MEI STRUCTURED MATHEMATICS

A Credit Accumulation Scheme

for

Advanced Mathematics

Contents

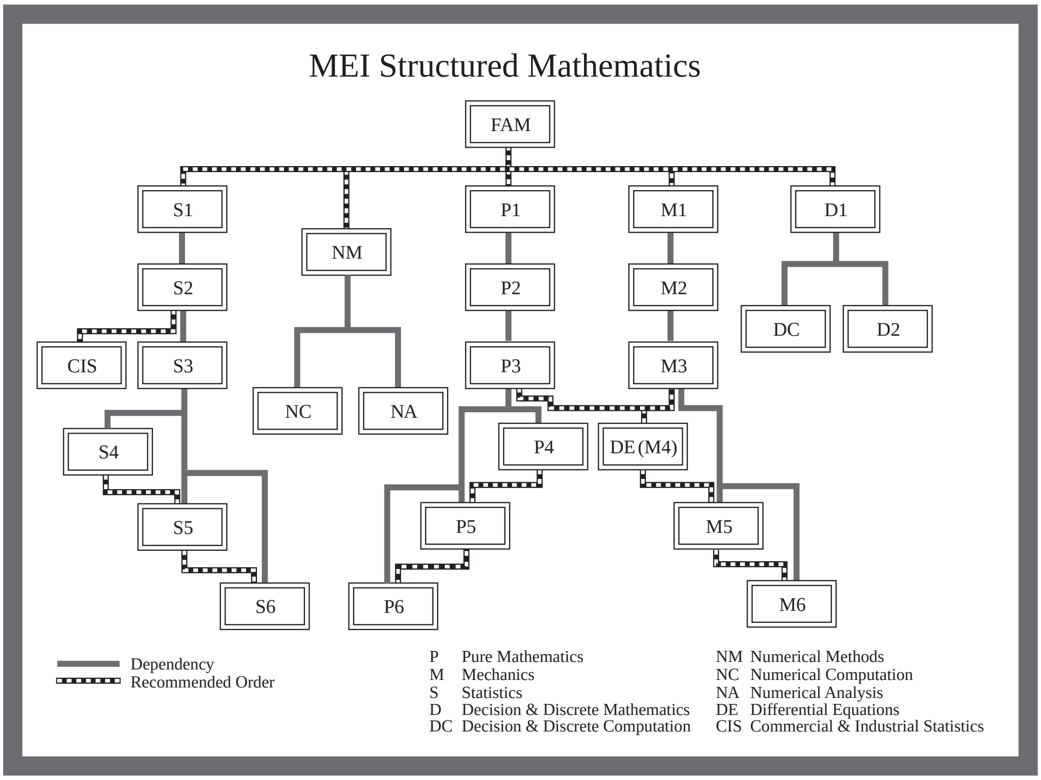
This syllabus was devised by Mathematics in Education and Industry (MEI) and is administered by OCR

MEI STRUCTURED MATHEMATICS

SPECIFICATION SUMMARY

The modules in MEI Structured Mathematics provide a step-by-step route of progression through the subject, from Intermediate Tier GCSE into the first year of university.

Suitable combinations of 3 and 6 units give rise to AS and Advanced GCEs in Mathematics, Further Mathematics and other related subject titles.



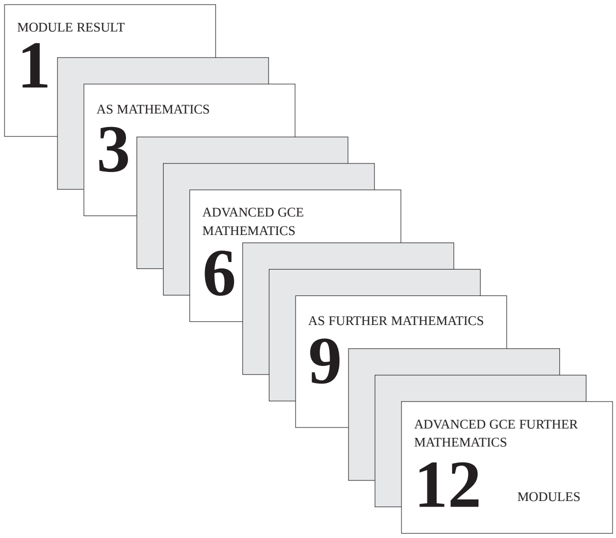
The term module describes specified teaching and learning requirements. The term unit describes a unit of assessment. Each teaching and learning module is assessed by its associated unit of assessment.

In the diagram each box represents one module. There are three types of modules.

pre-AS	Foundations of Advanced Mathematics (FAM)
AS	Pure Mathematics 1, Mechanics 1, Statistics 1, Decision & Discrete Mathematics 1
A2	All other modules.

The diagram also shows dependency and recommended order. The meanings of these terms are described in Section 5.1.2

About the Course



* Each module is expected to take about 45 hours of teaching time.
* The normal method of assessment is by unit examinations, lasting 1 hour 20 minutes.
* Examinations are held in January and June each year.
* Certain units (for example Statistics 1) also have coursework requirements.
* Candidates are allowed to resit units once before seeking an Advanced Subsidiary GCE or Advanced GCE award, with the better mark standing.
* Students usually take their units at different stages through their course, accumulating credit as they do so.
* AS and Advanced GCE certification is available in Mathematics and Further Mathematics, as well as in related subjects such as Statistics.
* The units chosen by students must conform to rules ensuring: coverage of the Subject Criteria; mathematical consistency; a balance between pure and applied mathematics.
* The course is supported by text books and INSET is available.
* Several modules are also available as GNVQ Units.
* The course provides opportunities for developing and using Key Skills.

These specifications meet all the requirements of the Common Criteria (QCA, 1999), the GCE Advanced Subsidiary and Advanced Level Qualification-specific criteria (QCA, 1999) and the Subject Criteria for Mathematics (QCA, 1999).

These specifications will comply in all respects with the revised Code of Practice requirements for courses starting in September 2000.

1 INTRODUCTION

1.1 Rationale

1.1.1 Background

MEI Structured Mathematics was first introduced in 1990 and was then refined in 1994 to take account of a new subject core and advice from teachers and lecturers. This specification is a further refinement, made in the light of 1999 Subject Criteria and various QCA regulations.

The ideas underlying this specification were set out in the introduction to the 1990 syllabus. Since these ideas have not changed since then, this introduction is reproduced verbatim in the next section, 1.1.2.

In the first place MEI Structured Mathematics was devised for AS and Advanced GCE students but it has become increasingly clear that such a set of modules will also serve other purposes: Credit Transfer and Accreditation of Prior Learning, for example into Higher Education; contributing to the mathematics element in other courses, such as a General National Vocational Qualification (GNVQ); as the basis for other mathematical qualifications.

1.1.2 Broad Aims and Objectives: Introduction to the 1990 syllabus

“Our decision to develop this structure, based on 45-hour Components, for the study of Mathematics beyond GCSE stems from our conviction, as practising teachers, that it will better meet the needs of our students. We believe its introduction will result in more people taking the subject at both A and AS, and that the use of a greater variety of assessment techniques will allow content to be taught and learnt more appropriately with due emphasis given to the processes involved.

Mathematics is required by a wide range of students, from those intending to read the subject at university to those needing particular techniques to support other subjects or their chosen careers. Many syllabuses are compromises between these needs but the necessity to accommodate the most able students results in the content being set at a level which is inaccessible to many, perhaps the majority of, sixth formers. The choice allowed within this scheme means that in planning courses centres will be able to select those Components that are relevant to their students’ needs, confident that the work will be at an appropriate level of difficulty.

While there are some areas of Mathematics which we feel to be quite adequately assessed by formal examination, there are others which will benefit from the use of alternative assessment methods, making possible, for example, the use of computers in Numerical Analysis and of substantial sets of data in Statistics. Other topics, like Modelling and Problem Solving, have until now been largely untested because by their nature the time they take is longer than can be allowed in an examination. A guiding principle of this scheme is that each Component is assessed in a manner appropriate to its content.

We are concerned that students should learn an approach to Mathematics that will equip them to use it in the adult world and to be able to communicate what they are doing to those around them. We believe that this cannot be achieved solely by careful selection of syllabus content and have framed our Coursework requirements to develop skills and attitudes which we believe to be important. Students will be encouraged to undertake certain Coursework tasks in teams and to give presentations of their work. To further a cross-curricular view of Mathematics we have made provision for suitable Coursework from other subjects to be admissible.

We believe that this scheme will do much to improve both the quantity and the quality of Mathematics being learnt in our schools and colleges."

1.1.3 Progression

The scheme is thus designed not just to be a specification for AS or Advanced GCE Mathematics but to provide a route of progression through the subject starting at Intermediate Tier GCSE and going into what is first year work in some university courses.

The specification is also, by design, entirely suitable for those who are already in employment, or are intending to progress directly into it.

1.2 Certification titles

Advanced Subsidiary GCE and Advanced GCE certificates are available in the following subject titles.

Advanced Subsidiary GCE	
OCR Advanced Subsidiary GCE in Mathematics (MEI)	3850
OCR Advanced Subsidiary GCE in Further Mathematics (MEI)	3856
OCR Advanced Subsidiary GCE in Further Mathematics (*Additional*) (MEI)	3857
OCR Advanced Subsidiary GCE in Pure Mathematics (MEI)	3852
OCR Advanced Subsidiary GCE in Mechanics (MEI)	3854
OCR Advanced Subsidiary GCE in Statistics (MEI)	3853
OCR Advanced Subsidiary GCE in Discrete Mathematics (MEI)	3855
OCR Advanced Subsidiary GCE in Applied Mathematics (MEI)	3851
Advanced GCE	
OCR Advanced GCE in Mathematics (MEI)	7850
OCR Advanced GCE in Further Mathematics (MEI)	7856
OCR Advanced GCE in Further Mathematics (*Additional*) (MEI)	7857
OCR Advanced GCE in Pure Mathematics (MEI)	7852
OCR Advanced GCE in Statistics (MEI)	7853

SECTION 2 SPECIFICATION AIMS

2.1 Aims of MEI

'To promote the links between Education and Industry in Mathematics at Secondary School Level, and to produce relevant examination and teaching syllabuses and support material.'

2.2 Aims of this scheme

The course should encourage students to:

a) develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;

b) develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;

c) extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;

d) develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;

e) recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;

f) use mathematics as an effective means of communication;

g) read and comprehend mathematical arguments and articles concerning applications of mathematics;

h) acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;

i) develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;

j) take increasing responsibility for their own learning and the evaluation of their own mathematical development.

2.3 Key Skills

In accordance with the aims of MEI, this scheme has been designed to meet the request of industry (for example the CBI) that students be provided with opportunities to use and develop key skills. This was described in paragraph 4 of the introduction to the 1990 syllabus reproduced on page 5. The table below indicates which modules are particularly likely to provide opportunities for the various Key Skills at level 3.

Key Skill	element	P2	P3	M2	DE	S1	S2	CIS	D1	D2	DC	NM	NA	NC	FAM
Communication (C)	1a			✓	✓	✓	✓	✓	✓	✓		✓	✓		
	1b	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓		
	2		✓					✓							
	3	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓		
I. T. (IT)	1	✓			✓			✓	✓	✓	✓	✓	✓	✓	
	2	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
	3	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Application of Number (N)	1	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓		✓
	2	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓		✓
	3	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓		✓
Problem Solving (PS)	1			✓	✓	✓	✓	✓	✓	✓		✓	✓		
	2			✓	✓	✓	✓	✓	✓	✓		✓	✓		
	3			✓	✓	✓	✓	✓	✓	✓		✓	✓		
Working with others (WO)	1			✓	✓	✓	✓	✓	✓	✓		✓	✓		
	2			✓	✓	✓	✓	✓	✓	✓		✓	✓		
	3			✓	✓	✓	✓	✓	✓	✓		✓	✓		
Improving own learning and performance (LP)	By its nature a modular course fosters this Key Skill and all the modules can contribute to it.														

References are also given at various points in this syllabus. Thus C 3.2 indicates an opportunity for Communication Level 3 element 2.

2.4 Spiritual, Moral, Ethical, Social and Cultural Issues

Candidates are required to examine arguments critically and so to distinguish between truth and falsehood. They are also expected to interpret the results of modelling exercises and there are times, particularly in statistical work, when this inevitably raises moral and cultural issues. Such issues will not be assessed in examination questions; nor do they feature, per se, in the assessment criteria for the various coursework tasks.

2.5 Environmental issues

While the work developed in teaching this specification may use examples, particularly involving modelling and statistics, that raise environmental issues, these issues do not in themselves form part of the specification.

2.6 Health and Safety considerations

The work developed in teaching this specification may at times involve examples that raise health and safety issues. These issues do not in themselves form part of the specification.

There are health and safety considerations in conducting coursework, as outlined in Section 4.3.

2.7 European Dimension

This does not have any direct impact on this specification.

2.8 Calculators and Computers

Students are expected to make appropriate use of graphical calculators and computers.

SECTION 3

ASSESSMENT OBJECTIVES

Candidates should be able to demonstrate that they can:

1 recall, select and use their knowledge of appropriate mathematical facts, concepts and techniques in a variety of contexts;

2 construct rigorous mathematical arguments and proofs through appropriate use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form;

3 recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models;

4 comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications;

5 use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) effectively and appropriately; understand when not to use such technology, and its limitations; give answers to appropriate accuracy.

The table below gives the permitted allocation of marks to Assessment Objectives for the various units. The figures given are percentages.

Unit number	Unit	AO 1	AO 2	AO 3	AO 4	AO 5
2601	Pure Mathematics 1	45-50	45-50	0-5	0-7	0-7
2602	Pure Mathematics 2	35-40	35-40	0-5	0-7	20-27
2603	Pure Mathematics 3	35-40	35-40	5-10	15-22	0-7
2604	Pure Mathematics 4	45-50	45-50	0-5	0-7	0-7
2605	Pure Mathematics 5	45-50	45-50	0-5	0-7	0-7
2606	Pure Mathematics 6	45-50	45-50	0-5	0-7	0-7
2607	Mechanics 1	20-25	20-25	30-35	15-22	5-12
2608	Mechanics 2	30-35	30-35	25-30	0-7	5-12
2609	Mechanics 3	30-35	30-35	25-30	0-7	5-12
2610	Differential Equations (Mechanics 4)	30-35	30-35	25-30	0-7	5-12
2611	Mechanics 5	30-35	30-35	25-30	0-7	5-12
2612	Mechanics 6	30-35	30-35	25-30	0-7	5-12
2613	Statistics 1	20-30	20-25	30-35	15-22	5-12
2614	Statistics 2	30-35	30-35	25-30	0-7	5-12
2615	Statistics 3	30-35	30-35	25-30	0-7	5-12
2616	Statistics 4	30-35	30-35	25-30	0-7	5-12
2617	Statistics 5	30-35	30-35	25-30	0-7	5-12
2618	Statistics 6	30-35	30-35	25-30	0-7	5-12
2619	Commercial & Industrial Statistics	30-35	30-35	25-30	0-7	5-12
2620	Decision & Discrete 1	20-25	20-25	30-35	15-22	5-12
2621	Decision & Discrete 2	30-35	30-35	25-30	0-7	5-12
2622	Decision & Discrete Computation	25-30	25-30	25-30	0-7	15-25
2623	Numerical Methods	35-40	35-40	0-5	0-7	20-27
2624	Numerical Analysis	35-40	35-40	0-5	0-7	20-27
2625	Numerical Computation	30-35	30-35	0-5	0-7	20-30

Percentage allocation of marks to Assessment Objectives for each module

These allocations ensure that any allowable combination of units for AS or Advanced GCE Mathematics satisfies the weightings given in *Subject Criteria for Mathematics.*

SECTION 4

4.1 SCHEME OF ASSESSMENT

4.1.1 Summary Table

Unit Number	Title	Type	Questions	Examination		Coursework Tasks
				Time	Sessions	
2601	P1	AS	Section A: 8 – 10 Section B: 2	1 hr 20 mins	1, 6	-
2602	P2	A2	4	1 hr 20 mins	1, 6	1
2603	P3	A2	Section A: 4 Section B: 1	1 hr 20 mins 1 hr	1, 6 1, 6	-
2604	P4	A2	3 from 4	1 hr 20 mins	1, 6	-
2605	P5	A2	3 from 4	1 hr 20 mins	1, 6	-
2606	P6	A2	3 from 5	1 hr 20 mins	6	-
2607	M1	AS	4	1 hr 20 mins	1, 6	-
2608	M2	A2	4	1 hr 20 mins	1, 6	1
2609	M3	A2	4	1 hr 20 mins	1, 6	-
2610	DE (M4)	A2	3 from 4	1 hr 20 mins	1, 6	1
2611	M5	A2	3 from 4	1 hr 20 mins	6	-
2612	M6	A2	3 from 4	1 hr 20 mins	6	-
2613	S1	AS	4	1 hr 20 mins	1, 6	1
2614	S2	A2	4	1 hr 20 mins	1, 6	1
2615	S3	A2	4	1 hr 20 mins	1, 6	-
2616	S4	A2	3 from 4	1 hr 20 mins	1, 6	-
2617	S5	A2	3 from 4	1 hr 20 mins	6	-
2618	S6	A2	3 from 5	1 hr 20 mins	6	-
2619	CIS	A2	3 from 4	1 hr 20 mins	6	1
2620	D1	AS	Section A: 3 Section B: 3	1 hr 20 mins	1, 6	1
2621	D2	A2	4	1 hr 20 mins	6	1
2622	DC	A2	3 from 4	2 hours	6	-
2623	NM	A2	4	1 hr 20 mins	1, 6	1
2624	NA	A2	3 from 4	1 hr 20 mins	6	1
2625	NC	A2	3 from 4	2 hours	6	-
	FAM	pre-AS	30 multiple choice	1 hr 30 mins	1, 6	-

Availability Sessions 1: January 6: June

January 2001 P1, P2, M1, S1, D1
June 2001 P1, P2, P3, P4, M1, M2, S1, S2, D1, D2, NM

Resitting For units 2602, 2608, 2610, 2613, 2614, 2619, 2620, 2621, 2623 and 2624, Centres have the option of submitting new coursework (entry code option A) or carrying forward a coursework mark from a session within the previous 12 months (option B).

4.1.2 Language

The assessment of these specifications is in English.

4.1.3 Special Circumstances

Candidates with particular assessment requirements should apply to OCR for special consideration as detailed in the *Inter-Board Regulations and Guidance Booklet for Special Arrangements and Special Consideration.*

4.2 EXAMINATIONS

4.2.1 Timing

Examinations will be held in January and June each year, but certain units are examined in June only (see page 11). Unless stated otherwise, examinations last 1 hour 20 minutes. Details of unit examinations for each year will be included in the OCR timetable.

4.2.2. Use of Language

Candidates are expected to use clear, precise and appropriate mathematical language, as described in Assessment Objective 2.

4.2.3 Calculator Regulations

In the examinations for Pure Mathematics 1 and Pure Mathematics 2 candidates are permitted to use as a calculating aid, only a scientific calculator. Computers, graphical calculators and calculators with computer algebra facilities are *not* permitted in these unit examinations.

Standard

Candidates and centres must note that each A2 unit is assessed at Advanced GCE standard and that no concessions are made to any candidate on the grounds that the examination has been taken early in the course. Centres may disadvantage their candidates by entering them for a unit examination before they are ready.

Thresholds

At the time of setting, each examination paper will be designed so that 50% of the marks are available to a grade E candidates, 75% to grade C and 100% to grade A. Grade thresholds will be set making a reasonable allowance for examination performance, and for any features of a particular paper that only become apparent after it is taken. Typically, the candidates may be expected to achieve about three quarters of the marks available to achieve a given grade: for example grade A 75%, grade C 56%, grade E 38%.

4.3 COURSEWORK

4.3.1 Rationale

The requirements of the following units include a single piece of coursework, which will count for 20% of the assessment of the unit.

Pure Mathematics 2
Mechanics 2
Differential Equations (Mechanics 4)
Statistics 1 and 2
Numerical Methods, Numerical Analysis
Decision and Discrete Mathematics 1 and 2
Commercial & Industrial Statistics

In each case the coursework covers particular skills or topics that are, by their nature, unsuitable for assessment within a timed examination but are nonetheless important aspects of their modules.

The work undertaken in coursework is thus of a different kind from that experienced in examinations. As a result of the coursework, candidates should gain a better understanding of how mathematics is applied in real-life situations.

4.3.2 Use of Language

Candidates are expected to use clear, precise and appropriate mathematical language, as described in Assessment Objective 2.

4.3.3 Guidance

Teachers should give candidates such guidance and instruction as is necessary to ensure that they understand the task they have been given, and know how to set about it. They should explain the basis on which it will be assessed. Teachers should feel free to answer reasonable questions and to discuss candidates' work with them, until the point where they are working on their final write-up.

A candidate who takes up and develops advice offered by the teacher should not be penalised for so doing. Teachers should not leave candidates to muddle along without any understanding of what they are doing; if, however, a candidate needs to be led all the way through the work, this should be taken into account in the marking, and a note of explanation written on the assessment sheet.

Candidates may discuss a task freely among themselves and may work in small groups. The final write-up must, however, be a candidate's own work. It is not expected that candidates will work in larger groups than are necessary.

Coursework may be based on work for another subject (e.g. Geography or Economics), where this is appropriate, but the final write-up must be submitted in a form appropriate for Mathematics.

In order to obtain marks for the assessment domain Oral Communication, candidates must either give a presentation to the rest of the class, have an interview with the assessor or be engaged in on-going discussion.

4.3.4 Coursework Tasks

Centres are free to develop their own coursework tasks and in that case they may seek advice from OCR about the suitability of a proposed task in relation both to its subject content and its assessment. However Centres that are new to the scheme are strongly advised to start with tasks in the MEI folder entitled *Coursework Resource Material*.

4.3.5 Moderation

Coursework is assessed by the teacher responsible for the module or by someone else approved by the Centre. It should be completed and submitted within a time interval appropriate to the task.

Consequently the teacher has two roles. While the candidate is working on coursework, the teacher may give assistance as described above. However once the candidate has handed in the final write-up the teacher becomes the assessor and no further help may be given. Only one assessment of a piece of coursework is permitted; it may not be handed back for improvement or alteration.

The coursework is assessed over a number of domains according to the criteria laid down. The method of assessment of Oral Communication should be stated and a brief report on the outcome written in the space provided on the assessment sheet.

4.3.6 Internal Standardisation

Centres that have more than one teaching group for a particular module must carry out internal standardisation of the coursework produced to ensure that a consistent standard is being maintained across the different groups. This must be carried out in accordance with guidelines from OCR. An important outcome of the internal standardisation process will be the production of a rank order of all candidates.

4.3.7 External Moderation

After coursework is marked by the teacher and internally standardised by the Centre, the marks are then submitted to OCR by a specified date, after which postal moderation takes place in accordance with OCR procedures. Centres must ensure that the work of all candidates is available for moderation.

As a result of external moderation, the coursework marks of a Centre may be changed, in order to ensure consistent standards between Centres.

4.3.8 Resitting

If a unit is re-taken, Centres are offered the option of submitting new coursework (Entry Code option A) or carrying over the coursework mark from a previous session (option B).

4.3.9 Minimum Coursework Requirements

If a candidate submits no work for the coursework component, then the candidate should be indicated as being absent from that component on the coursework Mark Sheets submitted to OCR. If a candidate completes any work at all for the coursework component then the work should be assessed according to the criteria and marking instructions and the appropriate mark awarded, which may be 0 (zero).

4.3.10 Authentication

As with all coursework, Centres must be able to verify that the work submitted for assessment is the candidates' own work.

4.3.11 Health and Safety considerations

Teachers should be aware that students may be exposed to risks when doing coursework. They should apply usual laboratory precautions when experimental work is involved. Students should not be expected to collect data on their own when outside their Centre.

Teachers should be aware of the dangers of repetitive strain injury for any student who spends a long time working on a computer.

4.4 Special Consideration

For candidates who are unable to complete the full assessment or whose performance may be adversely affected through no fault of their own, teachers should consult the *Inter-Board Regulations and Guidance Booklet for Special Arrangements and Special Consideration*. In such cases advice should be sought from OCR as early as possible during the course.

4.5 DIFFERENTIATION

In the question papers, differentiation is achieved by setting questions which are designed to assess candidates at their appropriate levels of ability and which are intended to allow all candidates to demonstrate what they know, understand and can do.

In coursework, differentiation is by task and by outcome. Candidates undertake assignments which enable them to display positive achievement.

4.6 SYNOPTIC ASSESSMENT

Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the Advanced GCE course focusing on the use and application of methods developed at earlier stages of the course to the solution of problems. Making and understanding connections in this way is intrinsic to learning mathematics.

In this specification synoptic assessment is met in three ways

(i) The specification for any module includes the content of those that precede it in the same strand. This is indicated on the scheme diagram on page 2 and also stated on the cover sheet of each module.

(ii) Applied modules assume knowledge of relevant techniques in pure mathematics up to the same level. These requirements are stated on the cover sheets of the various applied modules but, for the sake of simplicity, are not shown on the scheme diagram.

(iii) The comprehension question in Pure Mathematics 3 is synoptic across the whole of Advanced GCE mathematics in that it may draw on any of the content in Pure Mathematics 1, 2 and 3, and on the ideas which pervade applied mathematics at this level of sophistication.

At least 20% of the assessment for all Advanced GCE students will be synoptic. In order to ensure this, Candidates taking Advanced GCE Mathematics must take Pure Mathematics 3 and another A2 unit at the sitting at which they complete the Advanced GCE in Mathematics.

4.7 AWARDING OF GRADES

4.7.1 Procedures

These specifications will comply in all respects with the revised GCE Code of Practice requirements for courses starting in September 2000.

