the LHS and RHS of the original equation, as shown below:

LHS = $5x - 4 = 5 \times 8 - 4 = 36$
RHS = $3x + 12 = 3 \times 8 + 12 = 36$
LHS = RHS, so the solution is correct.

Example 2

Solve $4(x + 3) - 2(x - 5) = 46$

$$4(x + 3) - 2(x - 5) = 46$$

Multiply out the brackets: $4x + 12 - 2x + 10 = 46$
Collect terms: $2x + 22 = 46$
Transpose 22: $2x = 24$
Divide by 2: $x = 12$

Exercise 15

Solve the following equations to find the value of x:

1. $7x + 3 = 5x + 11$
2. $5x - 2 = 2x + 7$
3. $6x - 4 = 10 - x$
4. $3 - 2x = 4 - 5x$
5. $3(x - 5) = 12$
6. $5 + 2(x + 1) = 11$
7. $2(x + 7) + 4 = 18$
8. $2(x - 4) = (x + 2)$
9. $3(2x - 1) = 7(x - 1)$
10. $(x + 5) + 2(3x - 2) = 8$
11. $4(x + 2) + 2(x + 3) = 32$
12. $2(2x + 1) - 3(3x - 4) = 40$

SOLVING QUADRATIC EQUATIONS

1. By Factorising

Example 1

Solve the equation $x^2 - 5x - 14 = 0$

$$x^2 - 5x - 14 = 0$$

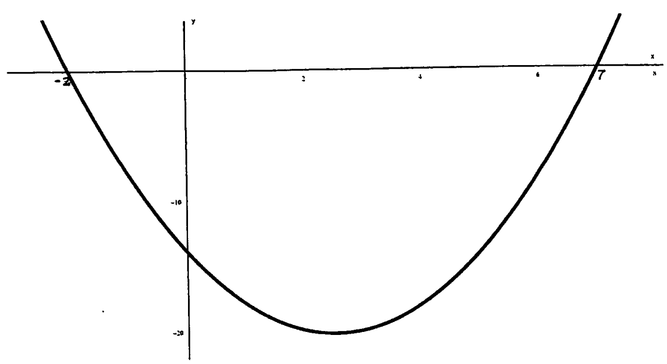
Factorising gives: $(x - 7)(x + 2) = 0$

$\therefore$ either $(x - 7) = 0$ or $(x + 2) = 0$

$\therefore$ either $x = 7$ or $x = -2$

$\therefore$ The solution is $x = 7$ or -2

This can be shown graphically:



Note: A quadratic equation always has *two* possible solutions. If the quadratic is a perfect square, the two solutions will be the same and they are called repeated roots.

Example 2

Solve the equation $2x^2 - 5x + 3 = 0$

$$2x^2 - 5x + 3 = 0$$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $(2x - 3) = 0$ or $x - 1 = 0$

$2x = 3$

$\therefore x = 1^1/_2$ or $x = 1$

Exercise 16

Solve the following equations:

1. $x^2 - 5x + 4 = 0$
2. $x^2 + 11x + 28 = 0$
3. $x^2 - 49 = 0$
4. $x^2 + 6x + 9 = 0$
5. $2x^2 + 7x + 3 = 0$
6. $x^2 + 3x = 0$
7. $3x^2 - 2x - 8 = 0$
8. $5x^2 - 2x = 0$
9. $3x^2 - 20x + 12 = 0$
10. $2x^2 + 3x - 14 = 0$
11. $2x^2 - 2x - 40 = 0$
12. $6x^2 + 3x - 3 = 0$

2. Using the Formula

To solve $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Solve $x^2 + 3x + 1 = 0$

Compare $x^2 + 3x + 1 = 0$
with $ax^2 + bx + c = 0$

Then $a = +1$, $b = +3$, $c = +1$
and substituting for *a, b* and *c* in the formula gives

$$x = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times 1}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

AS you can't use a calculator in C1, answers must be left in surd form

Example 2

Solve $2x^2 - 3x - 1 = 0$

In this equation, $a = 2$, $b = -3$, $c = -1$ and substituting in the formula gives:

$$x = \frac{+3 \pm \sqrt{9 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

Exercise 17

Solve the following equations using the quadratic formula.