4.7.2 Entries

Candidates enter for three units of assessment at Advanced Subsidiary, followed by three units in A2 to complete the Advanced GCE.

4.7.3 Shelf-Life

Individual unit results, prior to certification of the qualification, have a shelf-life limited only by that of the qualification.

4.7.4 Resitting

A candidate may resit a unit once before seeking an Advanced Subsidiary GCE or Advanced GCE award, with the better mark standing.

4.7.5 Extra Modules

A candidate may submit more than the required number of units for a Subject award (for example 7 instead of 6 for an Advanced GCE). In that case the legal combination for that award which is the most favourable to the candidate will normally be chosen.

4.7.6 Weighting

All units are equally weighted.

4.7.7 Aggregation and Grading

Each unit is given a grade and a Uniform Mark, using procedures laid down by QCA in the document “GCE A and AS Code of Practice”. The relationship between grades and Uniform Marks follows the national scheme.

4.7.8 Final Certification

Where a candidate has requested awards in both Mathematics and Further Mathematics, OCR will adopt the following procedure:

1. determine the best Mathematics grade available to the candidate;
2. select the combination of units which allows the least total Uniform Mark to be used in achieving that grade legally;
3. determine the best Further Mathematics grade subject to 1.

Candidates will receive their final unit results at the same time as their Subject results. In common with other Advanced GCEs results, the Subject results are at this stage provisional to allow Enquiries on results. Enquiries concerning marking are made at the unit level and so only those units taken at the last sitting may be the subject of such appeals. Enquiries are subject to OCR's general regulations.

4.7.9 Exclusions

No Advanced Subsidiary GCE qualification within these specifications may be taken with any other Advanced Subsidiary GCE having the same title.

No Advanced GCE qualification within these specifications may be taken with any other Advanced GCE having the same title.

Candidates may **not** obtain certification (under any title) based on units from this scheme and certification (under any title), based on units for other Mathematics specifications at the same examination session without prior permission from OCR.

Candidates may not enter a unit from this specification and a unit with the same title from another Mathematics specification at the same examination session.

Every specification is assigned to a national classification code indicating the subject area to which it belongs. Centres should be aware that candidates who enter for more than one GCE qualification with the same classification code will have only one grade (the highest) counted for the purpose of School and College Performance Tables.

The national classification codes for these specifications are as follows.

Mathematics	2210	Applied Mathematics	2250
Mechanics	2220	Statistics	2260
Pure Mathematics	2230	Further Mathematics	2330
Discrete Mathematics	2240		

4.8 AS AND ADVANCED GCE CERTIFICATION

Subject: Mathematics

A candidate's choice of units for these awards is subject to the following restrictions

(a) Mathematics Subject Criteria

Combinations of units leading to certifications entitled Mathematics are required to cover the Mathematics subject criteria. The content of this is covered by the following compulsory modules.

Advanced Subsidiary GCE:	Pure Mathematics 1, 2.
Advanced GCE:	Pure Mathematics 1, 2, 3.

(b) Dependency

A unit may only contribute to an award in Mathematics if those units upon which it is dependent are included in that award. Dependency is indicated in the diagram on page 2.

(c) Balance

There must be a balance between pure and applied mathematics. There must be one applied unit in AS Mathematics and at least two applied units in Advanced GCE Mathematics.

One area of the applied mathematics must be addressed at AS, and at least one area at Advanced GCE.

(d) A2 Units

There must be at least 3 A2 units in Advanced GCE.

These restrictions allow the following combinations of modules.

AS Level Mathematics (3850)

P1, P2, M1	P1, P2, S1	P1, P2, D1

Advanced GCE Mathematics (7850)

All Advanced GCE Mathematics combinations include

P1, P2, P3

The remaining 3 units consist of one of the following combinations

M1, M2, M3	NM, M1, M2	DE, M1, M2
M1, M2, S1	NM, M1, S1	DE, M1, S1
M1, M2, D1	NM, M1, D1	DE, M1, D1
S1, S2, S3	NM, S1, S2	DE, S1, S2
S1, S2, CIS	NM, S1, D1	DE, S1, D1
S1, S2, M1	NM, D1, D2	DE, D1, D2
S1, S2, D1	NM, D1, DC	DE, D1, DC

D1, D2, DC	P4, M1, M2	P4, DE, M1
D1, D2, M1	P4, M1, S1	P4, DE, S1
D1, D2, S1	P4, M1, D1	P4, DE, D1
D1, DC, M1	P4, S1, S2	
D1, DC, S1	P4, S1, D1	
	P4, D1, D2	
	P4, D1, DC	

For convenience, the unit entry codes are listed below.

P1	2601	M1	2607	S1	2613	D1	2620
P2	2602	M2	2608	S2	2614	D2	2621
P3	2603	M3	2609	S3	2615	DC	2622
P4	2604	M4	2610	S4	2616		
P5	2605	M5	2611	S5	2617	NM	2623
P6	2606	M6	2612	S6	2618	NA	2624
				CIS	2619	NC	2625

FURTHER MATHEMATICS

Advanced Subsidiary GCE Further Mathematics (3856),

Advanced GCE Further Mathematics (7856),

Advanced Subsidiary GCE Further Mathematics *(Additional)* (3857)

Advanced GCE Further Mathematics *(Additional)* (7857)

9 units:	Advanced GCE Mathematics + Advanced Subsidiary GCE Further Mathematics
12 units:	Advanced GCE Mathematics + Advanced GCE Further Mathematics
15 units:	Advanced GCE Mathematics + Advanced GCE Further Mathematics + Advanced Subsidiary GCE Further Mathematics (additional)
18 units:	Advanced GCE Mathematics + Advanced GCE Further Mathematics + Advanced GCE Further Mathematics (additional)

Candidates may be expected to have obtained, or to be obtaining concurrently, an Advanced GCE in Mathematics. The award is made on the basis of further modules.

The 9 units for Advanced GCE Mathematics + Advanced Subsidiary GCE Further Mathematics must include Pure Mathematics 4 (as well as Pure Mathematics 1, 2, and 3).

The 12 units for Advanced GCE Mathematics + Advanced GCE Further Mathematics must include Pure Mathematics 4 and 5 (as well as Pure Mathematics 1, 2 and 3).

At most one AS unit may contribute to a Further Mathematics Award. A further AS unit may be included in an award of Further Mathematics (Additional).

The six units encashed for the Mathematics Advanced GCE must comply with the rules of dependency.

Please note that a separate entry is required for each AS or Advanced GCE subject to be certificated.

OTHER SUBJECT TITLES

SUBJECT: PURE MATHEMATICS

The following combinations are acceptable.

Advanced Subsidiary GCE Pure Mathematics (3852)

P1, P2, P3	P1, P2, NM

Advanced GCE Pure Mathematics (7852)

An Advanced GCE in Pure Mathematics may not be awarded in combination with any other mathematics title.

P1, P2, P3, P4

and any of the following pairs

P5, P6	NM, DE	NM, NA
P5, NM	P5, DE	NM, NC
P6, NM	P6, DE	

SUBJECT: STATISTICS

The following combinations are acceptable.

Advanced Subsidiary GCE Statistics (3853)

S1, S2, S3	S1, S2, D1	S1, S2, P1
S1, S2, CIS		

Advanced GCE Statistics (7853)

S1, S2, S3, S4

and any of the following pairs

S5, S6	S5, D1	S5, P1
S5, CIS	S6, D1	S6, P1
S6, CIS	CIS, D1	CIS, P1

OTHER AS LEVEL SUBJECTS

Advanced Subsidiary GCE Mechanics (3854)

The following combinations are acceptable

M1, M2, M3   M1, M2, DE   M1, M2, P1

Advanced Subsidiary GCE Discrete Mathematics (3855)

The following combinations are acceptable

D1, D2, DC   D1, D2, P1   D1, DC, P1

Advanced Subsidiary GCE Applied Mathematics (3851)

The AS in Applied Mathematics consists entirely of AS units. Candidates who have completed this qualification and take additional units that, together with those taken for AS Applied Mathematics, meet the requirements for Advanced GCE in Mathematics, will be entitled to an award in Advanced GCE in Mathematics.

The following combinations are acceptable

P1, M1, S1   P1, M1, D1   P1, S1, D1

For convenience, the unit entry codes are listed below.

P1	2601	M1	2607	S1	2613	D1	2620
P2	2602	M2	2608	S2	2614	D2	2621
P3	2603	M3	2609	S3	2615	DC	2622
P4	2604	M4	2610	S4	2616		
P5	2605	M5	2611	S5	2617	NM	2623
P6	2606	M6	2612	S6	2618	NA	2624
				CIS	2619	NC	2625

4.9 SCIENTIFIC CALCULATORS

The scientific calculators permitted as a calculating aid in the examinations for Pure Mathematics 1 and 2 must meet the description set out below. They must incorporate all the *Required functions*, and must possess none of the *Functions which are not permitted*.

Required functions

- add, subtract, multiply, divide
- π
- brackets
- Square, square root
- *n*th power and root
- reciprocal
- sin, cos, tan and their inverses
- degrees and radians
- logarithms and exponentials
- standard index notation
- factorial
- ${}^nC_r$
- standard deviation
- sign change
- memory
- execute/enter or =
- cancel
- clear all

Functions which are not permitted

- graph plotting
- symbolic manipulation
- memory capable of storing formulae
- memory capable of storing expressions
- equation solving
- numerical integration
- complex numbers
- vector and matrix handling

4.10 GRADE DESCRIPTIONS

The following grade descriptions indicate the level of attainment characteristic of the given grade at Advanced GCE. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performance in others.

Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language and some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation; they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed, and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations into mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

SECTION 5 SUBJECT CONTENT

5.1 Specifications

5.1.1 Assumed knowledge

(a) Overall Starting Point

There is no formal prerequisite for a student wishing to embark on MEI Structured Mathematics.

The specifications are written with the same assumption about prior knowledge as that used for the Subject Criteria, that students embarking on AS and Advanced GCE Study in Mathematics are expected to have achieved at least grade C in GCSE Mathematics or equivalent, and to have covered all the material in the Intermediate Tier. Consequently everything which is in the National Curriculum up to and including that level is also implicit in these specifications. However, in order to present coherent specifications, some of this earlier work is included in the descriptions of certain topics.

In addition, knowledge of the following topics is assumed.

(a) The arithmetic of integers (including HCFs and LCMs) of fractions and of real numbers.

(b) The laws of indices for positive integer exponents.

(c) Solution of problems involving ratio and proportion (including similar triangles and links between length, area and volume of similar figures).

(d) Elementary algebra (including multiplying out brackets, factorising quadratics with integer coefficients - to include $a^2 - b^2$ - and solution of simultaneous linear equations by eliminating a variable).

(e) Changing the subject of a simple formula or equation.

(f) The equation $y = mx + c$ for a straight line; gradient and intercept.

(g) The distance between two points in 2-D with given co-ordinates.

(h) Solution of triangles using trigonometry, including the sine and cosine rules.

(i) Volume of cone and sphere.

(j) The following properties of a circle:

  (i) the angle in a semicircle is a right angle;

  (ii) the perpendicular from the centre to a chord bisects the chord;

  (iii) the perpendicularity of radius and tangent.

These topics will not be tested directly but will be expected to be freely available for use in other questions.

(b) Pervading Knowledge

Candidates are expected to develop an understanding of Proof and of Modelling, both of which pervade all the modules. These processes are stated in the syllabus for Pure Mathematics 1 since candidates taking any other module are expected to be familiar with its content. Modelling requires an understanding of errors and consequently that topic may also be regarded as assumed knowledge.

The modelling process is illustrated in the diagram on the facing page (page 29).

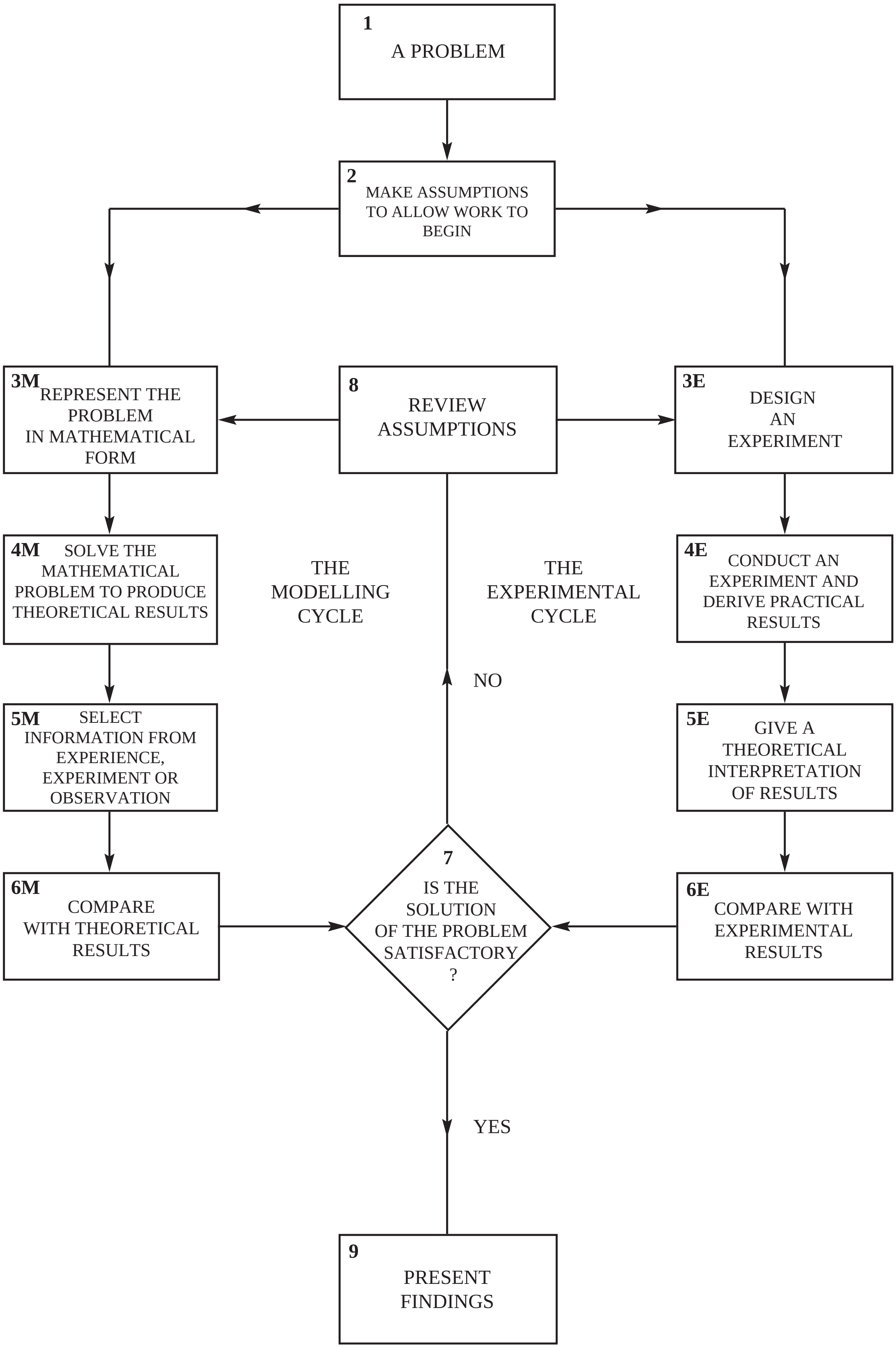
5.1.2 Dependency and Recommended Order

The syllabus of any module is written upon the assumption of knowledge of all syllabuses upon which it is dependent, and its assessment may include work from such earlier module syllabuses.

Where there is a *Recommended Order* for taking modules the assessment of a later module may use specific techniques within an earlier one but such techniques amount only to a minority of the content of the earlier module.

Module syllabuses have been designed so that the content of the Pure Mathematics syllabuses is sufficient to support the applied syllabuses at the same level. The same principle applies to the examination questions, so that, for example, a question will not be set in Mechanics 2 which requires a technique in Pure Mathematics 3.

MODELLING FLOWCHART



5.1.3 Competence Statements

The module syllabuses include Competence Statements. These are included to help users by clarifying the syllabus requirements, but the following three points need to be noted.

(i) Work that is covered by a Competence Statement may be asked in an examination question without further assistance being given.

(ii) Examination questions may require a candidate to use two or more Competence Statements at the same time without further assistance being given.

(iii) Where an examination question requires work that is not covered by a Competence Statement, sufficient guidance will be given within the question.

Competence Statements have an implied prefix of the words: “A candidate should ...”.

The letters used in assigning reference numbers to Competence Statements are as follows

a	algebra	A	algorithms
b	bivariate data	B	
c	calculus	C	confidence intervals
d	dynamics	D	data presentation
e	equations	E	estimation
f	functions	F	
g	geometry, graphs,	G	centre of mass
h	Hooke’s law	H	hypothesis testing
i	impulse & momentum	I	ill conditioning
j	complex numbers	J	
k	kinematics	K	
l	limiting processes	L	linear programming
m	matrices	M	matchings
n	newtons laws	N	networks, critical path analysis
o	oscillations (shm)	O	operators
p	mathematical processes (modelling, proof, etc.)	P	Poisson distribution
q	dimensions (quantities)	Q	quality control
r	rotation	R	random variables
s	sequences and series	S	sampling
t	trigonometry	T	time series
u	probability (uncertainty)	U	errors (Uncertainty)
v	vectors	V	analysis of variance
w	work, energy & power	W	
x	experimental design	X	critical path analysis
y	projectiles	Y	
z	regression	Z	simulation

5.2 Detailed Syllabus Content

This is contained in the following pages 31 to 209

PURE MATHEMATICS

Introduction

There are six pure mathematics modules.

The content of the first three is determined largely by the subject criteria. *Pure Mathematics 1* and *2* meet the requirements for Advanced Subsidiary GCE Mathematics, and *Pure Mathematics 1*, *2* and *3* those for Advanced GCE Mathematics.

These modules are designed to ensure that students understand the need both for mathematical rigour and for being able to use the various techniques within models of real world situations.

Thus all the modules are subject to the following statement:

> *"The construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language pervade the whole of mathematics at this level."*

Although the applied modules are also subject to the same statement, it is in the teaching of the pure mathematics that students are likely to find it first emphasised. However they will soon realise that the need for rigour covers all aspects of the subject.

A similar statement covers modelling.

> *Modelling pervades much of mathematics at this level and a basic understanding of the processes involved will be assumed in all modules.*

In this case students are likely to meet the idea of modelling in their applied mathematics in the first place, and to learn that the techniques they are meeting in pure mathematics support the work within modelling cycles.

In the later modules, *Pure Mathematics 4, 5 and 6*, there is rather less emphasis on applicability. Much of their content is a development of pure mathematics for its own sake.

Pure Mathematics 4 and *5* are both broadly based modules. They cover much of the material traditionally associated with Further Mathematics, particularly if taken in combination with *Differential Equations* (*Mechanics 4*).

Pure Mathematics 6 takes five topics to considerably greater depth, giving students an insight into what the subject is like at university. Students are not required to study all of the topics in this module.

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PURE MATHEMATICS 1 (2601) : AS

Objectives

To build on the student's GCSE work, extending the range of mathematical techniques and developing the basic concepts necessary for advanced study, for example calculus.

Assessment

Examination (60 marks)

1 hour 20 minutes.

The examination paper has two sections.

Section A:	8–10 questions, each worth no more than 4 marks. Section Total: 30 marks
Section B:	2 questions, each worth about 15 marks. Section Total: 30 marks

Assumed Knowledge

Candidates are expected to know the syllabus for Intermediate Tier GCSE and, in addition, the topics listed as Assumed Knowledge on page 26.

Subject Criteria

The modules *Pure Mathematics 1* and *2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The modules *Pure Mathematics 1, 2* and *3* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

Calculating Aids

Candidates are permitted to use only a scientific calculator in the examination for this module. Computers, graphical calculators and calulators with computer algebra facilities are **not** permitted.

PURE MATHEMATICS 1

Specification	Competence Statements

Competence statements marked with an asterisk ***** *are assumed knowledge and will not form the basis of any examination questions. These statements are included for clarity and completeness.*

MATHEMATICAL PROCESSES
Proof

The construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language pervade the whole of mathematics at this level. These skills, and the Competence Statements below, are requirements of all the modules in this specification.

Mathematical argument	P1	p1	Know and be able to use vocabulary and notation with precision.
		2	Be able to construct a direct mathematical argument or proof.
		3	Understand, and be able to use, proof by contradiction.
		4	Be able to disprove a conjecture by the use of a counter example.

Modelling

Modelling pervades much of mathematics at this level and a basic understanding of the processes involved will be assumed in all modules.

The modelling cycle.		5	Be able to recognise the essential elements in a modelling cycle.

PURE MATHEMATICS 1

Notes	Notation	Exclusions
Equals, does not equal, identically equals, therefore, because, implies, is implied by, is equivalent to, necessary, sufficient, proof, converse, counter example.	$=, \neq, \equiv, \therefore, \Rightarrow, \Leftarrow, \Leftrightarrow$	
The elements are illustrated on the diagram on page XX.		

PURE MATHEMATICS 1

Specification			Competence Statements
ALGEBRA			
The basic language of algebra.	P1a	1	Know and be able to use vocabulary and notation appropriate to the subject at this level.
Solution of equations.		2	* Be able to solve linear equations in 1 unknown.
		3	* Be able to change the subject of a formula.
		4	Know how to solve an equation graphically.
		5	Be able to solve quadratic equations.
		6	Be able to find the discriminant of a quadratic function and understand its significance.
		7	Know how to use the method of completing the square to find the line of symmetry and turning point of the graph of a quadratic function.
		8	* Be able to solve linear simultaneous equations in 2 unknowns.
		9	Be able to solve simultaneous equations in 2 unknowns where one equation is linear and one is of 2nd order.
		10	Know the significance of points of intersection of 2 graphs with relation to the solution of simultaneous equations.
Inequalities.		11	Be able to solve linear inequalities.
		12	Be able to solve quadratic inequalities.
		13	Understand the modulus function.
		14	Be able to solve simple inequalities containing a modulus sign.
CO-ORDINATE GEOMETRY			
The co-ordinate geometry of straight lines.	P1g	1	* Know the equation $y=mx+c$.
		2	Know how to specify a point in cartesian co-ordinates in 2 dimensions.
		3	Know the relationship between the gradients of parallel lines and perpendicular lines.
		4	* Be able to calculate the distance between 2 points.
		5	Be able to find the co-ordinates of the midpoint of a line segment joining two points.
		6	Be able to form the equation of a straight line.
		7	Be able to draw a line when given its equation.
		8	Be able to find the point of intersection of 2 lines.
The co-ordinate geometry of curves		9	* Know how to plot a curve given its equation.
		10	Understand the difference between plotting and sketching a curve.
		11	Know how to find the point of intersection of a line and a curve.

PURE MATHEMATICS 1

Notes	Notation	Exclusions
Expression, function, constant, variable, term, coefficient, index, linear, modulus, identity, equation.	f(x), $\lvert\ \rvert$	Formal treatment of functions.
Including those containing brackets and fractions.		
Including repeated roots.		
By factorising, completing the square, using the formula and graphically. The condition for distinct real roots is $d > 0$.		Numerical methods as given in Pure mathematics 2.
The graph of $y = a(x + p)^2 + q$ has a turning point at $(-p, q)$ and a line symetry $x = -p$.		
Analytical solution by substitution.		
Algebraic treatment of linear and quadratic inequalities.		
The use of inequalities to express upper and lower bounds for data which are not known (or stored) precisely. Candidates are expected to know the meanings of absolute and relative error.		Examples involving quadratics which cannot be factorised.
		Inequalities involving more than one modulus sign.
$y - y_1 = m(x - x_1)$, $ax + by + c = 0$, $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$		

PURE MATHEMATICS 1

Specification		Competence Statements
	P1g12	Know how to find the point(s) of intersection of 2 curves.
	13	Understand that the equation of a circle, centre (0, 0) radius r is $x^2 + y^2 = r^2$.
	14	Understand that $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle with centre (a,b) and radius r.
TRIGONOMETRY		
The sine, cosine and tangent functions.	P1t 1	* Know how to solve triangles using trigonometry.
	2	Be able to use the definitions of $\sin\theta$ and $\cos\theta$ for any angle.
	3	Understand the definition of a radian and be able to convert between radians and degrees.
	4	Be able to use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (for any angle).
	5	Know the graphs of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for all values of θ, their symmetries and periodicities.
	6	Know the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ when θ is 0°, 30°, 45°, 60°, 90° and 180°.
	7	Know the meanings of the reciprocal functions $\operatorname{cosec}\theta$, $\sec\theta$ and $\cot\theta$.
	8	Be able to use the identity $\sin^2\theta + \cos^2\theta = 1$, and the equivalent forms.
	9	Be able to solve simple trigonometric equations in given intervals.
	10	Be able to use the fact that the area of a triangle is given by $\frac{1}{2}ab\sin C$.
	11	Be able to find the arc length and area of a sector of a circle, when the angle is given in radians.
POLYNOMIALS		
Basic operations on polynomials.	P1f 1	Know how to add, subtract, multiply and divide polynomials.
	2	Understand the factor theorem and know how to use it to factorise a polynomial.
The factor theorem.	3	Know how to use the factor theorem to solve a polynomial equation.
	4	Know how to use the factor theorem to find an unknown coefficient.
The remainder theorem.	5	Understand the remainder theorem and know how to use it.
	6	Know how to sketch the graphs of polynomial functions.
Binomial expansions.	7	Know how to use Pascal's triangle in the binomial expansion of $(a + x)^n$ where n is a positive integer.
	8	Know the notations ${}^nC_r$ and $\binom{n}{r}$, and their relationship to Pascal's triangle.
	9	Know how to use $\binom{n}{r}$ in the binomial expansion of $(a + x)^n$ where n is a positive integer.