1. $2x^2 + 2x - 3 = 0$
2. $2\,x^2 + 4x + 1 = 0$
3. $x^2 + 2x - 2 = 0$
4. $3\,x^2 - x - 1 = 0$
5. $x^2 + 6x - 10 = 0$
6. $x^2 - 7x + 9 = 0$
7. $4x^2 - 8x - 16 = 0$
8. $3x^2 - 6x + 2 = 0$
9. $2x^2 + 5x = 6$
10. $3x^2 - 10x = -5$
11. $x(x + 2) = 5$
12. $x\,(x - 3) = -1$

Solving Simultaneous Equations

1. Linear

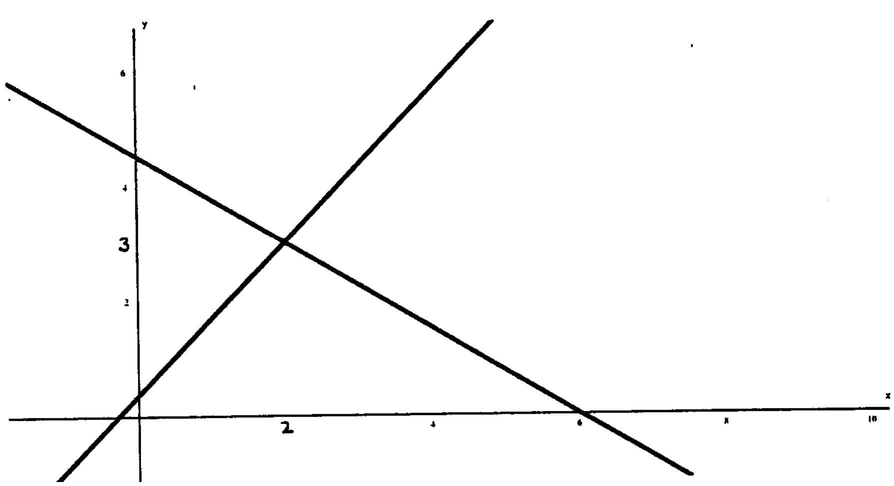
<u>Example</u>

Solve the equations $3x + 4y = 18$ (1)
$4x - 3y = -1$ (2)

Neither x nor y has the same coefficient in each equation, so this needs to be remedied first.

Method	Working
i) Decide which variable is to be eliminated.	Eliminate y
ii) Multiply one or both equations so that this variable has the same coefficient in each equation (not counting the signs).	Multiply equation (1) by 3 and equation (2) by 4 $9x + 12y = 54$ $16x - 12y = -4$
iii) Add or subtract the equations, depending on the signs of the variable to be eliminated.	As the coefficients of y have opposite signs, add the equations: $9x + 12y = 54$ $16x - 12y = -4$ $25x = 50$
iv)Solve for the remaining variable.	$x = 2$
v) Substitute into one of the original equations to find the eliminated variable and hence the complete solution.	Substitute $x = 2$ into (1) $3 \times 2 + 4y = 18$ $4y = 12$ $y = 3$ Solution is $x = 2$, $y = 3$
vi) Substitute into the other original equation to check the solution.	Substitute into (2) LHS = $4 \times 2 - 3 \times 3$ $= 8 - 9$ $= -1$ = RHS

You can see this on the following graph:



Exercise 18

Solve the following pairs of simultaneous equations:

1. $2x + y = 8$
   $x + y = 6$

2. $x + 3y = 12$
   $x = y = 10$

3. $x + y = 12$
   $x - y = 2$

4. $3x - 2y = 9$
   $x + 2y = 15$

5. $x + 3y = 6$
   $2x + y = 7$

6. $4x + 2y = 10$
   $3x + 5y = 11$

7. $3x + 2y = 18$
   $4x - y = 2$

8. $2x - 3y = 10$
   $3x + 4y = 15$

9. $x - y = -1$
   $2x + 3y = 28$

10. $7x + y = 9$
    $3x + y = -3$

11. $2x + 5y = 15\frac{1}{2}$
    $3x - 4y = -5\frac{1}{2}$

12. $6x + 5y = -17$
    $3x + 2y = -8$

2. One Linear One Non-Linear

Using Substitution method

<u>Example 1</u>

$y = 2x^2 + 5x - 3$

$5x - y + 5 = 0$

Stage 1: Make x or y the subject of the linear equation

i.e., $5x - y + 5 = 0$

$\therefore$ $y = 5x + 5$

It's easier to make y the subject here

Stage 2: Substitute this rearranged equation into the other one

$y = 2x^2 + 5x - 3$

$\therefore$ $5x + 5 = 2x^2 + 5x - 3$

Stage 3: Rearrange and solve the quadratic

$0 = 2x^2 - 8$

$0 = x^2 - 4$

$0 = (x - 2)(x + 2)$

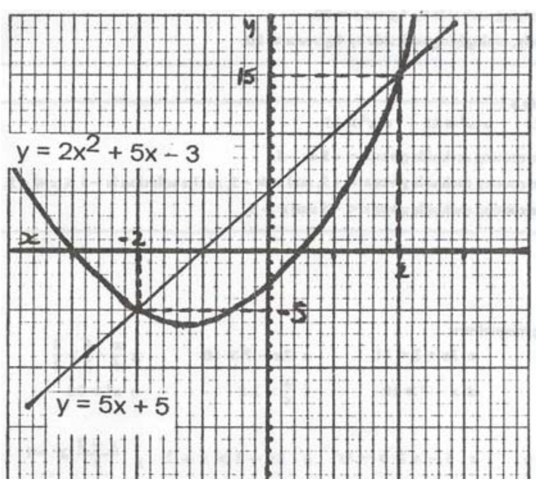
$\therefore$ $x = 2$ or $x = -2$

Stage 4: Find the corresponding values of y using the simplest equation

$x = 2$ $y = 5 \times 2 + 5 = 15$

$x = -2$ $y = 5 \times -2 + 5 = -5$

You can see this on the following graph.



Example 2

Solve simultaneously
$$y - x = 2 \quad (1)$$
$$x^2 + y^2 = 10 \quad (2)$$

$y - x = 2$ is the linear equation so re=write as $y = x + 2$

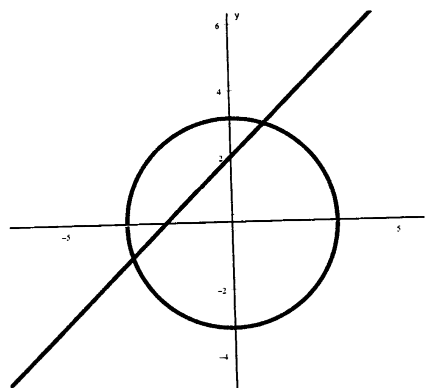
Now substitute for y in the non-linear equation

$$x^2 + (x+2)^2 = 10 \qquad \text{which is a quadratic in } x \text{ only}$$
$$x^2 + x^2 + 4x + 4 = 10$$
$$2x^2 + 4x - 6 = 0$$
$$x + 2x - 3 = 0$$
$$(x+3)(x-1) = 0 \qquad \text{so } x = -3 \text{ or } 1$$

Substitute $x = -3$ in (1) $\quad y + 3 = 2 \quad$ therefore $y = -1$

Substitute $x = 1$ in (1) $\quad y - 1 = 2 \quad$ therefore $y = 3$

So the solutions are $\underline{x = -3,\ y = -1}$ or $\underline{x = 1,\ y = 3}$



CHECK: in equation (2):

$$(-3)^2 + (-1)^2 = 9 + 1 = 10$$
$$1^2 + 3^2 = 1 + 9 = 10$$

Exercise 19

Solve the simultaneous equations:

1. $x + y = 1$, $16x^2 + y^2 = 65$

2. $2x + y = 1$, $x^2 + y^2 = 1$

3. $y - x = 2$, $2x^2 + 3xy + y^2 = 8$

4. $x - 2y = 7$, $x^2 + 4y^2 = 37$

5. $x + y = 9$, $x^2 - 3xy + 2y^2 = 0$

6. $x = 2y$, $x^2 + 3xy = 10$

7. $y = x^2 + 3$, $y = 4x$

8. $u - v = 3$, $u^2 + v^2 = 89$

9. $x + 2y = -3$, $x^2 - 2x + 3y^2 = 11$

10. $y - x = 4$, $2x^2 + xy + y^2 = 8$

ANSWERS

Exercise 1

1.
(i) 8.69
(ii) 26.13
(iii) 0.09
(iv) 0.10
(v) 2.02
(vi) 2.19
(vii) 1.00

2.
(i) 86.4
(ii) 187
(iii) 0.09
(iv) 2.00
(v) 170
(vi) 3.00
(vii) 8 690
(vii) 10 000

Exercise 2

Check these yourself.

Exercise 3

1. $\frac{3x}{5}$
2. a
3. $\frac{x}{4}$
4. $\frac{7x}{4}$
5. $\frac{7x}{12}$
6. $\frac{x}{15}$
7. $\frac{13x}{12}$
8. $\frac{a}{4}$
9. $\frac{y}{3}$
10. $\frac{y}{15}$
11. $\frac{3}{x}$
12. $\frac{2x+1}{2}$
13. $\frac{3(x-5)}{10}$
14. $\frac{1}{2y}$
15. $\frac{3}{20a}$