PURE MATHEMATICS 1

Notes	Notation	Exclusions
Including the use of the sine and cosine rules.		
eg by reference to the unit circle.		
Their use to find angles outside the 1st quadrant.		
The equivalent forms $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \mathrm{cosec}^2\theta$		
eg $\sin\theta = 0.5 \Rightarrow \theta = 30°, 150°$ in [0°, 360°]	$\arcsin x$, $\arccos x$, $\arctan x$	Principal values. General solutions.
The results $s = r\theta$ and $A = \frac{1}{2}r^2\theta$.		
Expanding and collecting like terms. Division by linear expressions only.		
$\mathrm{f}(a) = 0 \Leftrightarrow x = a$ is a root of $\mathrm{f}(x) = 0$.		Equations of degree $\geqslant 5$.
Use of factors to determine zeros.		
		Functions of degree $\geqslant 5$.
	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$. $n! = 1.2.3...n$ ${}^nC_0 = {}^nC_n = 1$	

PURE MATHEMATICS 1

Specification		Competence Statements
DIFFERENTIATION		
The basic process of differentiation. P1c	1	Know that the gradient of a curve at a point is given by the gradient of the tangent at the point.
	2	Know that the gradient of the tangent is given by the limit of the gradient of a chord.
	3	Know that the gradient function $\frac{\mathrm{d}y}{\mathrm{d}x}$ gives the gradient of the curve and measures the rate of change of y with respect to x.
Applications of differentiation to the graphs of functions.	4	Be able to differentiate $y = kx^n$ where k is a constant and n a positive integer or 0, and the sum of such functions.
	5	Be able to use differentiation to find stationary points on a curve: maxima, minima and points of inflection.
	6	Be able to find the equation of a tangent and normal at any point on a curve.
INTEGRATION		
Integration as the inverse of differentiation.	7	Know that integration is the inverse of differentiation.
	8	Be able to integrate functions of the form kx^n where k is a constant and n a positive integer or 0, and the sum of such functions.
	9	Know what are meant by indefinite and definite integrals.
	10	Be able to evaluate definite integrals.
	11	Be able to find a constant of integration given relevant information.
Integration to find the area under a curve.	12	Know that the area under a graph can be found as the limit of a sum of areas of rectangles.
	13	Know that an approximate value of a definite integral can be obtained using the trapezium rule, and comment sensibly on its accuracy.
	14	Be able to use integration to find the area between a graph and the x-axis.
	15	Be able to use integration to find the area between a graph and the y-axis.
	16	Be able to use integration to find the area between 2 graphs.
Volumes of revolution	17	Be able to calculate the volumes of the solids generated by rotating a plane region about an axis.

PURE MATHEMATICS 1

Notes	Notation	Exclusions
	$\dfrac{dy}{dx} = \lim_{\delta x \to 0} \dfrac{\delta y}{\delta x}$.	
The terms increasing function and decreasing function.		
Simple cases of differentiation from first principles.	$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.	
Stationary points, by considering the sign of $\dfrac{dy}{dx}$ in the region of the point.		
Comments on the accuracy should be made with reference to the shape of the curve and/or calculations involving different numbers of strips.		
The area between intersecting curves.		
Rotation about the x- and y-axes only.		

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PURE MATHEMATICS 2 (2602) : A2

Objectives

To extend the work in the module *Pure Mathematics 1* to harder examples, and to broaden the range of techniques available to the student, including an introduction to numerical methods for solving equations. To ensure that students appreciate how to use appropriate technology, such as computers and calculators, as a mathematical tool, and have awareness of its limitations.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

Candidates answer four compulsory questions of approximately equal weight.

Component 2: Coursework (15 marks)

Candidates are required to undertake a piece of coursework on the solution of equations by numerical methods (see page 50).

Assumed Knowledge

Knowledge of the module *Pure Mathematics 1* is assumed.

Subject Criteria

This module is required for both Advanced Subsidiary GCE and Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

The modules *Pure Mathematics 1* and *2* are required for Advanced Subsidiary GCE Mathematics.

The modules *Pure Mathematics 1*, *2* and *3* are required for Advanced GCE Mathematics.

Calculating Aids

Candidates are permitted to use only a scientific calculator in the **examination** for this module. Computers, graphical calculators and calculators with computer algebra facilities are **not** permitted.

The use of graphical calculators and computer software (e.g. spreadsheets) is encouraged in the **coursework**.

PURE MATHEMATICS 2

Specification		Competence Statements
ALGEBRA		
Surds, indices and logarithms.	P1a 1	Be able to use and manipulate surds.
	2	Be able rationalize the denominator of a surd.
	3	Understand and be able to use the laws of indices for all rational exponents.
	4	Understand the meaning of negative, fractional and zero indices.
	5	Understand the meaning of the word logarithm.
	6	Understand the laws of logarithms and how to apply them.
	7	Know the value of $\log_a a$ and $\log_a 1$.
	8	Know how to convert from index form to logarithmic form and vice versa.
	9	Know how to reduce the equations $y = ax^n$ and $y = ab^x$ to linear form and, using experimental data, to draw a graph to find values of a, n and a, b.
	10	Understand and be able to use the simple properties of exponential and logarithmic functions including the functions e^x and $\ln x$.
	11	Be able to solve problems involving exponential growth and decay.
	12	Know the relationship between $\ln x$ and e^x.
	13	Be able to solve equations of the form $a = b^x$.
SEQUENCES AND SERIES		
Definitions of sequences.	P2s 1	Know what a sequence of numbers is and the meaning of finite and infinite sequences.
	2	Know that a sequence can be generated using a formula for the k^{th} term, or a recurrence relation of the form $a_{k+1} = \mathrm{f}(a_k)$.
	3	Know what a series is.
	4	Be familiar with Σ notation.
	5	Know and be able to recognise the periodicity of sequences.
	6	Know the difference between a convergent and divergent series.
	7	Know what is meant by arithmetic series and sequences.
	8	Be able to use the standard formulae associated with arithmetic series and sequences.
	9	Know what is meant by geometric series and sequences.
	10	Be able to use the standard formulae associated with geometric series and sequences.
	11	Know the condition for a geometric series to be convergent and be able to find its sum to infinity.
	12	Be able to solve problems involving arithmetic and geometric series and sequences.

PURE MATHEMATICS 2

Notes	Notation	Exclusions
e.g. $\frac{1}{5+\sqrt{3}} = \frac{5-\sqrt{3}}{22}$		
$x^a \times x^b = x^{a+b}$, $x^a \div x^b = x^{a-b}$, $(x^a)^n = x^{an}$		
$x^{-a} = \frac{1}{x^a}$, $x^{\frac{1}{a}} = \sqrt[a]{x}$, $x^0 = 1$		
$\log(ab) = \log a + \log b$ $\log\left(\frac{a}{b}\right) = \log a - \log b$ $\log(a^n) = n\log a$		Change of base of logarithms.
$x = a^n \Leftrightarrow n = \log_a y$		
Including graphs.	$\log_e x = \ln x$.	
Candidates studying *Statistics 2* will need to know series expansion for e^x for the Poisson distribution.		
eg $a_k = 2 + 3k$; $a_{k+1} = a_k + 3$, $a_1 = 5$.		
Including the sum of the first n natural numbers.		
The term arithmetic progression may also be used.		
The term geometric progression may also be used.		

PURE MATHEMATICS 2

Specification		Competence Statements
FUNCTIONS		
The language of functions.	P2f	1 Understand the definition of a function, and the associated language.
		2 Know the effect of transformations on a graph and be able to form the equation of the new graph.
		3 Be able, given the graph of $y = f(x)$, to sketch related graphs.
		4 Be able to apply transformations to the basic trigonometrical functions.
		5 Know how to find a composite function, $gf(x)$.
		6 Know the conditions necessary for the inverse of a function to exist and how to find it (algebraically and graphically).
		7 Understand what is meant by the terms odd and even functions and the symmetries associated with them.
		8 Understand the relationship between the graphs of the sin, cos, tan, cosec, sec and cot functions; and the inverse functions arcsin, arccos, arctan with the appropriate restricted domains.
The techniques of curve sketching.		9 Know the techniques of curve sketching.

PURE MATHEMATICS 2

Notes	Notation	Exclusions
Many-to-one, one-to-many, one-to-one, mapping, object, image, domain, co-domain, range, odd, even, periodic.		
Translation parallel to the x-axis. Translation parallel to the y-axis. Stretch parallel to the x-axis. Stretch parallel to the y-axis. Reflection in the x-axis. Reflection in the y-axis.	Translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$	
$y = \mathrm{f}(x \pm a)$ $y = \mathrm{f}(x) \pm a$ $y = \mathrm{f}(ax)$ $y = a\mathrm{f}(x)$ for a>0 $y = \mathrm{f}(-x)$ $y = -\mathrm{f}(x)$		
$\sin(x+30°)$. $2\cos(x-45°)$. $\sin(2x+60°)$.		
The use of reflection in the line $y = x$. eg $\ln x$ (x>0) is the inverse of e^x.		
eg x^n for integer values of n.		
Use of symmetry. Points of intersection with the x- and y- axes. Behaviour of the function as x tends to ± infinity. Points where the function is undefined. Turning points. Asymptotes parallel to the axes.		

PURE MATHEMATICS 2

Specification		Competence Statements
DIFFERENTIATION		
Differentiation of further functions.	P2c	1 Be able to differentiate kx^n for rational values of n, and the sum of such functions.
		2 Be able to differentiate e^{ax} and $\ln x$.
The product, quotient and chain rules.		3 Be able to differentiate the product of two functions.
		4 Be able to differentiate the quotient of two functions.
		5 Be able to differentiate composite functions using the chain rule.
		6 Be able to find rates of change using the chain rule.
Higher derivatives.		7 Be able to find the higher derivatives of a function, and to know the associated terminology.
		8 Be able to use the second derivatives in determining the nature of a stationary point.
		9 Know the meaning of a non-stationary point of inflection, and how to locate it.
Inverse functions.		10 Be able to differentiate an inverse function.
INTEGRATION		
Integration of further functions.	P2c	10 Be able to integrate kx^n for rational values of n, and the sum of such functions.
		11 Be able to integrate $\frac{1}{x}$.
		12 Be able to integrate e^{ax}.
Integration by substitution.		13 Be able to use integration by substitution in cases where the process is the reverse of the chain rule.
		14 Be able to use integration by substitution in other cases.
NUMERICAL METHODS		
Change of sign.	P2e	1 Be able to locate the roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.
Fixed point iteration.		2 Be able to carry out a fixed point iteration after rearranging an equation into the form $x = g(x)$.
		3 Understand that not all iterations converge to the roots of an equation.
The Newton-Raphson method.		4 Be able to use the Newton-Raphson method to solve an equation.
Error bounds.		5 Appreciate the need to establish error bounds when applying a numerical method.
Geometrical interpretation.		6 Be able to give a geometrical interpretation both of the processes involved and of their algebraic representation.

PURE MATHEMATICS 2

Notes	Notation	Exclusions
eg $y=\dfrac{3}{x^2}, y=\sqrt{x}, y=\dfrac{3}{x^2}+\sqrt{x}$.		
	$f''(x), \dfrac{d^2y}{dx^2}$.	
$\dfrac{dy}{dx}=\dfrac{1}{\frac{dx}{dy}}$.		
eg $n=-2, n=\dfrac{1}{2}$.		
eg $(1+2x)^8$, $x(1+x^2)^8$, xe^{x^2}, $\dfrac{1}{2x+3}$; Where appropriate recognition may replace substitution.		
Simple cases only eg $\dfrac{x}{2x+1}$		
eg the method of bisection.		This topic will not be assessed in the examination for *Pure Mathematics 2*, since it is the subject of the coursework.
eg $x^3 - x - 4 = 0$ written as $x=\sqrt[3]{x+4}$.		
Error bounds should be established within the numerical method and not by reference to an already known solution.		

PURE MATHEMATICS 2 COURSEWORK: SOLUTION OF EQUATIONS BY NUMERICAL METHODS
(IT) 3.1, 3.2, 3.3 (C) 3.3

Rationale

The assessment of Pure Mathematics 2 includes a coursework task (Component 2) involving the solution of equations by three different numerical methods.

The aims of this coursework are that candidates should appreciate the principles of numerical methods and at the same time be provided with useful solving techniques.

The objectives are:

(i) that candidates should be able to solve equations efficiently, to any required level of accuracy, using numerical methods;

(ii) that in doing so they will appreciate how to use appropriate technology, such as calculators and computers, as a mathematical tool and have an awareness of its limitations;

(iii) that they show geometrical awareness of the processes involved.

This task represents 20% of the assessment and the work involved should be consistent with that figure, both in quality and level of sophistication.

Numerical methods should be seen as complementing analytical ones and not as providing alternative (and less accurate) ways of doing the same job. Thus, equations which have simple analytical solutions should not be selected. Accuracy should be established from within the numerical working and not by reference to an exact solution obtained analytically.

The intention of this piece of coursework is not merely to solve equations; candidates should be encouraged to treat it as an investigation and to choose their own equations.

Requirements

1 Candidates must solve equations by the following three methods.

(i) Systematic search for change of sign using one of the methods: bisection; decimal search; linear interpolation. One root is to be found.

(ii) Fixed point iteration using the Newton-Raphson method. The equation selected must have at least two roots and all roots are to be found.

(iii) Fixed point iteration after rearranging the equation $f(x) = 0$ into the form $x = g(x)$. One root is to be found.

A different equation must be used for each method.

In addition, a candidate's write-up must meet the following requirements.

2 One root of one of the equations must be found by all three methods. The methods should then be compared in terms of their efficiency and ease of use.

3 The write up must include a graphical illustration of how the methods work on the candidate's equations.

4 A candidate is expected to be able to give error bounds for the value of any root. This must be demonstrated in the case of the change of sign method (maximum possible error 0.5×10^{-3}), and for one of the roots found by the Newton Raphson method (required accuracy 5 significant figures).

5 For each method an example should be given of an equation where the method fails: that is an expected root is not obtained, a root is not found or a false root is obtained. There must be an explanation, illustrated graphically, of why this happens. In this situation it is acceptable to use equations with known analytical solutions provided they are not trivial.

Oral Communication

Each candidate must talk about the task; this may take the form of a class presentation, an interview with the assessor or ongoing discussion with the assessor while the work is in progress. Topics for discussion may include strategies used to find suitable equations and explanations, with reference to graphical illustrations, of how the numerical methods work.

Use of Software

The use of existing computer of calculator software is encouraged, but candidates must:

(i) edit any print-outs and displays to include only what is relevant to the task in hand;

(ii) demonstrate understanding of what the software has done, and how they could have performed the calculations themselves;

(iii) appreciate that the use of such software allows them more time to spend on investigational work.

Selection of equations

Centres may provide candidates with a list of at least 10 equations from which they can, if they wish, select those that they are going to solve or to use to demonstrate failure of a method. Such a list of equations should be forwarded to the Moderator with the sample of coursework. A new set of equations must be supplied with each examination season.

Centres may however exercise the right not to issue a list, on the grounds that students stand to benefit from the mathematics they learn while finding their own equations.

MEI STRUCTURED MATHEMATICS
PURE MATHEMATICS 2 (2602)
COURSEWORK ASSESSMENT SHEET
SOLUTION OF EQUATIONS BY NUMERICAL METHODS

TASK: Candidates will investigate the solution of equations using the following three methods.

(i) Systematic search for change of sign using one of the methods: bisection, decimal search, linear interpolation.
(ii) Fixed point iteration using the Newton-Raphson method.
(iii) Fixed point iteration after rearranging the equation $f(x) = 0$ into the form $x = g(x)$.

Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Change of Sign Method	1 1 1	The method is applied successfully to find one root of an equation and illustrated graphically. Error bounds are established for the roots An example is given of an equation where one of the roots cannot be found by your chosen method. There is an illustrated explanation of why this is the case.		
Newton-Raphson method	1 1 1	The method is applied successfuly to find all the roots of a second equation and illustrated graphically for one of them. Error bounds are established for at least one of the roots. An example is given of an equation where this method fails to find a particluar root despite taking a starting value close to it. There is an illustrated explanation why this has happened.		
Rearranging $f(x) = 0$ in the form $x = g(x)$	1 1 1 1	A rearrangement is applied successfuly to find a root of a third equation. Convergence of this rearrangement to a root is demonstrated graphically and the magnitude of $g'(x)$ is discussed. A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root. This failure is demonstrated graphically and the magnitude of $g'(x)$ is discussed.		
Comparison of methods	1 1 1	One of the equations used above is selected and the other two methods are applied successfully to find the same root. There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence. There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software.		
Oral Communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

PURE MATHEMATICS 3 (2603) : A2

Objectives

To complete the work necessary both to provide Advanced GCE students with an appropriate set of techniques for handling a range of problems and to give them a basis for further study in the subject.

To ensure that students are able to read and comprehend a mathematical argument or an example of the application of mathematics.

Assessment

Examination (75 marks)

The examination has two parts:

Part A	Four questions of approximately equal weight. 1 hour 20 minutes) Part Total : 60 marks
Part B	One comprehension task. (Further details on pages 56 and 58). Up to 1 hour. Part Total : 15 marks

Assumed Knowledge

Knowledge of the modules *Pure Mathematics 1* and *2* is assumed.

Subject Criteria

This module is required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

PURE MATHEMATICS 3

Specification		Competence Statements
ALGEBRA		
The general binomial expansion.	P3a 1	Be able to form the binomial expansion of $(1 + x)^n$ where n is any rational number and find a particular term in it.
	2	Be able to write $(a + x)^n$ in the form $a^n\left(1+\frac{x}{a}\right)^n$ prior to expansion.
Rational expressions.	3	Be able to simplify rational expressions.
Partial fractions.	4	Be able to solve equations involving algebraic fractions.
	5	Know how to express algebraic fractions as partial fractions.
	6	Know how to use partial fractions with the binomial expansion to find the power series for an algebraic fraction.
TRIGONOMETRY		
Compound angle formulae.	P3t 1	Be able to use the identities for $\sin(\theta \pm \phi)$, $\cos(\theta \pm \phi)$, $\tan(\theta \pm \phi)$.
	2	Be able to use the identities for $\sin 2\theta$, $\cos 2\theta$ (3 versions), $\tan 2\theta$.
	3	Be able to use expressions for $\cos^2\theta$ and $\sin^2\theta$ in terms of $\cos 2\theta$.
Solution of trigonometrical equations.	4	Be able to solve simple trigonometrical equations within a given range including the use of any of the trigonometrical identities above.
	5	Know how to write the function $a\cos\theta \pm b\sin\theta$ in the forms $R\sin(\theta \pm \alpha)$, $R\cos(\theta \pm \alpha)$, and how to use these to sketch the graph of the function, find its maximum and minimum values and to solve equations.
Small angle approximations.	6	Know and be able to use appropriately the small angle approximations for $\sin\theta$, $\cos\theta$ and $\tan\theta$.
CO-ORDINATE GEOMETRY		
The use of parametric equations.	P3g 1	Understand the meaning of the terms parameter and parametric equations.
	2	Be able to find the equivalent cartesian equation for parametric equations.
	3	Recognise the parametric form of circles and ellipses centred on the origin.
	4	Be able to find the gradient at a point on a curve defined in terms of a parameter by differentiation.

PURE MATHEMATICS 3

Notes	Notation	Exclusions
For $\lvert x \rvert < 1$ where n is not a positive integer.		
$\left\lvert \frac{x}{a} \right\rvert < 1$ where n is not a positive integer.		
Including factorising and cancelling, and algebraic division.		
Proper fractions with the following denominators $(ax + b)(cx + d)$, $(ax + b)(cx + d)^2$ and $(ax + b)(x^2 + c^2)$.		
Including identities from earlier modules. Knowledge of principal values.		
$\sin\theta \approx \theta, \tan\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2$.		
		Oblique ellipses.
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$		

PURE MATHEMATICS 3

Specification		Competence Statements
CALCULUS		
Differentiation of further functions.	P3c	1 Be able to differentiate the trigonometrical functions: $\sin x$; $\cos x$; $\tan x$.
Differentiation of simple functions defined implicitly.		2 Be able to differentiate a function defined implicitly.
Integration of further functions.		3 Be able to integrate $\sin x$ and $\cos x$.
		4 Use trigonometric identities to integrate functions.
Integration by parts.		5 Be able to use the method of integration by parts in cases where the process is the reverse of the product rule.
		6 Be able to apply integration by parts to $\ln x$.
		7 Be able to use the method of partial fractions in integration.
Differential equations.		8 Be able to formulate first order differential equations.
		9 Be able to solve first order differential equations.
VECTORS		
Vectors in two and three dimensions.	P3v	1 Understand the language of vectors in two and three dimensions.
		2 Be able to add vectors, multiply a vector by a scalar, and express a vector as a combination of others.
The scalar product.		3 Know how to calculate the scalar product of two vectors, and be able to use it to find the angle between two vectors.
Co-ordinate geometry in two and three dimensions.		4 Be able to find the distance between two points, the midpoint and other points of simple division of a line.
The equations of lines and planes.		5 Be able to form and use the equation of a line and a plane.
The intersection of a line and a plane.		6 Know that a vector which is perpendicular to a plane is perpendicular to any line in the plane.
		7 Know that the angle between two planes is the same as the angle between their normals.
		8 Be able to find the intersection of a line and a plane.
COMPREHENSION		
The ability to read and comprehend a mathematical argument or an example of the application of mathematics.	P3p	1 Be able to follow mathematical arguments and descriptions of the solutions of problems when given in writing.
		2 Understand the modelling cycle, and realise that it can be applied across many branches of mathematics.

PURE MATHEMATICS 3

Notes	Notation	Exclusions
Including their sums and differences.		
eg $\sin y = 1 - x^2$		
eg $\cos^2 x$, $\tan x$. eg xe^x		Integrals requiring the substitution $t = \tan\frac{\theta}{2}$. Integrals involving arctan and arcsin forms.
From given information about rates of change.		
Differential equations with separable variables only.		
Scalar, vector, modulus, magnitude direction, unit vector, position vector, cartesian components, equal vectors, parallel vectors.	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, $\hat{\mathbf{r}}$, $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$.	
Including test for perpendicular vectors.		
	$\mathbf{a}\,.\,\mathbf{b} = a_1b_1+a_2b_2+a_3b_3$ $= \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$.	
In vector and cartesian form.	Line: $\mathbf{r} = \mathbf{a} + t\mathbf{u}$ $\dfrac{x-a_1}{u_1} = \dfrac{y-a_2}{u_2} = \dfrac{z-a_3}{u_3} (= t)$ Plane: $(\mathbf{r} - \mathbf{a})\,.\,\mathbf{n} = 0$ $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}\,.\,\mathbf{n}$	
If a vector is perpendicular to two non-parallel lines in a plane it is perpendicular to the plane.		
This may be tested using a real-world modelling context.		
Abstraction from a real-world situation to a mathematical description; approximation, simplification and solution; check against reality; progressive refinement.		

PURE MATHEMATICS 3: COMPREHENSION (C) 3.2

Rationale

The aim of the comprehension question is to foster the view among students that in learning Mathematics, they are acquiring skills which transcend the particular items of specification content which have made up their course.

The objectives are that candidates should be able to:

(i) read and comprehend a mathematical argument or an example of the application of mathematics;

(ii) respond to a synoptic piece of work covering ideas permeating their whole course;

(iii) appreciate the relevance of particular techniques to real world problems.

Description and conduct

The examination for Section B of Pure Mathematics 3 includes a comprehension question on which candidates are expected to take no more than about 40 minutes. The question takes the form of a written article followed by most candidates questions designed to test how well they have understood it.

Candidates are allowed to bring standard English dictionaries into the examination and those for whom English is a second language are also allowed a translation dictionary. However care will be taken in preparing the question to ensure that the language is readily accessible.

Content

By its nature, the content of the written piece of mathematics cannot be specified in the detail of the rest of the specification. However, knowledge of *GCSE (Intermediate Tier)* and *Pure Mathematics 1*, *2* and *3* will be assumed, as will the topics listed on page 26 as Assumed Knowledge. Candidates are expected to be aware of ideas concerning accuracy and errors. The written piece may follow a modelling cycle and in that case candidates will be expected to recognise it. No knowledge of Mechanics will be assumed.

PURE MATHEMATICS 4 (2604) : A2

Objectives

To introduce the detailed study of a number of topics in algebra, geometry and complex numbers.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions out of four set.

Assumed Knowledge

Knowledge of the modules *Pure Mathematics 1*, *2* and *3* is assumed.

FURTHER MATHEMATICS

The module *Pure Mathematics 4* must be included in the 9 modules offered for the award of Advanced GCE Mathematics + Advanced Subsidiary GCE Further Mathematics.

The modules *Pure Mathematics 4* and *5* must be included in the 12 modules offered for the award of Advanced GCE Mathematics + Advanced GCE Further Mathematics.