Exercise 4

1.	x^4	2.	a^6	3.	$4a^3$	4.	$30a^2b^2$	5.	x^{10}
6.	y^{12}	7.	$12y^7$	8.	$24y^8$	9.	$60a^5b^4$	10.	$48a^5b^5$
11.	$30p^5q^9$	12.	$5p^3$	13.	4	14.	$48a^4b^3c^7$	15.	$12a^7$
16.	$\frac{6a^2c^2}{b^2}$								

Exercise 5

1. 10
2. 9
3. 2
4. $\frac{1}{144}$
5. $\frac{1}{64}$
6. $\frac{1}{10000}$
7. $\frac{1}{3}$
8. $\frac{1}{100}$
9. $\frac{1}{3}$
10. 125
11. 32
12. 8
13. 9
14. $\frac{1}{100}$
15. $\frac{1}{128}$
16. $\frac{1}{16}$

Exercise 6

1.	$3\sqrt{3}$	2.	$3\sqrt{5}$	3.	$9\sqrt{2}$	4.	$4\sqrt{3}$	5.	$5\sqrt{3}$
6.	$7\sqrt{3}$	7.	$9\sqrt{7}$	8.	$4\sqrt{7}$	9.	$\sqrt{3}$	10.	2
11.	3	12.	3	13.	$17\sqrt{3}$	14.	$\sqrt{2}$	15.	$-4\sqrt{5}$
16.	$10\sqrt{6}$	17.	$4\sqrt{7}$						

Exercise 7

1. $\frac{\sqrt{2}}{2}$ 2. $\frac{\sqrt{7}}{7}$ 3. $\frac{7\sqrt{5}}{5}$ 4. $\frac{\sqrt{6}}{9}$ 5. ½ 6. 1/3

7. $\frac{\sqrt{7}}{7}$ 8 $\frac{\sqrt{3}}{6}$

Exercise 8

a) -15 **b)** 88 **c)** -20 **d)** 3 **e)** $-119\,^1/_6$ **f)** -25

Exercise 9

1.	$20x + 8y$	**6.**	$3x^2 - xy + 2xz$	**11.**	$4x^2 - 25$	**16.**	$4x^2 - 12x + 9$
2.	$6a - 12b$	**7.**	$-a^2 - ab + ac$	**12.**	$9x^2 - 16$	**17.**	$2x^2 + 9x + 4$
3.	$-6p + 12q$	**8.**	$2x^3 + 3x^2 + 2x$	**13.**	$x^2 + 4x + 4$	**18.**	$3x^2 - 4x - 4$
4.	$5x - 15y + 10z$	**9.**	$3x + 5y$	**14.**	$x^2 - 2x + 1$	**19.**	$6x^2 + 7x + 2$
5.	$-a + b + c$	**10.**	$5x^2y + xy^2$	**15.**	$9x^2 + 6x + 1$	**20.**	$12x^2 + x - 6$

Exercise 10

1.	$4(x + 3y)$	**6.**	$4(3s + 5t)$	**11.**	$3x\,(2y + 1)$
2.	$3(p - 2q)$	**7.**	$x\,(y + z)$	**12.**	$5x\,(1 - 2x)$
3.	$5(a + 2)$	**8.**	$x(y + x)$	**13.**	$2(a + 4b - 2c)$
4.	$5(2b - 1)$	**9.**	$y(y - 2)$	**14.**	$xy(x + z + y)$
5.	$7(2m - 3n)$	**10.**	$2y(y - 2)$	**15.**	$4pq(r - 3p)$

Exercise 11

1.	$(x - 1)(x + 1)$	**5.**	$(3x - 1)(3x + 1)$	**9.**	$(5 - 7x)(5 + 7x)$
2.	$(x - 4)(x + 4)$	**6.**	$(4x - 5)(4x + 5)$	**10.**	$2(x - 2)(x + 2)$
3.	$(x - 5)(x + 5)$	**7.**	$(7 - x)(7 + x)$	**11.**	$9(x - 2)(x + 2)$
4.	$(2x - 3)(2x + 3)$	**8.**	$(6 - 5x)(6 + 5x)$	**12.**	$3(2x - 5)(2x + 5)$

Exercise 12

1.	$(x+1)(x+7)$	**7.**	$(x+3)(x-5)$	**13.**	$3x\,(2x-1)$
2.	$(x+2)(x+5)$	**8.**	$(x-3)(x-3)$	**14.**	$(x-6)(x+1)$
3.	$(x-2)(x+3)$	**9.**	$2x\,(x+3)$	**15.**	$(x-8)(x+2)$
4.	$(x-2)(x-3)$	**10.**	$(x-6)(x+2)$	**16.**	$x\,(1-2x)$
5.	$x\,(x-7)$	**11.**	$(x-3)(x-4)$	**17.**	$(x+6)(x+6)$
6.	$(x+4)(x-2)$	**12.**	$(x+12)(x-1)$	**18.**	$(x - 4)(x - 4)$