PURE MATHEMATICS 4

Specification		Competence Statements
PROOF		
Meaning and use of the terms if, only if, necessary and sufficient.	P4p	1 Be able to use the terms *if*, *only if*, *necessary*, *sufficient* correctly in any appropriate context.
Mathematical induction.		2 Be able to construct and present a correct proof using mathematical induction.
CURVES		
Treatment and sketching of graphs of the form $y = \mathrm{f}(x)$, $y^2 = \mathrm{f}(x)$.	P4g	1 Be able to sketch the graph of $y = \mathrm{f}(x)$ obtaining information about symmetry, asymptotes (parallel to the axes or oblique), stationary points, intercepts with the co-ordinate axes, behaviour near $x = 0$ and for numerically large x.
		2 Be able to sketch the graph $y^2 = \mathrm{f}(x)$.
		3 Be able to sketch the graph of $y = \lvert \mathrm{f}(x) \rvert$, dealing correctly with the shape of the graph near $y = 0$.
ALGEBRA		
Summation of simple finite series.	P4a	1 Be able to sum a simple finite series.
The manipulation of simple algebraic inequalities.		2 Be able to manipulate simple algebraic inequalities, to deduce the solution of such an inequality.
COMPLEX NUMBERS		
Addition, subtraction, multiplicaton and division of complex numbers.	P4j	1 Be able to add, subtract, multiply and divide complex numbers given in the form $x + y\mathrm{j}$.
		2 Understand the language of complex numbers.
		3 Know that a complex number is zero if and only if both the real and imaginary parts are zero.
		4 Be able to solve linear simultaneous equations with complex coefficients.
Application of complex numbers to the solution of polynomial equations with real coefficients.		5 Be able to solve any quadratic equation with real coefficients.
		6 Be able to solve equations of higher degree with real coefficients in simple cases.
		7 Know that the complex roots of real polynomial equations with real coefficients occur in conjugate pairs.
Representation of complex numbers in the Argand diagram.		8 Know how to represent complex numbers on an Argand diagram.
The polar form of a complex number.		9 Understand the polar (modulus-argument) form of a complex number, and the meaning of modulus, argument.
		10 Be able to multiply and divide complex numbers in polar form.
Simple loci in the Argand diagram.		11 Be able to represent simple sets of complex numbers as loci in the Argand diagram.

PURE MATHEMATICS 4

Notes	Notation	Exclusions

$f(x)$ may be a simple rational function, or a simple modulus function.

Including determining the shape of the graph near $y = 0$.

Use of gradient at points near $y = 0$.

Using standard formula for $\sum r$, $\sum r^2$, $\sum r^3$.

Using partial fractions, mathematical induction or the method of differences eg $\sum r^4$, $\Sigma \dfrac{1}{r(r+1)(r+2)}$.

Including those reducible to the form $f(x) > 0$ where $f(x)$ can be expressed in factors.

Division using complex conjugates.

Notation: $j^2 = -1$, $z = x + yj$
$\text{Re}(z) = x$, $\text{Im}(z) = y$
z^* for the complex conjugate of z.

Real part, imaginary part, complex conjugate, real axis, imaginary axis.

eg to solve a cubic equation given one complex root.

$z = r(\cos\theta + j\sin\theta)$ where $r = |z|$, $\theta = \arg z$

Notation: $|z|$ for the modulus of z.
$\arg z$ for the principal argument of z, $-\pi < \arg z \le \pi$.

$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2))$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2))$.

eg $|z - z_0| = c$, $|z - z_1| = |z - z_2|$,
$\arg(z - z_0) = \alpha$, and their associated inequalities.

PURE MATHEMATICS 4

Specification		Competence Statements
VECTORS		
Vector (cross) product of two vectors. P4v	1	Be able to form the vector product of two vectors in magnitude and direction, and in component form.
	2	Understand the anti-commutative and distributive properties of the vector product.
	3	Know the significance of $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.
The intersection of two planes.	4	Be able to find the line of intersection of two planes.
The intersection of lines in three dimensions.	5	Be able to determine whether two lines in three dimensions are skew or intersect, and to find the point of intersection if there is one.
MATRICES		
Matrix addition and multiplication. P4m	1	Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar.
	2	Know the zero and identity matrices, and what is meant by equal matrices.
	3	Know that matrix multiplication is associative but not commutative.
Linear transformations in a plane, their associated 2×2 matrices and their manipulation.	4	Be able to find the matrix associated with a linear transformation and vice-versa.
	5	Understand successive transformations and the connection with matrix multiplication.
	6	Understand the meaning of invariant points and lines and how to find them.
	7	Be able to find the determinant of a 2×2 matrix.
	8	Know that the determinant gives the area scale factor of the transformation, and understand the significance of a zero determinant.
The meaning of the inverse of a square matrix.	9	Understand what is meant by an inverse matrix.
	10	Be able to find the inverse of a non-singular 2×2 matrix.
The product rule for inverses.	11	Appreciate the product rule for inverse matrices.
The solution of the matrix equation $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is a 2×2 or 3×3 matrix (singular or non-singular).	12	Know that a matrix equation can be written as a set of simultaneous linear equations.
Geometrical interpretation of the solution in terms of configurations of lines (2×2 case) or planes (3×3 case).	13	Be able to solve a matrix equation or the equivalent simultaneous equations, and to interpret the solution geometrically.

PURE MATHEMATICS 4

Notes	Notation	Exclusions
$\mathbf{a} \times \mathbf{b} = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert \sin\theta\,\hat{\mathbf{n}}$ $= \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$		Proof of the distributive law.
	O for zero matrix. **I** for identity matrix.	
eg rotation through an angle θ about O, reflection in the line $y = x\tan\theta$.	The position of a point and its image to be written as column vectors.	
The terms singular and non-singular.	$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ or det**M**	
Candidates will not be asked to calculate the inverse of a 3×3 matrix without help, but should know, eg that $\mathbf{AB} = k\mathbf{I} \Rightarrow \mathbf{A}^{-1} = \frac{1}{k}\mathbf{B}$. $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.	$\mathbf{M}^{-1}$ for inverse.	

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PURE MATHEMATICS 5 (2605) : A2

Objectives

To continue the detailed study of a number of topics in algebra, geometry, complex numbers and calculus.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions out of four set.

Assumed Knowledge

Knowledge of the modules *Pure Mathematics 1, 2, 3* and *4* is assumed.

FURTHER MATHEMATICS

The modules *Pure Mathematics 4* and *5* must be included in the 12 modules offered for the award of Advanced GCE Mathematics and Advanced GCE Further Mathematics.

PURE MATHEMATICS 5

Specification		Competence Statements
GEOMETRY		
Polar co-ordinates in two dimensions.	P5g	1 Understand the meaning of polar co-ordinates (r, θ) and be able to convert from polar to cartesian co-ordinates and vice-versa.
		2 Be able to sketch curves with simple polar equations.
		3 Be able to find the area enclosed by a polar curve.
Simple co-ordinate treatment of the conics in their standard cartesian or parametric forms.		4 Be able to recognise and sketch conics in their standard forms.
		5 Be able to find the equations of chords, tangents and normals.
		6 Be able to find the asymptotes of a hyperbola.
Conics.		7 Be able to recognise and sketch a conic from its polar equation.
		8 Understand the terms focus, directrix, eccentricity, latus rectum.
		9 Appreciate that a conic is an ellipse, parabola, hyperbola according to whether $0 < e < 1$, $e = 1$, $e > 1$.
		10 Know the properties of conics.
		11 Be able to explain how the different types of conic can be obtained as sections of a cone.
COMPLEX NUMBERS		
De Moivre's theorem for integer index, and simple applications.	P5j	1 Understand de Moivre's theorem for an integer index.
		2 Be able to apply de Moivre's theorem to finding multiple angle formulae or summing a suitable series.
Expression of complex numbers in the form $z = re^{j\theta}$.		3 Understand the definition $e^{j\theta} = \cos\theta + j\sin\theta$ and hence the form $z = re^{j\theta}$.
The n n^{th} roots of a complex number.		4 Know that every non-zero complex number has n n^{th} roots, and that in the Argand diagram these are the vertices of a regular n-gon.
		5 Know that the n^{th} roots of $re^{j\theta}$ are $r^{1/n}[\cos\left(\frac{\theta+2k\pi}{n}\right)+j\sin\left(\frac{\theta+2k\pi}{n}\right)]$ for $k = 0, 1, \ldots, n-1$.
		6 Be able to explain why the sum of all the n^{th} roots is zero.
Applications of complex numbers in geometry.		7 Appreciate the effect in the Argand diagram of multiplication by a complex number.
		8 Be able to represent complex roots of unity on Argand diagram.
		9 Be able to apply complex numbers to geometrical problems.

PURE MATHEMATICS 5

Notes	Notation	Exclusions

eg $r = a(1 + \cos\theta)$, $r = a\cos 2\theta$.

A continuous line for $r > 0$ and a broken line for $r < 0$.

area $= \frac{1}{2}\int r^2 \,\mathrm{d}\theta$.

$y^2 = 4ax$; $x = at^2$, $y = 2at$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; x = a\cos\theta, y = b\sin\theta$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; x = a\sec\theta, y = b\tan\theta$

$xy = c^2; c = ct, y = \frac{c}{t}$

$\frac{l}{r} = 1 + e\cos\theta$.

e for eccentricity, l for the semi-latus rectum.

Any appropriate methods may be used.

SP = ePM.
The tangent at P is equally inclined to PS and PS' (ellipse or hyperbola) or to PS and PM (parabola).

SP + PS' = constant (ellipse) and
$|$SP – PS'$|$ = constant (hyperbola).

Where P is a point on the curve, S and S' are foci and M is the foot of the perpendicular from P to the appropriate directrix.

Proof that sections of a cone are conics.

eg the expression of $\tan 4\theta$ as a rational function of $\tan\theta$,
finding $\sum_{r=0}^{n} \binom{n}{r} \cos r\theta$.

eg multiplication by j corresponds to a rotation of $\frac{\pi}{2}$,
multiplication by $re^{j\theta}$ corresponds to enlargement scale factor r with rotation through θ.

eg relating to the geometry of regular polygons.

PURE MATHEMATICS 5

Specification		Competence Statements
CALCULUS		
The inverse functions of sine, cosine and tangent.	P5c	1 Understand the definitions of inverse trigonometric functions.
Differentiation of arcsin*x*, arccos*x* and arctan*x*.		2 Be able to differentiate inverse trigonometric functions.
Integration of $(a^2 - x^2)^{-½}$ and $(a^2 + x^2)^{-1}$.		3 Recognise these integrals, and be able to integrate associated functions by using trigonometric substitutions.
Hyperbolic functions: definitions, graphs, differentiation and integration.		4 Understand the definitions of hyperbolic functions and be able to sketch their graphs.
		5 Be able to differentiate and integrate hyperbolic functions.
Inverse hyperbolic functions, including the logarithmic forms. Use in integration.		6 Understand and be able to use the definitions of the inverse hyperbolic functions.
		7 Be able to use the logarithmic forms of the inverse hyperbolic functions.
		8 Be able to integrate $(a^2 + x^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$ and related functions.
Maclaurin series. Approximate evaluation of a function.		9 Be able to find the Maclaurin series of a function, including the general term in simple cases.
		10 Appreciate that the series may converge only for a restricted set of values of x.
		Identify and be able to use the Maclaurin series of standard functions.
ALGEBRA		
Division of a polynomial by a linear or quadratic divisor.	P5a	1 Know what is meant by an identity.
		2 Be able to use an algorithm for establishing polynomial identities of the form $P(x) \equiv D(x)\,Q(x) + R(x)$, where $D(x)$ is a linear or quadratic polynomial.
		3 Be able to calculate the remainder for an algebraic division.
		4 Be able to use the remainder theorem to calculate an unknown coefficient.
Relations beween the roots and coefficients of quadratic, cubic and quartic equations.		5 Appreciate the relationships between the roots and coefficients of quadratic, cubic and quartic equations.
		6 Be able to form a new equation whose roots are related to the roots of a given equation.

PURE MATHEMATICS 5

Notes	Notation	Exclusions
arcsin: $-\frac{\pi}{2} \leqslant \text{arcsin} \leqslant \frac{\pi}{2}$. arccos: $0 \leqslant \text{arccos} \leqslant \pi$. arctan: $-\frac{\pi}{2} < \text{arctan} < \frac{\pi}{2}$.		
sinh$x$, cosh$x$ and tanh$x$.		
arcosh$x \geq 0$. arsinh$x = \ln(x + \sqrt{(x^2 + 1)})$ arcosh$x = \ln(x + \sqrt{(x^2 - 1)})$ $x \geqslant 1$ artanh$x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ $-1 < x < 1$.		
		Taylor series
Candidates will be expected to identify the series for e^x, cos$x$, sin$x$, cosh$x$, sinh$x$, ln$(1 + x)$ and $(1 + x)^m$. The ranges of values of x for which these series are valid should be understood.		
	$\equiv$ for identity.	

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PURE MATHEMATICS 6 (2606) : A2

Objectives

To give able students an introduction to a number of more sophisticated areas of Pure Mathematics, with a choice of options.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Five questions are set, one on each option; candidates answer three questions.

Assumed Knowledge

Knowledge of the modules *Pure Mathematics 1, 2, 3, 4* and *5* is assumed.

PURE MATHEMATICS 6

Specification		Competence Statements
OPTION 1 **VECTORS AND MATRICES**		
Distance of a point from a line or from a plane.	P6v1	Be able to find the shortest distance from a point to a line in 2 or 3 dimensions.
	2	Be able to find the shortest distance from a point to a plane.
Scalar triple product. Geometrical interpretation. Volume of parallele-piped and tetrahedron. Shortest distance between two skew lines; condition in three dimensions for two lines to intersect.	3	Be able to find the scalar triple product of three vectors, and appreciate that its value is unchanged by cyclic permutation of the vectors.
	4	Be able to use the scalar triple product to find the volume of a parallepiped or tetrahedron, and the shortest distance between two skew lines.
	5	Be able to use the scalar triple product to determine the handedness of a triple of vectors, and whether or not two lines in 3 dimensions intersect.
Determinant and inverse of a 3×3 matrix.	6	Be able to calculate the determinant of any 3×3 matrix and the inverse of a non-singular 3×3 matrix.
Eigenvalues and eigenvectors of 2×2 and 3×3 matrices. Diagonalisation and powers of 2×2 and 3×3 matrices.	7	Understand the meaning of eigenvalue and eigenvector, and be able to find these for 2×2 or 3×3 matrices.
	8	Be able to form the matrix of eigenvectors and use this to reduce a matrix to diagonal form.
	9	Be able to find powers of a 2×2 or 3×3 matrix.
The use of the Cayley-Hamilton Theorem.	10	Understand the term *characteristic equation* of a 2×2 or 3×3 matrix.
	11	Understand that every 2×2 or 3×3 matrix satisfies its own characteristic equation, and be able to use this.

PURE MATHEMATICS 6

Notes	Notation	Exclusions
Distance of point (α, β, γ) from plane $n_1x+n_2y+n_3z+d = 0$ is $\dfrac{\lvert \alpha n_1 + \beta n_2 + \gamma n_3 + d \rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$		
Shortest distance between the two skew lines $\mathbf{r}=\mathbf{a}+t\mathbf{d}$ and $\mathbf{r}=\mathbf{b}+s\mathbf{e}$ is $\left\lvert (\mathbf{a}-\mathbf{b}) \cdot \dfrac{(\mathbf{d}\times\mathbf{e})}{\lvert \mathbf{d}\times\mathbf{e} \rvert} \right\rvert$.		
		Repeated eigenvalues. Complex eigenvalues.
$\det(\mathbf{M} - \lambda\mathbf{I}) = 0$ eg to find relations between powers of a matrix, or to find the inverse matrix.		Proof of the Cayley-Hamilton Theorem. Knowledge of the Jordan form.

PURE MATHEMATICS 6

Specification		Competence Statements
OPTION 2 **LIMITING PROCESSES**		
The limit of a function of a continuous variable.	P6l	1 Be able to differentiate a simple function from first principles.
		2 Be able to find limits of f(x) as $x \to a$ and as $x \to \infty$.
		3 Be aware when results on the limits of sums, products and quotients are being used.
L'Hôpital's rule.		4 Be able to use L'Hôpital's rule to find a limit as $x \to a$.
The fundamental theorem of calculus.		5 Appreciate the inverse relationship between differentiation and integration as expressed in the fundamental theorem.
Improper integrals.		6 Be able to evaluate integrals over an infinite domain or integrals where the integral becomes infinite by taking suitable limits.
Convergence of series.		7 Understand how the convergence of an infinite series depends on the limiting behaviour of the sequence of partial sums.
		8 Be able to decide whether or not a series converges in simple cases.
		9 Be able to find the limiting sum in simple cases.
Connections between the summation of series and definite integrals.		10 Be able to obtain connections between series and definite integrals by arguing from diagrams of the area under a curve and approximations to this area by upper or lower rectangles.
		11 Be able to use such connections to evaluate or estimate sums or integrals.

PURE MATHEMATICS 6

Notes	Notation	Exclusions

$$\mathrm{f}'(a) = \lim_{h \to 0} \frac{\mathrm{f}(a+h) - \mathrm{f}(a)}{h}$$

Knowledge of the results: $\lim\limits_{x \to \infty} x^n e^{-x} = 0$,

$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$, $\lim\limits_{x \to \infty} \frac{\ln x}{x^n} = 0$ for $n > 0$, $\lim\limits_{x \to 0+} (x^n \ln x) = 0$ for $n > 0$

and limits obtainable from these by simple substitution.

Exclusions: Formal definitions and theorems.

$$\lim_{x \to a}(\mathrm{f}(x) + \mathrm{g}(x)) = \lim_{x \to a} \mathrm{f}(x) + \lim_{x \to a} \mathrm{g}(x)$$

$$\lim_{x \to a}(\mathrm{f}(x)\mathrm{g}(x)) = \left(\lim_{x \to a} \mathrm{f}(x)\right)\left(\lim_{x \to a} \mathrm{g}(x)\right)$$

$$\lim_{x \to a}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\lim\limits_{x \to a} \mathrm{f}(x)}{\lim\limits_{x \to a} \mathrm{g}(x)} \text{ if } \lim_{x \to a} \mathrm{g}(x) \neq 0$$

f(a) = g(a) = 0 and g'(a) $\neq$ 0

$$\Rightarrow \lim_{x \to a} \frac{\mathrm{f}(x)}{\mathrm{g}(x)} = \frac{\mathrm{f}'(a)}{\mathrm{g}'(a)}.$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x \mathrm{f}(t)\,\mathrm{d}t\right) = \mathrm{f}(x).$$

eg $\int_0^\infty x e^{-x}\,\mathrm{d}x, \int_0^1 \frac{1}{\sqrt{x}}\,\mathrm{d}x.$

Simple cases only,

eg $\sum\limits_1^\infty \frac{1}{n(n+1)}, \ \sum\limits_1^\infty x^{n-1}, \ \sum\limits_1^\infty (-1)^n \frac{x^{2n+1}}{2n+1}.$

eg $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{n} \frac{n}{n^2 + r^2} \to \int_0^1 \frac{1}{1+x^2}\,\mathrm{d}x$

eg $\int_1^{n+1} \frac{1}{x}\,\mathrm{d}x < \sum\limits_{r=1}^{n} \frac{1}{r} < \int_1^n \frac{1}{x}\,\mathrm{d}x + 1.$

PURE MATHEMATICS 6

Specification		Competence Statements
OPTION 3 **MULTI-VARIABLE CALCULUS**		
$z = \mathrm{f}(x, y)$ and its interpretation as a surface. Contour lines, and sections of the form $z = \mathrm{f}(a, y)$ or $z = \mathrm{f}(x, b)$. Sketching of surfaces.	P6c	1 Appreciate that the relation $z = \mathrm{f}(x, y)$ defines a surface in three dimensions. 2 Be able to sketch contours and sections, and know how these are related to the surface.
First order partial derivatives.		3 Be able to find first order partial derivatives.
Simple applications to surfaces and stationary points.		4 Be able to use the conditions $\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$ to find the coordinates of stationary points on a surface.
Surfaces in three dimensions defined by $\mathrm{g}(x, y, z) = c$.		5 Appreciate that the relation $\mathrm{g}(x, y, z) = c$ defines a surface in three dimensions. 6 Be able to find **grad** g, and to evaluate this at a point on the surface to give a normal vector.
Applications to finding the normal line and the tangent plane at a point.		7 Be able to find the equations of the normal line and tangent plane at a point on the surface.
$\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$ and its application to errors.		8 Appreciate that the tangent plane gives a local approximation to the surface, and hence that $\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$, and be able to use this, or the similar result for functions of more than two variables, to estimate the consequence of errors in these variables.
OPTION 4 **DIFFERENTIAL GEOMETRY**		
Arc length.	P6g	1 Be able to calculate arc length using cartesian, parametric and polar co-ordinates.
Curved surface area of a solid of revolution.		2 Be able to calculate the volume and curved surface area of a solid of revolution using cartesian or parametric co-ordinates.
Envelopes.		3 Be able to find the envelope of a family of curves by eliminating p between $\mathrm{f}(x, y, p) = 0$ and $\frac{\partial \mathrm{f}}{\partial p}(x, y, p) = 0$.
Intrinsic co-ordinates and intrinsic equations.		4 Understand the use of arc length and inclination of tangent as intrinsic co-ordinates. Be able to work with intrinsic equations in simple cases.
Curvature, radius of curvature.		5 Be able to use the definitions of curvature and radius of curvature.
Centre of curvature.		6 Be able to find the centre of curvature.
Evolute.		7 Be able to find the evolute as the locus of the centre of curvature and as the envelope of the normals.

PURE MATHEMATICS 6

Notes	Notation	Exclusions
	$\frac{\partial z}{\partial x}$ or $\frac{\partial f}{\partial x}$ $z = f(x, y)$,	
If investigation of the nature of the stationary point is required the method will be given.		Second order partial derivative tests to determine the nature of a stationary point.
Surfaces may be defined by $z = f(x, y)$ or $g(x, y, z)=c$.	$\mathbf{grad}\, g = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{bmatrix}$	
Rotation about *x*- or *y*-axis only.		
eg $s = c\tan\psi$ (catenary), $s = 4a\sin\psi$ (cycloid).	s, ψ. Unit vectors are $\hat{t} = \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}, \hat{n} = \begin{bmatrix} -\sin\psi \\ \cos\psi \end{bmatrix}$	
$\kappa = \frac{d\psi}{ds}$, $\rho = \frac{ds}{d\psi}$; the cartesian and parametric forms.	ρ, κ.	
$\mathbf{c} = \mathbf{r} + \rho\hat{\mathbf{n}}$.		

PURE MATHEMATICS 6

Specification		Competence Statements
OPTION 5 **ABSTRACT ALGEBRA**		
The axioms of a group.	P6a	1 Understand the group axioms and the associated language.
Illustrations of groups.		2 Be familiar with examples of groups, and of the use of group tables.
Cyclic groups.		3 Understand the meaning of the term cyclic group, and how a single element can generate such a group.
The order of a finite group; the order of an element of a group.		4 Understand the terms order of a finite group, order of an element.
Subgroups.		5 Understand the term subgroup.
Lagrange's theorem.		6 Understand and be able to use Lagrange's theorem.
Isomorphism.		7 Understand that different situations can give rise to essentially the same structure.
		8 Be able to specify an isomorphism in simple cases.
The axioms of a real vector space.		9 Understand the axioms of a real vector space, and be familiar with simple examples.
Bases. Dimensions.		10 Understand the terms linearly independent set, spanning set, basis, dimension.
Linear mappings and their associated matrices.		11 Be able to find the matrix of a particular linear transformation with respect to particular bases and be aware that change of basis affects the matrix.

PURE MATHEMATICS 6

Notes	Notation	Exclusions
The terms binary operation, closed, associative, identity, inverse, abelian.		
e.g. symmetries of geometrical figures, residue classes, permutations, matrices.		
In a finite group the order of a sub-group divides the order of the group. The corollary that the order of an element divides the order of the group.		Proof of Lagrange's theorem.
Concept and illustrations only.		
eg be able to decide whether two examples of groups of order 4 are isomorphic.		
Finite dimensions only.		
		$\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for change of basis.

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MECHANICS

INTRODUCTION

The six modules in this section provide a thorough grounding in the principles and applications of mathematics to the concepts of force, motion and change.

From the outset there is an emphasis on the processes of mathematical modelling, which finds expression in both the written papers and the coursework element, following the outline of the flowchart on p.29.

Through the six modules there is a development in the use of pure mathematics techniques, which enables students to appreciate the power of these techniques and solving problems in the real world. In addition students should understand something of the role of mechanics in the history and growth of mathematics, and in technological advances.

In summary, the following topics are addressed.

Mechanics 1

Introduction to mathematical modelling and to the basic concepts in kinematics, statics and dynamics: the particle model; the language of vectors; motion in 1, 2 and 3 dimensions; Newton's Laws of Motion; the motion of a projectile.

Mechanics 2

Extension of *Mechanics 1* techniques to cover: the laws of friction; equilibrium of a rigid body; centres of mass; work, energy, power, momentum and impulse; elastic impact; conservation of energy and momentum.

Mechanics 3

Building on the work in *Mechanics 1* and *2,* and working at a more advanced level in the use of algebra and calculus techniques: circular motion; Hooke's Law; simple harmonic motion; centres of mass by integration; dimensional analysis.

Differential Equations (Mechanics 4)

Extending the work in *Pure Mathematics 3* on the formulation and solution of differential equations, with an emphasis on modelling and interpretation: separation of variables; intergrating factor, second and higher order equations; damped and forced oscillations; simultaneous equations; numerical methods of solution.

Mechanics 5

Development of calculus and vector methods, as applied to: motion under variable force; relative motion; motion described in polar co-ordinates; calculation of moments of inertia and the rotation of a rigid body.

Mechanics 6

Extending the range of techniques in dealing with the mechanics of bodies: angular momentum; vector product and systems of forces; stability of equilibrium and small oscillations; the motions of bodies with variable mass.

COURSEWORK

(C) 3.1a, 3.1b, 3.3 (N) 4.1, 4.2, 4.3 (IT) 3.2, 3.3
(PS) 3.1, 3.2, 3.3, (WO) 3.1, 3.2, 3.3

Rationale

The aims of the coursework in these modules are that students should learn how mathematics is used to solve real-world problems and that they should appreciate how the theory they have learnt for the examination helps them to do this.

The objectives are that they should be able to undertake the various steps in the problem solving procedure shown in the accompanying flow chart. The assessment criteria are closely related to these steps.

Description

The assessment for the modules Mechanics 2 and Differential Equations (Mechanics 4) includes one coursework task. The work undertaken requires mathematical modelling of a real-world situation. This is described in the section entitled in **Modelling** in the Specification for Mechanics 1 (pages 86 and 87) and pervades all the mechanics modules. The process is illustrated in the flow chart on page 29; it is divided into two aspects.

(i) The modelling cycle consists of pen and paper development of the consequences of the basic assumptions made, leading to a predicted outcome which must then be tested against reality.

(ii) In the experimental cycle, results are collected in order to give insight into the situation under investigation, so that a realistic model can be developed.

Some problems involve work within both of these cycles.

Content

The task for Mechanics 2 must be based on the mechanics within the specifications for Mechanics 1 and 2. That for Differential Equations (Mechanics 4) must be based on differential equations but the application need not necessarily be mechanics-based. A student whose knowledge of the subject is wider than these modules need not artificially ignore topics outside them, providing that the majority of the work lies within the stated specifications.

Each task represents 20% of the assessment and the work involved should be consistent with that figure, both in quantity and level of sophistication. Tasks which allow only superficial or trivial treatment should be avoided (for example kinematics involving only constant velocity).

Assessment

Each task for Mechanics 2 must be assessed on one of the coursework assessment sheets, A, B or C, which follow the module specification. The decision as to which is the appropriate sheet is made by the assessor according to the way the candidate has approached the particular task.

A In this case the modelling cycle is investigated in some depth, whilst the check against reality may use data from published sources, from experiments which the candidate has not actually performed or from experience; there must however be a quantitative element in such data.

B The work presented is approximately evenly divided between developing the model, and one or more experiments conducted by the candidate to verify the quality of predictions from it and/or to inform its development.

C The experimental work is thorough and detailed. It is used in the testing and/or development of a model (which may be a standard application of a part of the specification) including verification of the quality of predicted results.

No other mark sheet may be used, nor may these be amended in any way.

One mark is available for each criterion statement. Half marks may be awarded, but the overall total must be rounded (up or down) to an integer.

In the case of the Oral Communication domain two marks are available. These are given for the ability of the candidate to discuss or explain the work orally with conviction, and with broadly correct use of appropriate mechanics terms and theory. Assessors should use this stage of the assessment to assure themselves that the candidate understands what he/she has written.

The coursework assessment sheets for Differential Equations (Mechanics 4) are given at the end of the specification for that module. Note that in the case of Mark Scheme A, the marks for “Manipulating the Model” may be awarded for the quality of the work either on the first or the second modelling cycles.

Task Selection

Centres are encouraged to develop their own coursework tasks. If they have any doubt about the suitability of a proposed task, they are recommended to submit details of it to the Principal Coursework Moderator for Mechanics, via OCR.

However, Centres which are new to the scheme are strongly recommended to use the tasks published by MEI while they are familiarising themselves with the nature of the coursework. They should ensure that the material they have is that published for this specification and not that for an earlier syllabus.

Centres are advised that the choice of suitable tasks is crucial to the success of their candidates' coursework.

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MECHANICS 1 (2607) : AS

OBJECTIVES

To introduce the student to mathematical modelling and to the basic concepts in kinematics, statics and dynamics which underlie the study of mechanics.

Candidates will be expected to formulate models, using the mechanics within the specification, and to show an appreciation of any assumptions made; they will also be expected to make simple deductions from the model and to comment on its usefulness. They will understand the meaning of the particle model.

The examination will test candidates' knowledge of principles without excessive emphasis on algebraic or calculus skills.

ASSESSMENT

Examination (60 marks)

1 hour 20 minutes. Four compulsory questions of approximately equal weight to be answered.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly 9.8 $\mathrm{m\,s^{-2}}$.

ASSUMED KNOWLEDGE

Knowledge of the techniques covered in *Pure Mathematics 1* is assumed.

MECHANICS 1

Specification		Competence Statements
MODELLING - *This section is fundamental to all the Mechanics syllabuses.*		
The modelling cycle applied to real-world problems.	M1p1	Understand the concept of a mathematical model.
	2	Be able to abstract from a real world situation to a mathematical description (model).
	3	Know the language used to describe simplifying assumptions.
	4	Understand the particle model.
	5	Be able to analyse the model appropriately.
	6	Be able to interpret and communicate the implications of the analysis in terms of the situation being modelled.
	7	Appreciate the importance of a check against reality.
	8	Appreciate that a model may need to be progressively refined.
S.I. Units.	9	Know the relevant S.I. units.
VECTORS		
The properties of vectors and techniques associated with them in 2 or 3 dimensions.	M1v1	Understand the language of vectors.
	2	Be able to find the magnitude and direction of a vector given in component form.
	3	Be able to express a vector in component form given its magnitude and direction.
	4	Be able to carry out elementary operations on vectors.
	5	Be able to apply vectors to mechanics problems.
KINEMATICS		
Motion in 1 dimension.	M1k1	Understand the language of kinematics.
The accurate use of terminology.	2	Know the difference between position, displacement and distance.
	3	Know the difference between velocity and speed, and between acceleration and magnitude of acceleration.
Kinematics graphs.	4	Be able to draw and interpret kinematics graphs, knowing the significance (where appropriate) of their gradients and the areas underneath them.
The use of calculus in kinematics.	5	Be able to differentiate position and velocity with respect to time and know what measures result.
	6	Be able to integrate acceleration and velocity with respect to time and know what measures result.
The use of constant acceleration formulae.	7	Be able to recognise when the use of constant acceleration formulae is appropriate.
	8	Be able to solve kinematics problems using constant acceleration formulae and calculus

MECHANICS 1

Notes	Notation	Exclusions
Flow chart on page 29.		
The words: light; smooth; uniform; particle; inextensible; thin; rigid.		
Manipulation of the mathematical model.		
The implications in real world terms. The need for estimation of accuracy.		
A modelling exercise which is not in some way checked against reality is of little or no value.		
Metre (m), kilogram (kg), second (s), metre/second (ms^{-1}), metre/second/second (ms^{-2}), newton (N).		
Vector, scalar, unit vector, position vector, component, magnitude, direction, resultant.	Vectors printed in **bold**. Unit vectors **i**, **j**, **k**. The magnitude of the vector **a** is written $\lvert\mathbf{a}\rvert$ or *a*. Position vector $\overrightarrow{OP}$ or **r**.	
	Column vector eg $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$	The direction of a vector in 3 dimensions.
Addition, subtraction, multiplication by a scalar carried out algebraically and geometrically.		Product of two vectors.
Problems involving motion and forces.		
Position, displacement, distance; speed, velocity; acceleration, magnitude of acceleration; relative velocity (in 1 dimension).		
Position-time, distance-time, velocity-time, speed-time, acceleration-time.		
		Cases where the time dependency is not of polynomial form will not be tested in this module.

MECHANICS 1

Specification		Competence Statements
Motion in 2 and 3 dimensions.	M1k9	Understand the language of kinematics appropriate to motion in 2 and 3 dimensions.
	10	Be able to extend the scope of techniques from motion in 1 to that in 2 and 3 dimensions by using vectors.
	11	Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time.
	12	Be able to use vectors to solve problems in kinematics.
FORCE		
The identification of the forces acting on a body and their representation in a diagram.	M1d1	Understand the language relating to forces.
	2	Be able to identify the forces acting on a system and represent them in a force diagram.
Vector treatment of forces.	3	Be able to resolve a force into components and be able to select suitable directions for resolution.
	4	Be able to find the resultant of several concurrent forces by vector addition.
	5	Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero.
	6	Know that forces in equilibrium form a closed polygon.
	7	Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by way of a polygon of forces.
NEWTON'S LAWS OF MOTION		
The application of Newton's laws of motion to a particle.	M1n1	Know and understand the meaning of Newton's three laws.
	2	Understand the term equation of motion.
	3	Be able to formulate the equation of motion for a particle in 1-dimensional motion.
	4	Be able to formulate the equation of motion for a particle in 2- and 3-dimensional motion.
	5	Be able to formulate and solve separate equations of motion for connected particles.

MECHANICS 1

Notes	Notation	Exclusions
Position vector; relative position.		
The use of calculus and the use of constant acceleration formulae.	$\mathbf{a} = \dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt}$, $\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$ $\mathbf{r} = \int \mathbf{v}\,dt$, $\mathbf{v} = \int \mathbf{a}\,dt$,	
	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$, $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$.	Vector form of $v^2 - u^2 = 2as$.
At least one of the components will be a linear function of time.		Cases involving motion in 3 dimensions.
Weight, tension, thrust, normal reaction (or normal contact force), frictional force, resistance.		Calculations involving coefficient of friction.
eg horizontally and vertically or parallel and perpendicular to an inclined plane.		
Graphically or by adding components.		
Lami's Theorem may be used where appropriate.		
	$\mathbf{F} = m\mathbf{a}$.	Variable mass.
Including motion under gravity.		
eg simple pulley systems, trains.		

MECHANICS 1

Specification		Competence Statements
PROJECTILES		
The motion of a projectile.	M1y1	Be able to formulate the equations of motion of a projectile.
	2	Know how to find the position and velocity at any time of a projectile, including the maximum height and range.
	3	Be able to find the initial velocity of a projectile given sufficient information.
	4	Be able to eliminate time from the component equations that give the horizontal and vertical displacement in terms of time.
	5	Be able to obtain the cartesian equation of the trajectory of a projectile.
	6	Be able to form an equation in $\tan\alpha$ (where α is the angle of projection) given the initial speed and a point on the trajectory.
	7	Be able to solve such a quadratic equation to find α.
	8	Be able to explain the meanings of α having 2, 1 or 0 values.
	9	Be able to solve problems involving projectiles.

MECHANICS 1

Notes	Notation	Exclusions
		Air resistance. Inclined plane. Recall of formulae.
$y = x\tan\alpha - \frac{1}{2}\frac{g\,x^2}{v^2\cos^2\alpha}$.		
Within range, at extreme range or out of range.		

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MECHANICS 2 (2608) : A2

OBJECTIVES

To build on the work in *Mechanics 1* by extending the range of mechanics concepts which the student is able to use in modelling situations. Candidates will be able to use the rigid body model in simple cases involving moments.

Candidates will be expected to apply the modelling principles detailed in the *Mechanics 1* specification in the context of this module.

The examination will test candidates' understanding of principles and of when they should be applied. The examination will avoid excessive emphasis on algebraic or calculus skills, but candidates will be expected to interpret simple algebraic expressions.

ASSESSMENT

Examination (60 marks)

1 hour and 20 minutes. Four compulsory questions of approximately equal weight to be answered.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly $9.8\,\mathrm{m\,s^{-2}}$.

Coursework (15 marks)

There is one assignment involving modelling.

ASSUMED KNOWLEDGE

Knowledge of the modules *Mechanics 1, Pure Mathematics 1* and *Pure Mathematics 2* is assumed.

MECHANICS 2

Specification		Competence Statements
FORCE		
Frictional force.	M2d1	Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to draw an appropriate force diagram.
	2	Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction).
	3	Understand that the frictional force may be modelled by $F \leqslant \mu R$.
	4	Be able to apply Newton's Laws to problems involving friction.
Rigid bodies in equilibrium subject to forces in two dimensions.	5	Be able to draw a force diagram for a rigid body.
	6	Understand that a system of forces can have a turning effect on a rigid body.
	7	Be able to calculate the moment about a fixed axis of a force acting on a body.
	8	Understand and be able to apply the conditions for equilibrium of a rigid body to the solution of problems.
	9	Be able to identify whether equilibrium will be broken by sliding or toppling in simple cases.
Light frameworks.	10	Be able to find the internal forces in a framework of light, pin-jointed rods by applying the conditions for equilibrium at the pin joints.
	11	Be able to determine which rods in a framework are in compression and which in tension.
WORK, ENERGY AND POWER		
Concepts of work and energy.	M2w1	Be able to calculate the work done, both by a force which moves along its line of action and by a force which moves at an angle to its line of action.
	2	Be able to calculate kinetic energy.
	3	Understand the term mechanical energy.
The work-energy principle.	4	Understand the work-energy principle.
	5	Understand the terms conservative and dissipative forces.
	6	Be able to calculate gravitational potential energy.
	7	Be able to solve problems using the principle of conservation of energy.
Power.	8	Understand that the power of a force is the rate at which it does work.
	9	Be able to apply the concept of power to the solution of problems.

MECHANICS 2

Notes	Notation	Exclusions
Smooth is used to mean frictionless.		
Limiting friction.		The term angle of friction
In cases where the particle model is not appropriate.		
eg simple levers.		
Both as the product of force and perpendicular distance of the axis from the line of action of the force and by first resolving the force into components.		Vector treatment.
The resultant of all the applied forces is zero and the sum of their moments about any axis is zero. Three forces in equilibrium must be concurrent.		
eg a cube on an inclined plane.		
The term thrust may be used to describe a compression force.		Bow’s notation.
		Continuously variable forces. **F.s**.
	$\frac{1}{2}mv^2$.	
The total work done by all the external forces acting on a body is equal to the increase in the kinetic energy of the body.		
Relative to some arbitrary zero level.	mgh.	
eg the maximum height of a projectile, a particle sliding down a curved surface, a child swinging on a rope.		
Power is Force × Speed in the direction of the force.		
The concept of average power.		
eg finding the maximum speed of a vehicle.		

MECHANICS 2

Specification		Competence Statements
MOMENTUM AND IMPULSE		
Momentum and impulse treated as vectors.	M2i1	Be able to calculate the impulse of a force as a vector.
	2	Understand the concept of momentum and appreciate that it is a vector quantity.
	3	Understand and be able to apply the Impulse-Momentum equation to problems.
Conservation of linear momentum.	4	Understand that a system subject to no external force conserves its momentum.
	5	Be able to derive the conservation of momentum equation for a collision between two particles in one dimension.
	6	Be able to apply the principle of conservation of momentum to direct impacts within a system of bodies.
Coefficient of restitution.	7	Understand Newton's Experimental Law and the meaning of coefficient of restitution, and be able to apply it in modelling impacts.
	8	Be able to solve problems using both momentum conservation and Newton's Experimental Law.
	9	Understand that mechanical energy is not conserved during impacts (unless $e = 1$) and be able to find the loss of mechanical energy.
Oblique impact.	10	Understand that in an oblique impact between an object and a plane, the impulse acts in a direction normal to the plane.
	11	Know that velocity parallel to the plane is unchanged by impact.
	12	Know that the direction of the component of the velocity perpendicular to plane is reversed and that its magnitude is multiplied by the coefficient of restitution.
	13	Be able to calculate the loss of kinetic energy in an oblique impact.
	14	Be able to solve problems involving oblique impact.
CENTRE OF MASS		
Centre of mass of a set of point masses.	M2G1	Be able to find the centre of mass of a system of particles of given position and mass.
Centre of mass of simple shapes.	2	Appreciate how to locate centre of mass by appeal to symmetry.
Centre of mass of composite bodies.	3	Be able to find the centre of mass of a composite body by considering each constituent part as a particle at its centre of mass.
Applications of the centre of mass.	4	Be able to use the position of the centre of mass in problems involving the equilibrium of a rigid body.

MECHANICS 2

Notes	Notation	Exclusions
Impulse = Force × time.		The use of calculus for variable forces.
The total impulse of all the forces acting on a body is equal to the change in the momentum of the body. Problems may involve an understanding of relative velocity in 1 dimension.		
eg colliding railway trucks.		
eg between two spheres, or between a sphere and a wall.	*e* for coefficient of restitution.	
eg a ball bouncing following projectile motion.		
In 1, 2 and 3 dimensions.	$(\bar{x}, \bar{y}, \bar{z})$ $(\sum_i m_i)\bar{x} = \sum_i (m_i x_i)$	Calculus methods. Non-uniform bodies.
Rod, rectangular lamina, circular lamina, cuboid, sphere.		
Composite bodies may be formed by the addition or subtraction of parts. Where a composite body includes parts whose centre of mass the student is not expected to know (eg triangle, semicircle, cone), the centre of mass will be given.		
eg a suspended object or an object standing on an inclined plane.		

MEI STRUCTURED MATHEMATICS

MECHANICS 2 (2608)

COURSEWORK ASSESSMENT SHEET

MARK SCHEME A - Work based on the Modelling Cycle

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Making assumptions and relating them to the model	1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The relevance of the assumptions to the initial equations of the model is explained clearly.		
Manipulating the model	1 1 1 1	Using appropriate mechanics theory a set of initial equations is obtained. The equations are manipulated clearly and correctly. Values for the parameters of the model are chosen and justified. A set of predictions is produced from the model.		
The collection of data to verify the model	1	The source and means of collecting the data are clearly described and their relevance to the model is demonstrated.		
Discussion of variation in the parameters	1 1	There is quantitative consideration of the effects of possible variation in the values of the parameters of the model. The effects of this variation are illustrated (by tables and graphs, as appropriate).		
Comparison between the data collected and the predictions of the model	1 1	There is a clear comparison made, using diagrams and graphs where appropriate, and taking into account the effects of variation as above. Bearing in mind this variation, there is sensible discussion of the quality of the predictions made by the model.		
Revision of the process	1 1	In the light of the comparison, proposals are made for amending the initial assumptions to improve the quality of the model. There is a discussion of the effects that these amendments may have on the model (but these are not necessarily worked through in detail, unless the initial modelling was trivial).		
Oral Communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.
Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

MEI STRUCTURED MATHEMATICS

MECHANICS 2 (2608)

COURSEWORK ASSESSMENT SHEET

MARK SCHEME B - Work where Modelling and Experiment are evenly matched

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Making assumptions and relating them both to the model and the experiment	1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The assumptions are shown to be relevant to the initial equations of the model used and to the design of the experiment.		
Manipulating the model	1 1	Using appropriate mechanics theory a set of initial equations is obtained. The equations are manipulated clearly and correctly to produce a set of predictions.		
Conducting the experiment	1 1 1	The conduct of the experiment is described clearly, including diagrams of the apparatus used. Any steps taken to reduce experimental error are also described. Sufficient initial results are obtained and they are presented clearly and concisely. The initial results are processed into a form suitable for comparison with the model.		
Discussion of variation in the experimental results	1 1	There is quantitative consideration of the effects of possible variation in the experimental measurements. Graph(s) are drawn to illustrate the effects of this variation.		
Comparison between the experimental results and the predictions of the model.	1 1	There is a clear comparison made, using diagrams and graphs where appropriate, and taking into account the effects of variation as above. Bearing in mind this variation, there is sensible discussion of how good a match there is between the experiment and the model.		
Revision of the process	1 1	In the light of the comparison, to improve the match between model and experiment, proposals are made for amending the initial assumptions and/or the conduct of the experiment. There is a discussion of the effects that these amendments may have (but these are not necessarily worked through in detail, unless the initial work was trivial).		
Oral Communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

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Signed
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MEI STRUCTURED MATHEMATICS

MECHANICS 2 (2608)

COURSEWORK ASSESSMENT SHEET

MARK SCHEME C - Work based on the Experimental Cycle

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Making assumptions and relating them to both the model and the experiment	1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The assumptions are shown to be relevant to the design of the experiment and to the mechanics theory used.		
Conducting the experiment	1 1 1 1	The conduct of the experiment is described clearly, including diagrams of the apparatus used. There is a description of the steps taken to ensure the quality of the experimental measurements. Sufficient results are obtained and they are presented clearly and concisely. The results are processed into a form suitable for comparison with the predictions of the model.		
Presenting the model	1	The appropriate mechanics theory is presented clearly and correctly to produce a set of predictions.		
Discussion of variation in the experimental results	1 1	There is quantitative consideration of the effects of possible variation in the experimental measurements. Graph(s) are drawn to illustrate the effects of this variation.		
Comparison between the experimental results and the predictions of the model	1 1	There is a clear comparison made, using diagrams and graphs where appropriate, and taking into account the effects of variation as above. Bearing in mind this variation, there is sensible discussion of how well the experimental results are explained by the model.		
Revision of the process	1 1	In the light of the comparison, to improve the match between model and experiment, proposals are made for amending the initial assumptions and/or the conduct of the experiment. There is a discussion of the effects that these amendments may have (but these are not necessarily worked through in detail, unless the initial work was trivial).		
Oral Communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

TOTAL | 15

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

MECHANICS 3 (2609) : A2

OBJECTIVES

To build on the work in *Mechanics 1* and *2*, further extending the range of Mechanics concepts which the student is able to use in modelling situations.

Candidates will be expected to apply the modelling principles detailed in the *Mechanics 1* specification in the context of this module.

The examination questions will be designed to test candidates' understanding of the principles involved rather than a high degree of manipulative skill, but candidates will be expected to interpret simple expressions written in algebra and in the language of calculus.

ASSESSMENT

Examination (60 marks)

1 hour 20 minutes. Four compulsory questions of approximately equal weight to be answered. The questions may involve work on the specification for *Mechanics 1* and *2* but requiring techniques in *Pure Mathematics 2* and *3*. Questions will not necessarily be on a single topic.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly $9.8\,\mathrm{m\,s^{-2}}$.

ASSUMED KNOWLEDGE

Knowledge of the modules *Mechanics 1* and *2*, *Pure Mathematics 1* and *2* is assumed. In addition it is recommended that this module is studied after, or at the same time as, *Pure Mathematics 3*.

MECHANICS 3

Specification		Competence Statements
DIMENSIONAL ANALYSIS		
Dimensional consistency.	M3q1	Be able to find the dimensions of a quantity in terms of M, L, T.
	2	Understand that some quantities are dimensionless.
	3	Be able to determine the units of a quantity by reference to its dimensions.
	4	Be able to change the units in which a quantity is given.
	5	Be able to use dimensional analysis as an error check.
Formulating models using dimensional arguments.	6	Use dimensional analysis to determine unknown indices in a proposed formula.
CIRCULAR MOTION		
The language of circular motion.	M3r1	Understand the language associated with circular motion.
Modelling circular motion.	2	Identify the force(s) acting on a body in circular motion.
	3	Be able to calculate acceleration towards the centre of circular motion.
Circular motion with uniform speed.	4	Be able to solve problems involving circular motion with uniform speed.
Circular motion with non-uniform speed.	5	Be able to solve problems involving circular motion with non-uniform speed.
	6	Be able to calculate tangential acceleration.
	7	Be able to solve problems involving motion in a vertical circle.
	8	Identify the conditions under which a particle departs from circular motion.
HOOKE'S LAW		
Extension of an elastic string and extension or compression of a spring.	M3h1	Be able to calculate the stiffness or modulus of elasticity in a given situation.
	2	Be able to calculate the tension in an elastic string or spring.
	3	Be able to calculate the equilibrium position of a system involving elastic strings or springs.
	4	Be able to calculate energy stored in a string or spring.
	5	Be able to use energy principles to determine extreme positions.

MECHANICS 3

Notes	Notation	Exclusions
eg density, energy, momentum.	M, L, T, [ ].	
eg density from kg m^{-3} to g cm^{-3}.		
eg for the period of a pendulum.		
The terms tangential, radial and angular speed, radial component of acceleration, tangential component of acceleration.	$\dot{\theta}$, ω for angular speed.	
Candidates will be expected to set up equations of motion in simple cases.	$v = r\dot{\theta}$ or $r\omega$	
Using the expressions v^2/r and $r\dot{\theta}^2$.		
eg a conical pendulum, a car travelling horizontally on a cambered circular track.		
Tangential component of acceleration = $r\ddot{\theta}$. Use of Newton's 2nd law in tangential direction.		
The use of conservation of energy, and of $F = ma$ in the radial direction.		
eg when a string becomes slack, when a particle leaves a surface.		
eg a weight suspended by a spring.	$T = kx$ where k is the stiffness. $= \dfrac{\lambda x}{l_0}$ where λ is the modulus of elasticity and l_0 the natural length.	
	$\dfrac{1}{2}\dfrac{\lambda x^2}{l_0}$ or $\dfrac{1}{2}kx^2$.	
Application to maximum extension for given starting conditions in a system, whether horizontal or vertical.		

MECHANICS 3

Specification		Competence Statements
SIMPLE HARMONIC MOTION		
The Simple Harmonic Motion equation and its solution.	M3o1	Recognise situations which may be modelled by SHM.
	2	Be able to recognise the standard form of the equation of motion of SHM and formulate it as appropriate.
	3	Be able to recognise the SHM equation expressed in non-standard forms and to transform it into the standard form by means of substitution.
	4	Recognise the solution of the SHM equation in the form $x = a\sin(\omega t + \varepsilon)$ and be able to interpret it.
	5	Recognise other forms of the solution of the SHM equation, and be able to relate the various forms to each other.
	6	Be able to select a form of the solution of the SHM equation appropriate to the initial conditions.
	7	Be able to verify solutions of the SHM equation using calculus.
	8	Be able to apply standard results for SHM in context.
Applications of Simple Harmonic Motion.	9	Be able to analyse motion under the action of springs or strings as examples of SHM.
	10	Be able to calculate suitable constants to model given data by SHM equations.
SOLID BODIES AND PLANE LAMINAE		
Centres of Mass.	M3g1	* Be able to calculate the volume generated by rotating a plane region about an axis.
	2	Be able to use calculus methods to calculate the centre of mass of solid bodies formed by rotating a plane area about an axis.
	3	Be able to find the centre of mass of a compound body, parts of which are solids of revolution.
	4	Be able to use calculus methods to calculate the centres of mass of plane laminae.
	5	Apply knowledge of centres of mass to simple cases of equilibrium.