Exercise 13

1.	$(2x+1)(x+2)$	**5.**	$(3x+5)(x+1)$	**9.**	$(2x-3)(x-4)$
2.	$(2x+3)(x+2)$	**6.**	$3(x+1)(x-3)$	**10.**	$(3x+2)(x+6)$
3.	$(2x-1)(x+5)$	**7.**	$(3x-2)(x-2)$	**11.**	$(3x+2)(x-8)$
4.	$(2x+1)(x-7)$	**8.**	$(3x+2)(x-5)$	**12.**	$(2x-7)(x+2)$

Exercise 14

1.	$(x+4)^2 - 9$	**6.**	$(x - 1)^2 - 16$	**11.**	$\left(x-\frac{3}{2}\right)^2-\frac{9}{4}$
2.	$(x+3)^2 + 1$	**7.**	$(x - 5)^2 - 16$	**12.**	$(x+6)^2 + 64$
3.	$(x+1)^2 - 7$	**8.**	$(x+3)^2 - 14$	**13.**	$(x - 3)^2 - 25$
4.	$(x-2)^2 + 2$	**9.**	$(x - 2)^2 - 16$	**14.**	$\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$
5.	$(x - 4)^2 - 16$	**10.**	$(x - 4)^2 - 5$	**15.**	$\left(x+\frac{11}{2}\right)^2-\frac{1}{4}$

Exercise 15

1.	4	**5.**	9	**9.**	4
2.	3	**6.**	2	**10.**	1
3.	2	**7.**	0	**11.**	3
4.	$^1/_3$	**8.**	10	**12.**	−5.2

Exercise 16

1.	$x = 4$ or 1	**5.**	$x = -3$ or $-^1/_2$	**9.**	$x = {}^2/_3$ or 6
2.	$x = -7$ or -4	**6.**	$= 0$ or -3	**10.**	$x = -^7/_2$ or 2
3.	$x = 7$ or -7	**7.**	$x = -^4/_3$ or 2	**11.**	$x = 5$ or -4
4.	$x = -3$ or -3	**8.**	$x = 0$ or $^2/_5$	**12.**	$x = {}^1/_2$ or -1

Exercise 17

1. $\frac{-2 \pm \sqrt{28}}{4} = \frac{-1 \pm \sqrt{7}}{2}$ note: can cancel by 2
2. $\frac{-2 \pm \sqrt{2}}{2}$
3. $-1 \pm \sqrt{3}$
4. $\frac{1 \pm \sqrt{13}}{6}$
5. $-3 \pm \sqrt{19}$
6. $\frac{7 \pm \sqrt{13}}{2}$
7. $1 \pm \sqrt{5}$
8. $\frac{3 \pm \sqrt{3}}{3}$
9. $\frac{-5 \pm \sqrt{73}}{4}$
10. $\frac{5 \pm \sqrt{10}}{3}$
11. $-1 \pm \sqrt{6}$
12. $\frac{3 \pm \sqrt{5}}{2}$

Exercise 18

1.	$x = 2$ $y = 4$	**5.**	$x = 3$ $y = 1$	**9.**	$x = 5$ $y = 6$
2.	$x = 9$ $y = 1$	**6.**	$x = 2$ $y = 1$	**10.**	$x = 3$ $y = -12$
3.	$x = 7$ $y = 5$	**7.**	$x = 2$ $y = 6$	**11.**	$x = 1\frac{1}{2}$ $y = 2\frac{1}{2}$
4.	$x = 6$ $y = 4\frac{1}{2}$	**8.**	$x = 5$ $y = 0$	**12.**	$x = -2$ $y = -1$

Exercise 19

1.	$(2, -1), (-\frac{32}{17}, \frac{49}{17})$	**5.**	$(6, 3), (4\frac{1}{2}, 4\frac{1}{2})$	**9.**	$(1, -2), (-2\frac{3}{7}, -\frac{2}{7})$
2.	$(0, 1), (\frac{4}{5}, -\frac{3}{5})$	**6.**	$(2, 1), (-2, -1)$	**10.**	$(-1, 3), (-2, 2)$
3.	$(-2, 0), (\frac{1}{3}, \frac{7}{3})$	**7.**	$(3, 12), (1, 4)$		
4.	$(1, -3), (6, -\frac{1}{2})$	**8.**	$(8, +5),(-5, -8)$		

From:
http://www.uctc.e-sussex.sch.uk/maths_resources
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