MECHANICS 3

Notes	Notation	Exclusions
Including approximate cases such as a pendulum.		
The form $\ddot{x} = -\omega^2 x$.		
eg $\ddot{x} + cx = 0$, $\ddot{x} = -\omega^2(x + k)$.		
x can represent variables such as angles and population size.		
The sigificance of the constants a, ω and ε should be understood. $x = a\cos(\omega t + \varepsilon)$, $x = A\sin\omega t + B\cos\omega t$.	a = amplitude, T = period = $2\pi/\omega$. ε = phase. $a = \sqrt{(A^2 + B^2)}$.	Damped oscillations. Solution of the SHM equation other than by verification.
	$v^2 = \omega^2(a^2 - x^2)$, $T = \dfrac{2\pi}{\omega}$.	
Differentiation of sine and cosine. eg $v^2 = \omega^2(a^2 - x^2)$, $T = \dfrac{2\pi}{\omega}$.		
Rotation about the x- and y-axes only. * This topic also occurs in Pure Mathematics 3. It is included here for completeness.		The use of non-cartesian coordinates.
eg hemisphere, cone.		Variable density.
By treatment as equivalent to a finite system of particles.		Pappus' theorems.
Including composite bodies.		

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DIFFERENTIAL EQUATIONS (MECHANICS 4) (2610) : A2

Objectives

To extend the work in *Pure Mathematics 3* on the formulation and solution of Differential Equations. Candidates are expected to have a reasonable degree of manipulative competence and so to be able to handle more complicated problems.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions from four set.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly $9.8\,\mathrm{m\,s^{-2}}$.

Component 2: Coursework (15 marks)

One modelling assignment involving the use of differential equations at an appropriate level of sophistication. The context is not restricted by the syllabuses for *Mechanics 1, 2* and *3*.

Assumed Knowledge

Knowledge of basic kinematics, Newton's Second Law and of the techniques in *Pure Mathematics 1, 2* and *3* is assumed. In particular a knowledge of first order differential equations with separable variables, as covered in *Pure Mathematics 3*, is assumed.

DIFFERENTIAL EQUATIONS (MECHANICS 4)

Specification		Competence Statements
MODELLING WITH DIFFERENTIAL EQUATIONS		
Construction of models.	M4p1	Understand how to introduce and define variables to describe a given situation in mathematical terms.
	2	Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations.
	3	Know the language of kinematics, and the relationships between the various terms.
	4	Know Newton's 2nd Law of motion.
	5	Understand how to determine the order of a differential equation.
Interpretation of solutions.	6	Be able to interpret the solution of a differential equation in terms of the original situation.
	7	Appreciate the difference between a general solution and a particular solution, ie one which satisfies particular prescribed conditions.
	8	Understand the significance of the number of arbitrary constants in a general solution.
	9	Be able to investigate the effect of changing a differential equation on its solution.
Tangent fields	10	Be able to sketch the tangent field for a 1st order differential equation and be able to interpret it.
	11	Be able to sketch and interpret the curve of the solution corresponding to particular conditions.
	12	Be able to identify isoclines and use them in sketching and interpreting tangent fields.
FIRST ORDER DIFFERENTIAL EQUATIONS		
Equations with separable variables.	M4c1	Be able to find both general and particular solutions of a 1st order differential equation with separable variables.
First order linear differential equations.	2	Be able to solve 1st order linear differential equations with constant coefficients.
	3	Be able to distinguish differential equations where the integrating factor method is appropriate, and to rearrange such equations if necessary.
	4	Be able to find an integrating factor and understand its significance in the solution of an equation.
	5	Be able to solve an equation using an integrating factor and find both general and particular solutions.

DIFFERENTIAL EQUATIONS (MECHANICS 4)

Notes	Notation	Exclusions
Candidates will be expected to be able to model real-life situations using differential equations.		
The differential equations will not be restricted to those which candidates can solve analytically.		
Including acceleration = $v\dfrac{dv}{ds}$.	$v = \dfrac{ds}{dt}$. $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$	
In the form *F=ma*.		Variable mass.
eg changing the values of coefficients.		
	The term direction field has the same meaning as tangent field.	
eg the solution through a particular point.		
Equations of the form $y' + ay = 0$ and $y' + ay = f(x)$ in simple cases, where a is constant		
Equations which can be expressed in the form $\dfrac{dy}{dx} + P(x)y = Q(x)$.		
Integrating factor $e^{\int P dx}$.		
eg the solution through a particular point.		

DIFFERENTIAL EQUATIONS (MECHANICS 4)

Specification		Competence Statements
SECOND & HIGHER ORDER DIFFERENTIAL EQUATIONS		
Homogenous second order linear differential equations.	M4c6	Be able to solve homogeneous 2nd order differential equations, using the auxiliary equation and complementary function.
	7	Appreciate the relationship between different cases of the solution and the nature of the roots of the auxiliary equation, and be able to interpret these different cases graphically.
	8	Be able to find the particular solution in given contexts.
The general second order linear differential equation.	9	Be able to solve the general 2nd order linear differential equation, by solving the homogeneous case and adding a particular integral.
	10	Be able to find particular integrals in simple cases.
	11	Appreciate the relationship between different cases of the solution and the nature of the roots of the auxiliary equation, and be able to interpret these different cases graphically.
	12	Be able to solve the equation for simple harmonic motion, $\ddot{x} + \omega (x + k) = 0$, and be able to relate the various forms of the solution to each other.
Damped and forced oscillations.	13	Be able to model damped and forced oscillations (including resonance) using 2nd order linear differential equations, and understand the associated terminology.
	14	Be able to interpret the solutions of equations modelling damped and forced oscillations in words and graphically.
Higher order linear differential equations.	15	Appreciate that the same methods can be extended to higher order equations and be able to do so in simple cases.
SIMULTANEOUS DIFFERENTIAL EQUATIONS		
Simultaneous linear differential equations.	M4e1	Model situations with one independent variable and two dependent variables which lead to 1st order simultaneous differential equations, and know how to solve these by eliminating one variable to produce a single, 2nd order equation.
	2	Appreciate that the same method can be extended to more than two such equations, leading by elimination to a single higher order equation.
NUMERICAL METHODS		
Step by step methods.	M4s1	Be able to use step by step methods (eg Euler's method) to solve 1st order differential equations (including simultaneous equations) where appropriate.

DIFFERENTIAL EQUATIONS (MECHANICS 4)

Notes	Notation	Exclusions
Equations of the form $y'' + ay' + by = 0$ where a and b are constants.		
Discriminant > 0, = 0, < 0. A basic understanding of the complex roots of a quadratic equation is assumed.		
Equations of the form $y'' + ay' + by = \mathrm{f}(x)$ where a and b are constants.		
Where $\mathrm{f}(x)$ is a polynomial, trigonometric or exponential function.		
$\mathrm{f}(x)$ can be chosen to lead to forced or resonant oscillations.	The damping will be referred to as “over-”, “critical-”, or “under-”, according as the roots of the auxiliary equation are real distinct, equal or complex.	
In examination questions adequate guidance will be given.		
Applications such as predator-prey models.		
Adequate guidance will be given in any examination question involving more than two dependent variables.		
In examination questions adequate guidance will be given.		Specific step by step algorithms.

MEI STRUCTURED MATHEMATICS

DIFFERENTIAL EQUATIONS (MECHANICS 4) (2610)

COURSEWORK ASSESSMENT SHEET

MARK SCHEME A - Work based on the Modelling Cycle

TASK: Candidates will model a real-life situation of their own choice which requires the use of differential equations.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Simplifying the situation and setting up the model	1 1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The relevance of the assumptions to the initial equations of the model is explained clearly. Differential equation(s) to model the situation are established and justified.		
Manipulating the model	1 1 1 1 1	A correct method of solution is applied to the differential equation(s). A solution to the differential equation(s) is obtained. Values for the parameters of the equation(s) are chosen and justified. There is quantitative consideration of the effects of possible variation in the values of the parameters. A set of predictions is produced from the equation(s).		
The collection of data to verify the model	1 1	The source and means of collecting the data are clearly described and their relevance to the model is demonstrated. The data are presented in a form suitable for comparison with the predictions of the model.		
Comparison between the data collected and the predictions of the model	1	A clear comparison is made, using diagrams and graphs where appropriate, and taking into account the effects of variation in the parameters as above.		
Revision of the model	1 1	In the light of the comparison, proposals are made for amending the initial assumptions to improve the quality of the model. New equations are established as a result of the amended assumptions.		
Assessment of the improvement obtained	1 1	The new equations are manipulated to produce new predictions. A comparison is made between the data and the new predictions, and comments made on whether an improvement has been obtained.		
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

MEI STRUCTURED MATHEMATICS
DIFFERENTIAL EQUATIONS (MECHANICS 4) (2610)
COURSEWORK ASSESSMENT SHEET
MARK SCHEME B - Work based on a combination of the Modelling and Experimental cycles

TASK: Candidates will model a real-life situation of their own choice which requires the use of differential equations.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Simplifying the situation setting up the model	1 1 1	There are clear statements of the assumptions made and an awareness is shown of the relative importance of these assumptions. The relevance of the assumptions to the initial equations of the model is explained clearly. Differential equation(s) to model the situation are established and justified.		
Manipulating the model	1 1 1 1	A correct method of solution is applied to the differential equation(s). A solution to the differential equation(s) is obtained. Values for the parameters of the equation(s) are chosen and justified. A set of predictions is produced from the equations.		
Conducting the experiment	1 1 1 1	The relevance of the assumptions to the design of the experiment is explained clearly. The conduct of the experiment is described clearly, including diagrams of the apparatus used. Any steps taken to reduce experimental error are also described. Sufficient results are obtained, and these are presented clearly and concisely, in a form suitable for comparison with the predictions of the model. There is a discussion of the variability in the measurements taken.		
Comparison between the experimental results and the predictions	1	A clear comparison is made, using diagrams and graphs where appropriate, and taking into account the effects of variation in the parameters and in the measurements taken.		
Revision of the process	1 1	In the light of the comparison, a decision is made, and justified, on whether further revision is needed to the modelling process, or to the conduct of the experiment, or both. If revision is needed (or if the original assumptions led to a trivial piece of modelling), detailed proposals are made for amending the assumptions and/or the conduct of the experiment. If not, there is a serious discussion of how the work could be extended to related situations.		
Assessment of the improvement obtained.	1	There is an investigation into the effects of applying the proposed revision/extension. [N.B. It is not necessary for the candidate to rework the model completely unless the initial differential equation(s) were trivial.]		
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.
Coursework should be available for moderation by OCR.

Authentication by the Centre

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MECHANICS 5 (2611) : A2

Objectives

To develop the use of calculus and vector methods.

Candidates are expected to be technically competent in the use of calculus and able to apply it to a variety of situations.

Candidates are expected to apply the modelling principles detailed in the *Mechanics 1* specification in the context of this module.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions out of four set. One question will be set on each specification section.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly $9.8\,\mathrm{m\,s^{-2}}$.

Assumed Knowledge

Knowledge of the modules *Mechanics 1, 2, 3, Differential Equations (Mechanics 4)* and *Pure Mathematics 1, 2* and *3* is assumed.

MECHANICS 5

Specification		Competence Statements
VARIABLE FORCES		
Application of variable forces in 1 dimension.	M5d1	Be able to calculate measures involving variable forces, in given dynamic situations in 1 dimension.
	2	Be able to formulate and solve differential equations using an appropriate expression for acceleration.
Variable forces in 2 dimensions.	3	Be able to solve problems involving variable forces in 2-dimensional situations, using vectors.
	4	Be familiar with the scalar product and use it when appropriate.
RELATIVE MOTION		
Vector treatment of relative motion in 1, 2 or 3 dimensions.	M5k1	Understand relative displacement, velocity and acceleration and their interrelationships.
	2	Be able to apply graphical and analytical techniques, as appropriate.
MOTION DESCRIBED IN POLAR CO-ORDINATES		
Differentiation of vectors of constant or variable magnitude.	M5g1	Be able to find the rate of change of a unit vector of variable direction.
Radial and transverse components of velocity and acceleration.	2	Be able to derive and apply the polar expressions for velocity and acceleration.
Motion under a central force.	3	Understand the significance and use of the result $r^2\dot{\theta}$ = constant for central force systems.
	4	Be able to apply the substitution $r = \dfrac{1}{u}$ in appropriate cases.
ROTATION OF A RIGID BODY		
Calculation of moment of inertia.	M5r1	Understand the concept of moment of inertia as the analogue of mass, in rotational motion.
Rotation of a rigid body about a fixed axis.	2	Be able to calculate moments of inertia of simple plane shapes and solids of uniform density from first principles.
	3	Know and use the perpendicular and parallel axes theorems.
	4	Be able to calculate centres of mass and moments of inertia of bodies of variable density and of compound bodies.
The equation of motion.	5	Be able to formulate the equation of motion of a rigid body about a fixed axis.
Kinetic energy of rotation.	6	Be able to apply the principle of conservation of energy to rotational motion of a rigid body.

MECHANICS 5

Notes	Notation	Exclusions
Work, energy, power, impulse.	work = $\int F\mathrm{d}s$, energy = $\int P\mathrm{d}t$, impulse = $\int F\mathrm{d}t$, power = Fv.	
eg the terminal velocity of a particle falling in a resistive medium.	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$	
eg the work done by a particle moving along a given path through a force field; the motion of a projectile in a resistive medium.	work = $\int \mathbf{F}.\mathrm{d}\mathbf{s}$ power = $\mathbf{F}.\mathbf{v}$	
	${}_A\mathrm{V}_B$ = Velocity of A relative to B.	
Problems of interception and closest approach.		
	$\mathbf{v} = \begin{pmatrix} \dot{r} \\ r\dot{\theta} \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \ddot{r} - r\dot{\theta}^2 \\ 2\dot{r}\dot{\theta} + \ddot{\theta} \end{pmatrix}$	
Motion under forces of the form $k\mathbf{r}$ or $\frac{k\mathbf{r}}{r^3}$ (planetary motion).		
	$I = \sum mr^2$.	
	$L = I\ddot{\theta}$.	
eg small oscillations of a compound pendulum.		
	K.E. = $\frac{1}{2}I\dot{\theta}^2$.	

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MECHANICS 6 (2612) : A2

Objectives

To extend the range of techniques available to the student when dealing with the mechanics of bodies.

To study a number of more sophisticated ideas in order to give the student a feeling for some of the directions in which the subject extends.

Candidates are expected to apply the modelling principles detailed in the *Mechanics 1* specification in the context of this module.

Assessment

Examination (60 marks)

1 hour 20 minutes.

One question is set on each option. Candidates answer three questions out of the four set.

In the written papers, unless otherwise specified the value of the acceleration due to gravity should be taken to be exactly $9.8\,\mathrm{m\,s^{-2}}$.

Assumed Knowledge

Knowledge of the modules *Mechanics 1, 2, 3, 5* and *Differential Equations (Mechanics 4)* and *Pure Mathematics 1*, *2* and *3* is assumed.

MECHANICS 6

Specification		Competence Statements
OPTION 1 **ROTATION OF A RIGID BODY**		
Angular momentum.	M6r1	Be able to calculate the angular momentum of a rigid body and understand its significance.
Conservation of angular momentum.	2	Understand the conditions under which angular momentum is conserved, and apply the principle of conservation of angular momentum.
Motion following an impulsive blow.	3	Know how to calculate the angular velocity of a rotating body immediately after an impulsive blow.
Reaction at the support.	4	Be able to calculate the reaction at the support of a rotating rigid body.
General motion of a rigid body.	5	Be able to determine the general motion of a rigid body, by separating it into linear motion of the centre of mass and rotation about the centre of mass.
OPTION 2 **VECTORS**		
Vector product.	M6v1	Know the definition of $\mathbf{a} \times \mathbf{b}$ both in component form and in magnitude and direction form.
	2	Know the basic properties of the vector product.
	3	Be able to calculate the moment of a force about a point, or an axis, in 3 dimensions.
The treatment of couples, moments, angular velocity, angular momentum etc. in vector form.	4	Know that the moment of the sum of a system of forces is equal to the sum of their moments, and be able to express this in vectors.
	5	Know how to calculate angular momentum in vector form.
	6	Understand and be able to apply the equation: moment of force = rate of change of angular momentum.
Conditions for equilibrium of a system of forces, and for their reduction to a couple.	7	Understand and be able to apply the conditions for equilibrium, or reduction to a couple, of a system of forces in 3 dimensions.
OPTION 3 **STABILITY AND OSCILLATIONS**		
The stability of equilibrium of a system of particles where the position of each is determined by a single parameter.	M6o1	Know and apply the energy criteria for the stability of a system of particles.
	2	Appreciate that potential energy must be related to some fixed origin.
Small oscillations about a position of stable equilibrium.	3	Understand and use the relationship between the energy equation and the equation of motion.
	4	Be able to determine the period of small oscillations.
OPTION 4 **VARIABLE MASS**		
The application of Newton's Second Law to problems involving variable mass.	M6c1	Understand and apply Newton's 2nd Law in the form $F = \frac{\mathrm{d}}{\mathrm{d}t}(mv)$.
	2	Be able to set up and solve differential equations for situations involving variable mass.

MECHANICS 6

Notes	Notation	Exclusions
	$I\dot{\theta}$.	
eg a bullet striking a rigid body suspended from a fixed axis; a ring threaded on a smooth rotating rod.		
eg a cylinder rolling down a rough slope; a rod struck while lying on a smooth table.		2-dimensional motion of centre of mass.
	$\times$ for vector product.	
$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$; $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$.		
	$\mathbf{L} = \mathbf{r} \times \mathbf{F}$. $L = \hat{\mathbf{n}} . (\mathbf{r} \times \mathbf{F})$ about an axis with direction $\hat{\mathbf{n}}$.	
eg gravitational, elastic or electrical potential.		
eg the terminal speed of a raindrop falling through a mist; the motion of a rocket.		

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STATISTICS

Introduction

Statistics is a practical subject in constant everyday use and at the same time one with a strong theoretical background. The provision in this scheme is designed to show students that both are essential elements of the same discipline: that neither the practitioner with no theoretical knowlege nor the theoretician with no practical experience is a complete statistician. The idea of statistical modelling is crucial in understanding the connection between these aspects of the subject, and pervades all the modules. A second over-arching theme is that of statistical inference: how information in sample data can be extrapolated to the underlying population.

Important ideas are introduced in the early modules in a simple form, at times backed up by suitable coursework, and then built upon and developed more theoretically in later modules. Thus in *Statistics 1*, as well as data presentation, students meet elementary probability, sampling methods, the idea of a theoretical distribution and formal hypothesis testing.

These concepts are developed throughout the modules. Students become familiar with many distributions; some of these are used as models for real-life situations and others are introduced because of their importance in the theoretical development of the subject. Students also meet various aspects of statistical inference, learning how to estimate parameters and undertake hypothesis tests in a wide variety of situations. Those who take the higher level modules cover much of the background theory of statistical inference.

The course is designed with the intention that, however many modules they take, students will end up with a balanced and useful understanding of the subject.

COURSEWORK
(C) 3.1a, 3.1b, 3.3 (N) 4.1, 4.2, 4.3 (IT) 3.2, 3.3
(PS) 3.1, 3.2, 3.3 (WO) 3.1, 3.2, 3.3

Rationale

The aims of the coursework in these modules are that students should appreciate the practical nature of statistics, experiencing for themselves the problems in collecting and analysing real data, and that they should develop an understanding of the relationship between theory and practice.

The objectives are that they should be able to design experiments and surveys, using suitable sampling methods and collect raw data, analyse and interpret their data, and report their findings.

Description

The assessment for each of the modules *Statistics 1*, *Statistics 2* and *Commercial and Industrial Statistics* includes one coursework task. Students should be encouraged to decide on their own investigations and collect their own data. They are expected to process their data and to present their findings in a written report.

Content

Each task represents 20% of the assessment and the work involved should be consistent with that figure, both in quantity and level of sophistication. Tasks which allow only superficial or trivial treatment should be avoided.

The use of statistical packages in processing data is to be encouraged but candidates must:

(i) edit the print-out and displays to include only what is relevant to the tasks in hand;

(ii) demonstrate understanding of the meaning of what the package has done and how they could have performed any calculations themselves (full detailed calculations are not necessary);

(iii) appreciate that the use of a package allows them more time to spend on interpretation and more scope for investigational work.

Statistics 1

The theme for the coursework in *Statistics 1* is Data Exploration of single variable data. Students are required to collect a sample of raw data (at least 50 items) and to investigate their main features. Due care must be given to ensure that the sample data are of good quality. Suitable display techniques are expected to be used in the report, and these need not be restricted to those stated in the module specification. The report must include interpretation of the data. Students may compare two or more populations provided that the sample from each population contains a minimum of 30 items.

Statistics 2

In *Statistics 2* the coursework is based on bivariate data. Students are required to collect a sample of at least 50 pairs of raw data. They are then expected to investigate these data using the techniques within the syllabus. Students should be aware of the danger of carrying out work based on inappropriate modelling and are referred to the specification notes. The report must include interpretation of the data. Students may compare two or more populations provided that the sample from each population contains a minimum of 30 pairs of data values.

Commercial and Industrial Statistics

In *Commercial and Industrial Statistics* the coursework is based on data from business, commerce or other areas of industry. Students are expected to specify the problem, obtain the relevant data, analyse them using suitable techniques and report their findings in a written report and oral presentation.

Assessment

Each task must be assessed using the appropriate coursework assessment sheet.

One mark is available for each criterion statement. Half marks may be awarded, but the overall total must be rounded (up or down) to an integer.

In the case of the Oral Communication domain two marks are available. These are given for the ability of the candidate to discuss or explain the work orally with conviction, and with broadly correct use of statistics terms and theory. Assessors should use this stage of the assessment to assure themselves that the candidate understands what he/she has written.

Task Selection

Students are expected to develop their own ideas for coursework. A teacher who is uncertain about the suitability of a proposed task may submit details of the task to OCR for comment.

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STATISTICS 1 (2613) : AS

Objectives

To give the student the ability to represent single variable data in clear and manageable ways so that the main features of the data can easily be identified.

To enable the student to apply practical knowledge to theoretical situations using simple probability models.

To give the student an initial insight into the fundamental ideas and techniques of hypothesis testing.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

Candidates answer four compulsory questions of approximately equal weight.

Component 2: Coursework (15 marks)

There is one assignment on Data Exploration of single variable data.

Assumed Knowledge

Candidates are expected to have knowledge of the relevant techniques in *Pure Mathematics 1*.

STATISTICS 1

Specification		Competence Statements
MODELLING *This section is fundamental to all Statistics specifications (Statistics 1–6 and Commercial and Industrial Statistics).*		
Statistical modelling.	S1p1	Be able to abstract from a real world situation to a statistical description (model).
	2	Be able to apply an appropriate analysis to a statistical model.
	3	Be able to interpret and communicate results.
	4	Appreciate that a model may need to be progressively refined.
Sampling.	S1S1	Be aware of different sampling methods and the concepts of random sampling and bias.
	2	Understand the meanings of the terms population, sampling frame, parameter and statistic.
DATA PRESENTATION		
Classification and visual presentation of data.	S1D1	Know how to classify data as categorical, discrete or continuous.
	2	Understand the meaning of and be able to construct frequency tables for ungrouped and grouped data.
	3	Be able to define class intervals and class boundaries.
	4	Know how to display categorical and discrete data using a bar chart or a vertical line chart.
	5	Know how to display continuous data using a histogram for both unequal and equal class intervals.
	6	Know how to display and interpret data on a stem and leaf diagram.
	7	Understand the meaning and use of a box and whisker plot.
	8	Know how to display and interpret a cumulative frequency distribution.
	9	Know how to classify frequency distributions showing skewness.
Measures of central tendency and dispersion.	10	Know how to find median, mean, mode and midrange.
	11	Know the usefulness of each of the above measures of central tendency.
	12	Know how to find percentiles, quartiles and interquartile range.
	13	Know how to calculate and interpret range, standard deviation and variance.
	14	Be able to use the statistical functions of a calculator to find mean and standard deviation.
	15	Know how the mean and standard deviation are affected by linear coding.
	16	Understand the term outlier.

STATISTICS 1

Notes	Notation	Exclusions
Approximation and simplification involving appropriate distributions and probability models.		Formal definitions.
Their implications in real world terms.		
Check against reality.		
Cluster sampling, stratified sampling, systematic sampling, quota sampling (including opportunity sampling).		
Grouped data.		
Area proportional to frequency.		
Sorted and unsorted. The term stemplot is also widely used.		
The term boxplot is also widely used.		
Cumulative frequency curve.		
Positive and negative skewness.		Measures of skewness.
For raw data, frequency distribution and grouped frequency distribution.	Mean = $\bar{x}$.	
For raw data, frequency distributions and grouped frequency distributions.	Variance = s^2.	Correction for class interval in calculation of standard deviation for grouped data. Proof of equivalence will not be tested.
$y = a + bx \Rightarrow \bar{y} = a + b\bar{x}$, $s_y^2 = b^2 s_x^2$		
The term can be applied to data which are: (a) at least 2 standard deviations from the mean; (b) at least $1\frac{1}{2}$ X IQR beyond the nearer quartile.		

STATISTICS 1

Specification	Competence Statements
PROBABILITY	
Probability of events in a finite sample space.	S1u1 Know how to calculate the probability of one event.
	2 Understand the concept of a complementary event and know that the probability of an event may be found by finding that of its complementary event.
Probability of two or more events which are (i) mutually exclusive (ii) not mutually exclusive.	3 Know how to draw sample space diagrams to help calculate probabilities.
	4 Know how to calculate the expected frequency of an event given its probability.
	5 Understand the concepts of mutually exclusive events and independent events.
	6 Know to add probabilities for mutually exclusive events.
	7 Know to multiply probabilities for independent events.
	8 Know how to use tree diagrams to assist in the calculation of probabilities.
	9 Know how to calculate probabilities for 2 events which are not mutually exclusive.
Conditional probability.	10 Be able to use Venn diagrams to help calculations of probabilities for up to 3 events.
	11 Know how to calculate conditional probabilities by formula, from tree diagrams or sample space diagrams.
	12 Know that $P(B\|A) = P(B) \Leftrightarrow B$ and A are independent.

STATISTICS 1

Notes	Notation	Exclusions
	P(A). A' is event 'not A'.	
	Expected frequency = n P(A).	
		Formal notation and definitions.
Including the use of complementary events: eg finding the probability of at least one 6 in five throws of a die.		
Candidates should understand, though not ncessarily in this form, the relation: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.		Probability of a general or infinite number of events. Formal proofs.
$P(A \cap B) = P(A) \,.\, P(B \mid A)$	$P(B \mid A)$.	
In this case that $P(A \cap B) = P(A).P(B)$.		

STATISTICS 1

Specification	Competence Statements
THE BINOMIAL DISTRIBUTION AND ITS USE IN HYPOTHESIS TESTING	
Situations leading to a binomial distribution.	S1H1 Recognise situations which give rise to a binomial distribution.
	2 Be able to identify the binomial parameter p, the probability of success.
Calculations relating to binomial distribution.	3 Be able to calculate probabilities using the binomial distribution.
	4 Know that ${}^nC_r$ is the number of ways of selecting r objects from n.
	5 Know that $n!$ is the number of ways of arranging n objects in line.
Knowledge of mean.	6 Understand and apply mean = np .
Calculation of expected frequencies.	7 Be able to calculate the expected frequencies of the various possible outcomes from a series of binomial trials.
Hypothesis testing for a binomial probability p.	8 Understand the process of hypothesis testing and the associated vocabulary.
	9 Be able to identify Null and Alternative Hypotheses (H_0 and H_1) when setting up a hypothesis test on a binomial probability.
	10 Be able to conduct hypothesis tests at various levels of significance.
	11 Be able to draw a correct conclusion from the results of a hypothesis test on a binomial probability.
	12 Understand when to apply 1-tail and 2-tail tests.

STATISTICS 1

Notes	Notation	Exclusions
As a model for observed data.	$B(n, p)$, $q=1-p$ ~ means 'Has the distribution'	
Including use of tables of cumulative binomial probability.		
	${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$	
		Formal proof of variance of the binomial distribution.
eg five coins are tossed 100 times.		
Null hypothesis, alternative hypothesis; significance level, 1-tail and 2-tail tests; critical value, critical region.		
	H_0, H_1	
		Normal approximation.

MEI STRUCTURED MATHEMATICS

STATISTICS 1 (2613)

COURSEWORK ASSESSMENT SHEET: DATA EXPLORATION

TASK: Candidates will carry out an investigation of their own choice, collecting a sample of at least 50 items of single variable data which they will describe and interpret in a written report.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Aim	1	The aim of the investigation is stated in clear English and there is a convincing explanation of why the investigation is worth doing.		
Data collection	1 1	The population is defined and there is a clear explanation of the sampling method used and how efforts were made to ensure that the data are of good quality. The data are neatly and concisely presented.		
Displays	1 1	The diagrams are appropriate for the data, and there are no undue duplications. The diagrams are drawn and labelled correctly with suitable scales and titles.		
Analysis	1 1 1	The calculations are substantially correct and there are no obvious omissions. Answers are rounded appropriately. Calculations are attempted that are suitable for analysing data. The methods used (eg spreadsheet or calculator) are clearly indicated. No calculations are included that are of no relevance, and there are no undue duplications.		
Interpretation	1 1 1	The candidate indicates clearly what has been discovered. Conclusions are drawn which relate to the aim of the investigation. The candidate demonstrates why the data were worth collecting and the implications of the conclusions in relation to the population.		
Accuracy and refinements	1 1	The report includes a sensible discussion of possible sources of error and the restrictions imposed by the source of the data and the method of collection. The report includes a discussion of how the quality of the investigation could be improved.		
Oral communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

STATISTICS 2 (2614) : A2

Objectives

To extend the student's ability to represent data in bivariate situations, with an emphasis on linear and rank-order modelling. Further development of hypothesis testing is linked to this.

To extend the concept of modelling through discrete probability distributions and simple expectation theory.

To introduce continuous probability distributions through basic applications of the Normal distribution.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

Candidates answer four compulsory questions of approximately equal weight.

Component 2: Coursework (15 marks)

There is one assignment on Bivariate Data.

Assumed Knowledge

Knowledge of *Statistics 1* is assumed. Candidates are expected to have knowledge of the relevant techniques in *Pure Mathematics 1*. Candidates are also expected to know the series expansion of e^x.

STATISTICS 2

Specification		Competence Statements
BIVARIATE DATA		
Scatter diagram.	S2b1	Know how to draw a scatter diagram.
	2	Know the difference between dependent and independent variables.
Product-moment correlation coefficient (pmcc).	3	Know how to calculate the pmcc from raw data or summary statistics.
	4	Know how to carry out hypothesis tests using the pmcc and tables of critical values.
Spearman's Rank correlation coefficient.	5	Know how to calculate Spearman's Rank correlation coefficient from raw data or summary statistics.
	6	Know how to carry out hypothesis tests using Spearman's Rank correlation coefficient and tables of critical values.
Regression line for a random variable on a non-random variable.	7	Know how to calculate the equation of the least squares regression line using raw data or summary statistics.
	8	Know the meaning of the term residual and be able to calculate residuals.

STATISTICS 2

Notes	Notation	Exclusions
	Sample value = r	
Only 'H$_0$: no correlation' will be tested. Hypothesis tests using the pmcc require a modelling assumption that the data are drawn from a bivariate Normal distribution. Candidates will not be required to know the formal meaning of this term but will be expected to know that both variables must be random and that where one or both of the distributions is skewed, bimodal etc., the procedure is likely to be inaccurate. They will also be expected to recognise (from a scatter diagram) cases of non-linear association and, where appropriate, to apply a test based on Spearman's correlation coefficient.		Fisher's z transformation.
	Sample value = r_s	
Hypothesis tests using Spearman's Rank correlation coefficient require no modelling assumption about the underlying distribution. Only 'H$_0$: no association' will be tested.		
The goodness of fit of a regression line may be judged by looking at the scatter diagram. An informal measure which is often used is provided by the Coefficient of Determination, r^2, which measures the proportion of the total variation in the dependent variable, y, which is accounted for by linear regression. There is no standard hypothesis test based on the Coefficient of Determination.		Derivation of least squares regression line.
Informal checking a linear model by looking at residuals.		

Examination questions will be confined to cases in which a random variable, Y, and a non random variable, x, are modelled by a relationship in which the expected value of Y is a linear function of x.

Candidates' coursework may include the use of the same equation for the regression line in cases which are outside the syllabus because both variables are random. In such cases two modelling assumptions are required.

(i) The mean value of Y is a linear function of X : $\mathrm{E}(Y|X = x) = a + bx$.
(ii) The variance of Y is the same for all values of X : $\mathrm{Var}(Y|X = x)$ is constant.

These modelling assumptions are satisfied by a bivariate Normal distribution, which may be recognised on a scatter diagram by an approximately elliptical distribution of points.
Candidates may thus use their scatter diagrams to form a qualitative judgement on how well the model they are using fits their data.

STATISTICS 2

Specification		Competence Statements
DISCRETE RANDOM VARIABLES		
Probability distributions.	S2R1	Be able to use probability functions, given algebraically or in tables.
Calculation of probability, expectation (mean) and variance.	2	Be able to calculate the numerical probabilities for a simple finite distribution.
	3	Be able to calculate the expectation (mean), E(X), and understand its meaning.
	4	Be able to calculate the variance, Var(X), and understand its meaning.
	5	Be able to use the result $\mathrm{E}(a + bX) = a + b\mathrm{E}(X)$.
	6	Be able to use the result $\mathrm{Var}(a + bX) = b^2\,\mathrm{Var}(X)$.
	7	Understand and be able to use cumulative distribution functions.
POISSON DISTRIBUTION		
Situations leading to the Poisson distribution.	S2P1	Know the situations under which the Poisson distribution is likely to be an appropriate model.
Calculations of probability and of expected frequencies.	2	Be able to calculate the probabilities within a Poisson distribution.
	3	Be able to use the Poisson distribution as an approximation to the binomial distribution, and know when to do so.
The mean and variance of the Poisson distribution.	4	Know the mean and variance of a Poisson distribution.
The sum of independent Poisson distributions.	5	Know that the sum of two or more independent Poisson distributions is also a Poisson distribution.
NORMAL DISTRIBUTION		
The use of the Normal distribution in situations where the distribution is given as Normal.	S2N1	Be able to standardise a Normal variable and use the Normal Distribution Tables.
The use of the Normal distribution as an approximation to the binomial and Poisson distributions.	2	Be able to use the Normal distribution as an approximation to the binomial distribution and know when it is appropriate to do so.
	3	Be able to use the Normal distribution as an approximation to the Poisson distribution and know when it is appropriate to do so.
	4	Know when to use a continuity correction and be able to do so.

STATISTICS 2

Notes	Notation	Exclusions
	$P(X = x)$.	
	$E(X) = \mu$,	
Knowledge of $\text{Var}(X) = E(X^2) - \mu^2$.	$\text{Var}(X) = E[(X - \mu)^2]$.	
	$X \sim$ Poisson (λ).	
Use of tables of cumulative Poisson distribution.		
		Formal proofs.
$X \sim$ Poisson (λ), $Y \sim$ Poisson (μ) $\Rightarrow X + Y \sim$ Poisson $(\lambda + \mu)$.		Formal proof.
	$X \sim N(\mu,\sigma^2)$.	Proof.
Variance of $B(n, p)$ is npq.	$B(n, p) \approx N(np, npq)$.	Proof.
	Poisson $(\lambda) \approx N(\lambda, \lambda)$.	

MEI STRUCTURED MATHEMATICS

STATISTICS 2 (2614)

COURSEWORK ASSESSMENT SHEET: BIVARIATE DATA

TASK: Candidates will carry out an investigation of their own choice, collecting a sample at least 50 items (pairs) of bivariate data which they will describe and interpret in a written report.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Aim	1	The aim of the investigation is stated in clear English and there is a convincing explanation of why the investigation is worth doing.		
Data collection	1 1	The population is defined and there is a clear explanation of the sampling method used and how efforts were made to ensure that the data are of good quality. The data are neatly and concisely presented.		
Modelling procedures	1 1 1	A scatter diagram is drawn with the axes clearly and correctly labelled. Appropriate techniques are adopted. The report includes a discussion of why the modelling procedures are appropriate in terms of the nature of the two variables (random or non-random) and their relationship.		
Analysis	1 1	The data are processed to give a correlation coefficient or a regression line. The value of r or r_s is tested (correlation) or r^2 is calculated (regression) and discussed.		
Interpretation	1 1 1	The candidate indicates clearly what has been discovered. Conclusions are drawn which relate to the aim of the investigation. The candidate demonstrates why the data were worth collecting and the implications of the conclusion in relation to the population.		
Accuracy and refinements	1 1	The report includes a sensible discussion of possible sources of error and the restrictions imposed by the source of the data and the method of collection. The report includes a discussion of how the quality of the work could be improved.		
Oral communication	2	Presentation [ ] Interview [ ] Discussion [ ]	Please tick at least one box and give a brief report.	
			TOTAL	/ 15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

STATISTICS 3 (2615) : A2

Objectives

To introduce more sophistication in modelling techniques with the use of probability density functions (with the emphasis on concepts rather than calculus skills).

To introduce the ideas of estimation and very basic applications of the Central Limit Theory.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer four compulsory questions of approximately equal weight.

Assumed Knowledge

Knowledge of the modules *Statistics 1* and *2* is assumed. Candidates are also expected to have knowledge of the relevant techniques in *Pure Mathematics 1, 2* and *3*.

STATISTICS 3

Specification		Competence Statements
CONTINUOUS RANDOM VARIABLES		
The probability density function (pdf) of a random variable. Calculation of probability, expectation (mean), variance, median and mode.	S3R1	Understand the meaning of a pdf and be able to use one to find probabilities.
	2	Know the properties of a pdf.
	3	Find mean and variance from a given pdf.
	4	Find mode and median from a given pdf.
The cumulative distribution function (cdf) and its relationship to the probability density function.	5	Understand the meaning of a cdf and know how to obtain one from a given pdf.
	6	Know how to obtain a pdf from a given cdf.
	7	Use a cdf to calculate the median.
Rectangular (or continuous uniform) distribution.	8	Understand the properties of the rectangular (or continuous uniform) distribution and be able to find its pdf.
EXPECTATION ALGEBRA		
Mean and variance of a linear combination of two (or more) independent random variables.	S3a1	Know how to find the mean and variance of any linear combination of independent random variables.
The distribution of a linear combination of independent Normal variables.	2	Be able to use linear combinations of Normal random variables in solving problems.
ESTIMATION		
Sampling methods	S3E1	Know the definition of the term simple random sample.
	2	Understand the use of different sampling methods.
Estimation of population mean and variance from a simple random sample.	3	Be able to estimate population mean from sample data.
	4	Be able to estimate population variance from sample data with divisor (n – 1).
Distribution of the mean of a sufficiently large sample. Standard error of the mean.	5	Understand how and when the Central Limit Theorem may be applied to the distribution of sums and means.
Symmetric confidence intervals for the mean using Normal or *t*-distributions.	6	Be able to calculate and interpret the standard error of the mean.
	7	Understand the term confidence interval and be able to construct confidence intervals for the mean.

STATISTICS 3

Notes	Notation	Exclusions
Concepts, rather than expertise at calculus will be examined.	f(x)	pdfs comprising more than two (non-zero) parts.
	F(x)	
f(x) = F'(x).		
	$f(x) = \frac{1}{(b-a)} \quad a \le x \le b$	
	E($X \pm Y$) = E(X) ± E(Y) Var($X \pm Y$) = Var(X) + Var(Y) E($aX \pm bY$) = aE(X) ± bE(Y) Var($aX \pm bY$) = a^2Var(X) + b^2Var(Y)	Proof.
The distribution is Normal.		
Candidates are also reminded of this Note in *Statistics 1:* cluster sampling, stratified sampling, systematic sampling, quota sampling (including opportunity sampling).		
	$\hat{\mu} = \bar{x}$,	Proof.
	$\hat{\sigma}^2 = \Sigma \frac{(x-\bar{x})^2}{n-1}$	Proof.
The implication of the Central Limit Theorem.		Formal statement and derivation of the Central Limit Theorem.
	$\frac{\sigma}{\sqrt{n}}$.	
In situations where (a) the population variance is known; (b) the population variance is unknown but the sample size is large; (c) the population variance is unknown but the population may be assumed to have a Normal distribution.		

STATISTICS 3

Specification		Competence Statements	
HYPOTHESIS TESTING			
		S3H1	Understand the process of hypothesis testing and the associated vocabulary.
Tests for:			
(a)	A single mean using the Normal distribution.	2	Be able to carry out a hypothesis test for a single mean using the Normal distribution and know when it is appropriate to do so.
(b)	A single mean using the *t*-distribution.	3	Be able to carry out a hypothesis test for a single mean using the *t*-distribution and know when it is appropriate to do so.
(c)	For goodness of fit using χ^2 test.	4	Be able to carry out a χ^2 test for a discrete model, understanding and using degrees of freedom.
(d)	Type I and II errors.	5	Know what is meant by a Type I and a Type II error in a hypothesis test.
		6	Understand that the probability of a Type I error is the significance level of the test.
		7	Understand that the probability of a Type II error depends on the true population distribution, which is unknown, as well as the sample size and significance level.
		8	Be able to calculate the probability of a Type II error in a specific case.

STATISTICS 3

Notes	Notation	Exclusions
Null hypothesis; alternative hypothesis; significance level; 1- and 2-tail tests; critical value, critical region; test statistic.		
In situations where (a) the population variance is known; (b) the population variance is unknown but the sample size is large.		
In situations where the population variance is unknown but the population may be assumed to have a Normal distribution.		
eg uniform distribution; binomial distribution; Poisson distribution; distribution in given proportions.		Contingency tables.
It is not intended that this topic should form the subject of a whole question.		

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STATISTICS 4 (2616) : A2

Objectives

To lead the student further into the techniques and theory of inference and hypothesis testing, including introduction to some non-parametric procedures.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions from four set.

Assumed Knowledge

Knowledge of the modules *Statistics 1, 2* and *3* is assumed. Candidates are also expected to have knowledge of the relevant techniques in *Pure Mathematics 1, 2* and *3*.

Candidates are expected to know that $x^n e^{-x} \to 0$ as $x \to \infty$ for all $n \geqslant 0$.

STATISTICS 4

Specification		Competence Statements
ESTIMATION		
Expectation and variance of a function of a random variable.	S4E1	Be able to find the expectation and variance of a function of a discrete or continuous random variable.
Estimators as random variables.	2	Understand the idea of an estimator as a random variable and its sampling distribution.
Biased and unbiased estimators.	3	Understand the meaning of biased and unbiased estimators. Be able to determine whether a given estimator is biased or unbiased. Be able to construct estimators in simple cases.
Unbiased estimators for population mean and variance, from single and pooled samples.	4	Be able to find unbiased estimators for the mean and variance of a population from single and pooled samples.
Standard error of an estimator.	5	Understand and be able to obtain and use the standard error of an estimator.
Mean square error.	6	Understand the use of mean square error (MSE) for biased estimators.
	7	Be able to obtain a mean square error.
Relative efficiency of estimators.	8	Understand that estimators can be compared by considering their standard errors or mean square errors.

STATISTICS 4

Notes	Notation	Exclusions
	$\mathrm{E}[\mathrm{g}(X)] = \sum_i \mathrm{g}(x_i)\mathrm{p}(x_i)$ *(discrete random variable),* $\mathrm{E}[\mathrm{g}(X)] = \int \mathrm{g}(x)\mathrm{f}(x)\mathrm{d}x$ *(continuous random variable),* $\mathrm{Var}[\mathrm{g}(X)] = \mathrm{E}[(\mathrm{g}(X))^2] - (\mathrm{E}[\mathrm{g}(X)])^2$	
Calculation of the sampling distribution in simple cases only.		
E (estimator) ≠ or = parameter value.		
	$\hat{\mu} = \bar{x} = \sum_i \frac{x_i}{n}$	Proofs.
	$\hat{\sigma}^2 = \sum_i \frac{(x_i - \bar{x})^2}{n-1}$.	
	$\hat{\mu} = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y}$, $\hat{\sigma}^2 = \frac{(n_x - 1)\hat{\sigma}_x{}^2 + (n_y - 1)\hat{\sigma}_y{}^2}{n_x + n_y - 2}$.	
$\mathrm{MSE}(T) = \mathrm{Var}(T) + (\mathrm{Bias}\ (T))^2$.	$\mathrm{Bias}\ (\hat{\theta}) = \mathrm{E}(\hat{\theta}) - (\theta)$	

STATISTICS 4

Specification		Competence Statements
HYPOTHESIS TESTING		
	S4H1	Understand the difference between a two-sample test and a paired sample test.
(a) For the difference of means of paired and unpaired samples using the Normal or t-distribution.	2	Know when to use the Normal distribution and when to use the t-distribution in testing for differences of sample means.
	3	Know when to use pooled unbiased estimators for common mean and variance for samples from two populations and be able to do so.
(b) For goodness of fit for continuous distributions.	4	Be able to use the χ^2 test for goodness of fit for a continuous distribution.
(c) Contingency tables using the χ^2 test.	5	Be able to apply the χ^2 test to a contingency table.
	6	Be able to apply Yates' continuity correction.
(d) For location and difference in location, in paired and unpaired samples, using the Wilcoxon tests.	7	Recognise situations where the Wilcoxon single sample test is appropriate and be able to apply it.
	8	Recognise situations where the Wilcoxon paired sample test is appropriate and be able to apply it.
	9	Recognise situations where the Wilcoxon rank sum test, for two unpaired samples, is appropriate and be able to apply it.
	10	Be able to use appropriate Normal approximations including continuity corrections.
CONFIDENCE INTERVALS		
One-sided and asymmetric confidence intervals, using the Normal or t-distribution.	S4C1	Be able to construct and interpret a one-sided confidence interval for the population mean.
	2	Be able to construct and interpret an asymmetric confidence interval for the population mean.
Confidence intervals for the difference of means of paired and unpaired populations using the Normal or t-distribution.	3	Be able to construct and interpret confidence intervals for the difference in means of paired and unpaired populations.

STATISTICS 4

Notes	Notation	Exclusions
For 2×2 contingency tables only.		
		Recall of the mean and variance of the test statistic.

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STATISTICS 5 (2617) : A2

Objectives

To extend the statistical tools available to the student by introducing techniques such as the use of generating functions.

To introduce formal inference procedures for a population variance and for comparing two population variances.

To introduce inference procedures for binomial proportions using a Normal distribution.

Assessment

Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions from four set.

Assumed Knowledge

Knowledge of the modules *Statistics 1*, *2* and *3* is assumed. Candidates are also expected to have knowledge of the relevant techniques in *Pure Mathematics 1*, *2* and *3*, and *Statistics 4*.

Candidates are expected to know that $x^n e^{-x} \to 0$ as $x \to \infty$ for all $n \geqslant 0$.

STATISTICS 5

Specification		Competence Statements
PROBABILITY GENERATING FUNCTIONS		
Probability generating function (pgf).	S5u1	Know that the coefficients of a generating function represent probabilities for a discrete random variable.
	2	Understand the uniqueness of the relationship between a distribution and its probability generating function.
Derivation of the pgf of a discrete random variable with a given probability function.	3	Be able to derive a probability generating function for a discrete random variable from its probability distribution.
Use of the pgf to find the distribution of the sum of random variables	4	Know that the probability generating function of a sum of two or more independent discrete random variables is the product of their probability generating functions and be able to use this.
Use of the pgf to find the mean and variance of a random variable.	5	Be able to derive the mean and variance of a discrete random variable from its pgf.
	6	Be able to derive the probability generating function of a linear transformation of a random variable.
MOMENT GENERATING FUNCTIONS		
Moment generating function (mgf).	S5f1	Know that the coefficients of a generating function may be used to derive moments for a discrete or continuous random variable.
	2	Understand the uniqueness of the relationship between a distribution and its moment generating function.
Derivation of the mgf of a random variable with a given pdf or discrete probability function.	3	Be able to derive a moment generating function for a discrete or continuous random variable given its probability function or probability density function.
Use of the mgf to find the distribution of the sum of random variables.	4	Know that the moment generating function of a sum of two or more independent random variables is the product of their moment generating functions and be able to use this.
Use of the mgf to find the mean and variance of a random variable.	5	Be able to derive the mean and variance of a random variable by inspection of coefficients or differentiation.
	6	Be able to derive the moment generating function of a linear transformation of a random variable.
HYPOTHESIS TESTING		
(a) For the proportion or the difference in proportions of binomial distributions, using a Normal approximation.	S5H1	Be able to carry out a hypothesis test for the proportion in a binomial distribution using a Normal approximation, including continuity corrections and recognise when such a test is appropriate.
	2	Be able to carry out a hypothesis test for the difference in proportions in two binomial distributions using a Normal approximation, and recognise when such a test is appropriate.
(b) For the variance of a population, using a χ^2 test.	3	Be able to carry out a hypothesis test for the variance of a population using the χ^2 distribution and recognise when such a test is appropriate.
(c) For the comparison of two population variances, using the *F*-test.	4	Be able to carry out a hypothesis test for the comparison of two population variances using the *F*-distribution, and recognise when such a test is appropriate.

STATISTICS 5

Notes	Notation	Exclusions
	pgf : $\mathrm{G}_X(t) = \mathrm{E}(t^X)$ $\mathrm{G'}_X(t) = \frac{\mathrm{d}}{\mathrm{d}t}[\mathrm{G}_X(t)]$.	
	mgf: $\mathrm{M}_X(\theta) = \mathrm{E}[e^{\theta X}]$ $\mathrm{M'}_X(\theta) = \frac{\mathrm{d}}{\mathrm{d}\theta}\left[\mathrm{M}_X(\theta)\right]$	Characteristic functions.

STATISTICS 5

Specification			Competence Statements
HYPOTHESIS TESTING *(ctd.)*			
(d)	Operating characteristic, Power function.	S5H5	Be able to calculate, plot and interpret the operating characteristic and the power function of a hypothesis test.
CONFIDENCE INTERVALS			
(a)	For the proportion in a binomial distribution.	S5C1	Be able to construct and interpret confidence intervals for the proportion in a binomial distribution, using a Normal approximation.
(b)	For the difference in proportions of two binomial distributions.	2	Be able to construct and interpret confidence intervals for the difference in proportions in two binomial distributions, using a Normal approximation.
(c)	For the variance of a population, using a χ^2 distribution.	3	Be able to construct and interpret confidence intervals for the variance of a population, using the χ^2 distribution.

STATISTICS 5

Notes	Notation	Exclusions
Operating characteristic = P (Type II error $\mid$ the true value of the parameter) Power = 1 – Operating Characteristic.		

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STATISTICS 6 (2618) : A2

Objectives

To study a number of more sophisticated statistical ideas and techniques in order to give the most able students a feeling for some of the directions in which the subject extends. The module is structured so that students need study only some of the options.

Assessment

Examination (60 marks)

1 hour 20 minutes.

One question is set on each of the five options. Candidates answer three questions.

Assumed Knowledge

Knowledge of the modules *Statistics 1, 2* and *3* is assumed. Candidates are also expected to have knowledge of the relevant techniques in *Pure Mathematics 1, 2* and *3*, and *Statistics 4* and *5*.

Candidates are expected to know that $x^n e^{-x} \to 0$ as $x \to \infty$ for all $n \geqslant 0$.

STATISTICS 6

Specification		Competence Statements
OPTION 1 **MAXIMUM LIKELIHOOD ESTIMATION**		
Maximum likelihood.	S6E1	Understand the meaning of the likelihood of a set of outcomes for a discrete or continuous random variable.
	2	Be able to find the maximum likelihood estimator of a population parameter or parameters, for a discrete or continuous random variable, in simple cases.
OPTION 2 **BIVARIATE DISTRIBUTIONS**		
The joint distribution of two discrete random variables.	S6b1	Understand what is meant by the joint distribution of two discrete random variables and how it may be described.
Marginal and conditional probabilities.	2	Be able to find and use the marginal distribution of each variable.
	3	Be able to find the distribution of one variable, conditional on a particular value of the other, and interpret this.
Independence.	4	Know and be able to use the definition of independence of two random variables.
	5	Be able to determine whether or not two random variables are independent.
Covariance and coefficient of correlation.	6	Be able to calculate the covariance of two discrete random variables.
	7	Know the relationship between independence and zero covariance.
	8	Be able to calculate and interpret the coefficient of correlation for two discrete random variables.
Expectation and variance of a linear combination of two random variables, not necessarily independent.	9	Know how to find the expectation and variance of a linear combination of two random variables, given the expectations and variances of each and their covariance.
OPTION 3 **MARKOV CHAINS**		
Transition matrix.	S6m1	Know that a set of transition probabilities may be summarised in the form of a square matrix.
	2	Be able to use a transition matrix to calculate probabilities of future events.
Limit properties of a non-periodic transition matrix and calculation of the equilibrium probabilities.	3	Know that successive powers of a non-periodic transition matrix tend towards a limit.
	4	Be able to calculate directly equilibrium probabilities for a non-periodic transition matrix.
Run lengths; expected values.	5	Be able to calculate the expected value of a run length of a particular event.

STATISTICS 6

Notes	Notation	Exclusions
Use of logarithmic differentiation.		
As a function, or a table of probabilities.		
X, Y independent $\Rightarrow$ E(XY) = E(X) E(Y).		
	cov (X,Y) = E(XY) – E(X) E(Y)	
X, Y independent $\Rightarrow$ cov (X, Y) = 0 but cov (X, Y) = 0 $\not\Rightarrow$ X, Y independent.		
	$\rho(X,Y)=\dfrac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$.	
	E($aX \pm bY$) = aE(X) $\pm b$E(Y) Var($aX \pm bY$) = a^2Var(X) + b^2Var(Y) $\pm$ 2ab cov(X, Y)	
2×2 and 3×3 transition matrices only.	$\mathbf{P}$	
	$\mathbf{p}_{n+1} = \mathbf{p}_n\mathbf{P}$	Use of eigenvalues.

STATISTICS 6

Specification		Competence Statements
OPTION 4 **ANALYSIS OF VARIANCE**		
One way analysis of variance.	S6V1	Understand the usual one-way analysis of variance model, for the case of common population variance.
	2	Know that a one-way analysis of variance is used to test for equality of population means.
	3	Be able to carry out a one-way analysis of variance using an *F*-test.
	4	Be able to interpret the conclusions of such a test.
OPTION 5 **REGRESSION**		
Regression models.	S6z1	Understand the linear regression model and the assumptions that are necessary for it to apply.
The method of least squares.	2	Know how to derive estimators for the linear regression coefficients, using the method of least squares.
Non-linear regression.	3	Be able to use the method of least squares to find a curve of best fit for one variable dependent upon another.
Multiple regression.	4	Be able to use the method of least squares to find a plane of best fit for one variable dependent on two others.
Inference for regression coefficients.	5	Understand the sampling distributions of the estimators of the coefficients.
	6	Understand the meaning of residual and know the distribution of the residual sum of squares.
	7	Know how to calculate an unbiased estimate of the variance of the error term.
	8	Be able to test hypotheses about coefficients, in the cases where the variance of the error is known and where it must be estimated.
	9	Be able to construct confidence intervals for coefficients, in the cases where the variance of the error is known and where it must be estimated.

STATISTICS 6

Notes	Notation	Exclusions
Lines of best fit.	$Y_i = \alpha + \beta x_i + E_i$ where $E_i \sim \mathrm{N}(0, \sigma^2)$	
Normal equations.	a, b are estimates for α, β respectively.	
	eg $Y_i = \alpha + \beta x_i + \gamma x_i^2 + E_i$ where $E_i \sim \mathrm{N}(0, \sigma^2)$	
	$Y_i = \alpha + \beta x_i + \gamma z_i + E_i$ where $E_i \sim \mathrm{N}(0, \sigma^2)$	
	$\mathrm{RSS} = \sum (y_i - a - bx_i)^2$	
	eg $\mathrm{RSS} = \sum (y_i - a - bx_i - cx_i^2)^2$	
	$\mathrm{RSS} = \sum (y_i - a - bx_i - cz_i)^2$	
	$\hat{\sigma}^2 = \dfrac{\mathrm{RSS}}{n-2}$	

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COMMERCIAL AND INDUSTRIAL STATISTICS (2619) : A2

Objectives

To give the student experience of statistics in commerce and industry by providing theoretical and practical knowledge of relevant topics.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

Candidates answer three questions from four set.

Component 2: Coursework (15 marks)

One assignment (see page 168)

Assumed Knowledge

Knowledge of the modules *Statistics 1* and *2*. Candidates are also expected to have knowledge of the relevant techniques in *Pure Mathematics 1* and *2*.

Candidates are also expected to have knowledge of the test for a single mean using the Normal distribution for the cases of known population variance or large samples.

COMMERCIAL AND INDUSTRIAL STATISTICS

Specification		Competence Statements
QUALITY CONTROL AND MANAGEMENT		
On-line process control.	CISQ1	Be able to design and interpret Pareto charts.
	2	Be able to construct and interpret simple (Shewhart) charts for the mean and the range.
	3	Be able to construct and interpret CuSum charts.
Off-line inspection.	4	Understand the implications for managers of a *total quality* operation.
Sampling inspection.	5	Be able to explain the purposes and advantages of single and double sampling procedures and know how to conduct them.
	6	Be able to find and interpret the operating characteristic curve for a specified sampling procedure.
TIME-DEPENDENT VARIABLES		
Time series.	CIST1	Be able to identify the main features of a time series from a plot of the data.
	2	Be able to use appropriate measures to attempt to smooth the data.
	3	Be able to find and interpret, where appropriate, summary measures of time series.
	4	Be able to use residuals to comment on the fit of a model.
Autocorrelation.	5	Be able to calculate autocorrelation coefficients, and know how to plot and interpret them.
Forecasting.	6	Be able to estimate future values and understand the limitations of the forecasts obtained.
SAMPLING TECHNIQUES		
The use and comparison of different types of sampling techniques.	CISS1	Be able to explain the use, advantages and limitations of a variety of sampling techniques (possibly used in combination).
	2	Be able to construct estimators, in simple cases, from a given sampling design and to obtain simple properties of those estimators.
EXPERIMENTAL DESIGN		
Principles of randomisation and replication.	CISx1	Be able to explain the need for randomisation and replication in a particular experimental design.
	2	Be able to explain the purpose of particular techniques in a given situation.
	3	Be able to explain the purpose of particular experimental designs, and be able to make an initial interpretation of the data from them.
	4	Be able to suggest a suitable experimental design for a given situation and implement it.
Median Polish.	5	Be aware of the importance and use of random permutations in designing experiments.
	6	Be able to use median polish for an initial analysis of data in a two-way table.

COMMERCIAL AND INDUSTRIAL STATISTICS

Notes	Notation	Exclusions
To identify the causes of defects and count their frequencies.		
Warning lines and action lines.		
The basic idea only.		The mask.
The cost of quality control.		
Trend, seasonal and cyclic variations. Use of a historigram (time series plot).		
Running medians, Hanning and moving averages.		
Trend, seasonal and cyclic variations.		
Use of autocorrelogram.		
The use of exponential smoothing.	α values.	Double smoothing will not be tested in the examination.
Simple random, stratified, cluster, systematic, quota sampling.		
Paired comparisons. Randomised blocks. Latin squares. 2^n factorial experiments.		
Blind, double blind, cross-over.		
Tables of random permutations.		
To investigate the model: Observation = constant + row factor + column factor + random error.		

MEI STRUCTURED MATHEMATICS

COMMERCIAL AND INDUSTRIAL STATISTICS (2619)

COURSEWORK ASSESSMENT SHEET

This coursework should be based on data drawn from business, commerce or other areas of industry.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Aims	1 1	The aim of the investigation is stated in clear English. The relevance of the investigation to the situation is clearly explained.		
Data Collection	1 1	The source of the data is explained with an indication of their quality. Some justification is given as to the quantity of data collected.		
Statistical Formulation	1 1	Appropriate modelling techniques are adopted. The report includes a discussion as to why the modelling procedures are appropriate in terms of the nature of the investigation.		
Application of techniques	1 1 1	Appropriate techniques are selected but may be flawed in application except in the simplest of cases. Appropriate techniques are correctly applied. There is some evidence of consideration of efficiency. Efficiency, range of applications, etc., are dealt with as appropriate. Techniques are adapted or developed to meet specification needs of the problem.		
Interpretation	1 1	The results are interpreted in terms of the original problem. Their implications for the situation are considered.		
Review and Action	1 1	Work is reviewed critically. Suggestions are made as to how the investigation could usefully be refined and developed.		
Oral Communication	2	Presentation Interview Discussion	Please tick at least one box and give a report.	
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.
Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

DECISION AND DISCRETE MATHEMATICS

Introduction

Specification Structure

The specification is structured under a number of headings. These are selected from three themes which run through all three modules. The structure is as follows:

	Theme 1 **Graphs and Networks**	**Theme 2** **Linear Programming**	**Theme 3** **Discrete Topics**
Decision & Discrete Mathematics 1	Graphs Networks Critical Path Analysis	Linear Programming (introduction to LP)	Algorithms Simulation
Decision & Discrete Mathematics 2	Networks	Linear Programming (theory + applications)	Decision Trees Logic
Decision & Discrete Computation	Network Flows Matchings	Linear Programming (computing applications)	Recurrence Relations Simulation

Coursework
(C) 3.1a, 3.1b, 3.3 (N) 4.1, 4.2, 4.3 (IT) 3.1, 3.2, 3.3
(PS) 3.1, 3.2, 3.3 (WO) 3.1, 3.2, 3.3

Rationale

Decision and Discrete Mathematics is concerned with an area of mathematical modelling. It is therefore appropriate to assess coursework in domains which relate to the stages of the modelling process.

The modules aim to develop modelling skills:

- Problem identification
- Modelling
- Solving
- Interpreting
- Validating
- Refining
- Reporting

It is recognised that not every piece of coursework will map onto every skill, so some flexibility is provided by marking under fewer domains. In each domain descriptors are provided for each of the mark possibilities.

Description

The descriptors which are provided cannot be definitive. Rather they are indicative of what might be seen from work in the corresponding category. In all areas of this mark scheme there are likely to be candidates whose work does not match the criteria, who satisfy part of higher-scoring criteria and part of lower-scoring criteria. These candidates will qualify for a compromise mark; criteria cannot replace judgement.

Half marks are acceptable within the domains, but the total mark must be rounded (up or down) to an integer.

Problem identification and modelling (0-4)

1 An appropriate problem is identified, but the candidate's understanding of it is superficial. There is little or no appreciation shown for the scope or significance of the problem.

A basic model has been formulated, possibly with limited success. Typically one or two alternatives have been evaluated without any mathematical relationship having been established between inputs and outputs.

2 An appropriate problem is identified. The candidate has demonstrated a reasonable understanding of the problem. There is a limited understanding of its wider implications.

An appropriate model has been formulated in mathematical terms, but it lacks completeness or is inadequate in its level of sophistication.

3 There is a clear understanding of the problem and context, together with a sound understanding of its scope and importance.

The problem has been successfully and completely modelled in mathematical terms. The model is straightforward but adequate, typically one covered within the syllabus.

4 A good understanding of problem and context, together with a full analysis of its importance both in practical terms and in terms of the relationship between it and similar problems, possibly in other fields.

A successful, complete and appropriately sophisticated model has been produced. Extensions which go beyond the original problem have been considered.

Notes

Tasks need to be appropriate for the purpose. A candidate who has been given a task in which a well-defined problem is presented will not be in a position to gain credit for problem identification - unless another appropriate problem is identified and made explicit by that candidate.

If only a very elementary model is sufficient to achieve the required ends 'successfully and completely', then it will not score very highly in this section. Similarly top marks cannot be earned for merely selecting one of the commonly covered models, unless it is tailored in some way for the application, or unless the application is particularly complex. For instance, a coursework task leading to a mathematical model consisting of the shortest route through a simple network might qualify for between 2 and 3 marks in this section. To earn more marks the modelling will either need to be more sophisticated, or have an element of originality.

It is valid and laudable for a candidate to pursue excellence by developing models even if that development is not needed for the initial problem, and then to consider real world problems for which the development model might be appropriate. But this should not be confused with the refinement stage in the modelling process, in which the model is improved so as to mirror the real world problem more clearly (see Refinement below).

Solution of the Mathematical Problem (0-4)

1 Appropriate techniques are selected but flawed in application, except in the very simplest of cases.

2 There is a competent, straightforward application of the relevant standard techniques, correctly applied.

3 Relevant techniques are applied correctly. There is some evidence of the consideration of efficiency, range of applicability, adaptability, or other relevant factors.

4 Relevant techniques are applied correctly. Efficiency, optimality, or other related factors are dealt with as appropriate. Techniques are adapted or developed to meet specific needs of the problem.

Notes

The majority of relevant techniques in this module are algorithms which are designed to be efficient (and sometimes optimal) in dealing with very large problems. Coursework offers the opportunity for candidates to demonstrate an appreciation of that. Whilst it is easy (and undesirable) for candidates to become enmeshed in the work and to spend far longer on it than is recommended, good marks will not be earned by 'back-of-the-envelope' calculations to solve imitation problems. In most cases the problems tackled will necessarily be 'imitations', to a greater or lesser degree, but they should be solved algorithmically, and not by ad-hoc approaches relevant only to the particular problem under consideration. Of course, the development of a heuristic algorithm to solve a particular type of problem is valid, and, if appropriate and well done, can earn top marks.

Interpretation and Refinement (0-3)

1 There is a competent interpretation of the mathematical solution to a rather simple problem. Alternatively, there is a poor or only partially interpreted solution to an appropriately complex problem.

The limitations of the model are noted. Possible improvements are mooted, but not implemented.

2 There is a competent interpretation of the mathematical solution to an appropriately complex problem.

Simplifying or distorting assumptions are identified and their effects considered. Simple and obvious improvements are implemented.

3 A good interpretation of the solution to a complex problem in which the connection between the maths and the problem are made explicit.

Relevant factors such as the limitations, the sensitivity and the range of applicability of the solution are considered and analysed. Relevant refinements are implemented.

Notes

Whether or not a solution produced from the first loop through the modelling process is adequate, it is always the case that improvements are possible. If the solution is perceived to be inadequate in some respect then the need will be clear. If there is no such perceived need the assumptions made in building the model, explicit or implied, may be examined/relaxed. For instance, a common implied assumption is that input data is deterministic ('…painting takes 3 days…'), when, more realistically, data is usually stochastic.

Written Report (0-2)

1 The written report is readable and clear. The level of detail is sufficient for a straightforward problem. It is backed up with some, but not all, appropriate data, and by an analysis which is appropriate to the problem. The standard of presentation is good.

2 The written report is readable and clear. The problem tackled is of sufficient complexity to allow a high level of relevant detail. It is backed up with all relevant data and analysis, in appendices as appropriate. The standard of presentation is high.

Notes

Top marks would normally be awarded only to a report which brings clarity and enlightenment to a complex scenario. A beautifully presented and clear report of a simple, easy-to-describe piece of modelling will be worth 1 mark in this section, exceptionally 1½ marks. Again, the task needs to be appropriate for the purpose, and for the candidate. To earn top marks candidates must be succeeding with difficult tasks.

Oral Communication (0-2)

1 The candidate is comfortable with the work and can talk fluently about it. The mathematical content is handled confidently, if not always correctly. There is some perception of the generic issues, expressed through the candidate's ability to relate the work to the modelling cycle.

2 The candidate is completely at ease with all aspects of the work. Modelling issues are discussed confidently and fluently, and the limitations of the work are exposed and analysed. The attitude displayed is one of confident self-criticism, shortcomings being viewed as opportunities for further development.

Notes

The oral report should be conducted formally. It is particularly important that the assessor writes comments in the box on the assessment sheet since this is the only source of evidence available for this domain.

MEI STRUCTURED MATHEMATICS

DECISION AND DISCRETE MATHEMATICS 1/2 (2620/2621) (Please delete as appropriate)

COURSEWORK ASSESSMENT SHEET

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Comment	Mark
Problem identification and modelling	4		
Solution of the mathematical problem	4		
Interpretation and refinement	3		
Written report	2		
Oral Communication	2		
		TOTAL	15

Half marks may be awarded but the overall total must be an integer.
Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

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DECISION AND DISCRETE MATHEMATICS 1 (2620) : AS

Objectives

To give the student experience of modelling and of the use of algorithms in a variety of situations.

To develop modelling skills.

The problems presented are diverse and require flexibility of approach. In their coursework, candidates are expected to be able to evaluate the success or failure of their modelling, and to appreciate the limitations of their solutions.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

The examination paper will have two sections.

Section A:	3 questions, each worth about 5 marks. Section Total: 15 marks
Section B:	3 questions, each worth about 15 marks. Section Total: 45 marks.

Component 2: Coursework (15 marks)

One assignment involving the modelling of a realistic problem. The coursework should **not** be based on the section of the specification entitled Algorithms.

Assumed Knowledge

Knowledge of the relevant techniques in *Pure Mathematics 1* is assumed.

DECISION AND DISCRETE MATHEMATICS 1

Specification		Competence Statements
MODELLING *The three Decision and Discrete Mathematics modules are based on the use of the modelling cycle in solving problems.*		
	D1p1	Be able to abstract from a real world problem to a mathematical model.
	2	Be able to analyse the model appropriately.
	3	Be able to interpret and communicate results.
	4	Be able progressively to refine a model as appropriate.
ALGORITHMS *Note: The coursework task should not be based on this section of the specification.*		
Background and definition.	D1A1	Be able to interpret and apply algorithms presented in a variety of formats.
	2	Be able to develop and adapt simple algorithms.
Basic ideas of complexity.	3	Understand the basic ideas of algorithmic complexity.
	4	Be able to analyse the complexity of some of the algorithms covered in this syllabus.
GRAPHS		
Backgrounds and definitions.	D1g1	Understand notation and terminology.
Use in problem solving.	2	Be able to model appropriate problems by using graphs.
NETWORKS		
Definition.	D1N1	Understand that a network is a graph with weighted arcs.
Use in problem solving.	2	Be able to model appropriate problems by using networks.
The minimum connector problem.	3	Know and be able to use Kruskal's and Prim's algorithms.
The shortest path from a given node to other nodes.	4	Know and be able to apply Dijkstra's algorithm.

DECISION AND DISCRETE MATHEMATICS 1

Notes	Notation	Exclusions
Approximation and simplification.		
Solution.		
Implications in real world terms.		
Check against reality; adapt standard algorithms.		
Flowcharts; written English; pseudo-code.		
To include sorting and packing algorithms. Sorting: Bubble, Shuttle, insertion, quick sort. Packing: Full-bin, first-fit, first-fit decreasing.		
Worst case; size of problem; that for quadratic algorithms doubling the size of a large problem can quadruple the solution time, etc. Candidates may need to use logarithms.	Order notation, eg $O(n^2)$ for quadratic complexity.	
Kruskal; Prim; Dijkstra. The requirements will also apply to algorithms in later modules (D&D2W and D&D2C) at the stage when they are met.		
Nodes/vertices; arcs/edges; trees; digraphs.	Pictures (ie graphs), incidence matrices.	
eg Königsberg bridges; various river crossing problems; the tower of cubes problem; filing systems.		
Use in modelling 'geographical' problems and other problems: eg translating a book into different languages; the knapsack problem.		
Kruskal's algorithm in graphical form only. Prim's algorithm in graphical or tabular form.		

DECISION AND DISCRETE MATHEMATICS 1

Specification		Competence Statements
LINEAR PROGRAMMING		
Linear inequalities in two or more variables.	D1L1	Be able to manipulate inequalities algebraically.
	2	Be able to illustrate linear inequalities in two variables graphically.
Formulation of constrained optimisation problems.	3	Be able to formulate simple maximisation of profit and minimisation of cost problems.
Solution of constrained optimisation problems.	4	Be able to formulate shortest path and critical path analysis problems as linear programming problems.
	5	Be able to use graphs to solve 2-D problems, including integer valued problems.
Algebraic interpretation of the graphical solution in 2 dimensions.	6	Be able to convert $\leqslant$ inequalities into equalities with slack variables.
	7	Be able to manipulate the resulting inequalities.
CRITICAL PATH ANALYSIS		
Using networks in project management.	D1X1	Be able to construct and use a precedence network.
	2	Be able to construct and interpret a cascade chart.
	3	Be able to construct and interpret a resource histogram.
	4	Understand the use of alternative criteria in project optimisation.
	5	Be able to crash a network.
SIMULATION		
Random variables	D1Z1	Be able to generate realisations of a uniformly distributed random variable.
	2	Be able to use random variables to model discrete non-uniform random variables.
Simulation modelling	3	Be able to build and use simple models.
	4	Be able to interpret results.
	5	Understand the need for repetition.

DECISION AND DISCRETE MATHEMATICS 1

Notes	Notation	Exclusions
		Non-linear problems.
Showing alternative feasible points and their costs in a simple maximisation problem.	max $2x + 3y$ s.t. $x + y \le 6$ $5x + 2y \le 12$ $x \ge 0,\ y \ge 0$	Non-linear problems. Integer valued problems in more than 2 dimensions.
Including forward and backward passes, the identification of critical activities and the calculation of float (independent and interfering).	Activity on arc.	Knowledge of an algorithm for constructing a precedence network from a precedence table. Knowledge of an algorithm for numbering activities. Knowledge of any specific algorithm for resource smoothing.
Time; cost; use of resources.		
Checking critical activities and for activities becoming critical.		
Drawing numbers from a hat; coins; dice; pseudo-random numbers from a calculator; simple pseudo-random number generators; random number tables.		Continuous random variables.
Cumulative frequency methods, including rejecting values where necessary.		
Hand simulations, including queuing situations.		

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DECISION AND DISCRETE MATHEMATICS 2 (2621) : A2

Objectives

To give the student experience of modelling and of the use of algorithms in a variety of situations.

To develop modelling skills.

The problems presented are diverse and require flexibility of approach. In coursework candidates are expected to be able to evaluate the success or failure of their modelling, and to appreciate the limitations of their solutions.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes.

There are four compulsory questions.

Two of the questions are worth approximately 10 marks each.

The other two questions are worth approximately 20 marks each.

Component 2: Coursework (15 marks)

One assignment involving the modelling of a realistic problem.

Assumed Knowledge

Knowledge of the module *Decision and Discrete Mathematics 1* is assumed. Knowledge of the relevant techniques in *Pure Mathematics 1* is also assumed.

DECISION AND DISCRETE MATHEMATICS 2

Specification		Competence Statements
NETWORKS		
The shortest path between any two nodes in a connected network.	D2N1	Know and be able to apply Dijkstra's algorithm repeatedly.
	2	Know and be able to apply Floyd's algorithm.
	3	Be able to analyse the complexity of both algorithms.
The travelling salesperson problem (TSP)	4	Be able to convert the practical problem into the classical problem.
	5	Be able to interpret a solution to the classical problem in terms of a solution to an underlying practical problem.
	6	Be able to analyse the complexity of complete enumeration.
	7	Be able to construct an upper bound for the solution to the classical problem.
	8	Be able to construct a lower bound for the solution to the classical problem.
The route inspection (Chinese postperson) problem (CPP).	9	Know and be able to apply the route inspection algorithm.
	10	Be able to analyse the complexity of the algorithm.
LOGIC		
Propositions and connectivity.	D2P1	Know and understand how to form compound propositions by using $\sim$, $\wedge$, $\vee$, $\Leftrightarrow$ and $\Rightarrow$.
	2	Be able to use truth tables to analyse propositions.
Switching and combinatorial circuits.	3	Be able to model compund propositions with simple switching and combinatorial circuits.
Boolean algebra.	4	Be able to manipulate Boolean expressions involving $\sim$, $\wedge$ and $\vee$ using the distributive laws and de Morgan's law.

DECISION AND DISCRETE MATHEMATICS 2

Notes	Notation	Exclusions
Practical problem: revisiting vertices allowed; network not necessarily complete.		
Classical problem: no revisiting allowed; network complete.		
Problems of factorial complexity.		
Using the nearest neighbour algorithm.		
Using a minimum connector of a reduced network.		
Pairing of odd nodes and repeating shortest paths between the members of each pair.		
$\frac{(2n)!}{2^n n!}$ ways of pairing $2n$ odd nodes.		
	Propositions will be represented by lower case alphabetic characters.	
Proofs of simple equivalences will be required.		

DECISION AND DISCRETE MATHEMATICS 2

Specification		Competence Statements
LINEAR PROGRAMMING		
The simplex algorithm.	D2A 1	Be able to solve simple maximisation problems with $\leqslant$ constraints and with two or more variables.
Geometric interpretation	2	Be able to identify tableaus (initial, intermediate and final) with points, particularly in the case of problems involving two or three variables.
$\geqslant$ inequalities	3	Understand the use of two stage simplex, and of the big M methods to construct an initial feasible solution to problems involving $\geqslant$ constraints.
Equality constraints	4	Understand how to model an equality constraint by using a pair of inequality constraints.
Problem solving.	5	Be able to formulate and solve a variety of problems as linear programming problems.
DECISION TREES		
Using networks in decision analysis.	D2N 1	Be able to construct and interpret simple decision trees.
	2	Be able to use expected monetary values (EMVs) to compare alternatives.
	3	Understand the concept of utility.

DECISION AND DISCRETE MATHEMATICS 2

Notes	Notation	Exclusions
The tabular form of the algorithm.		Any consideration of the complexity of the algorithm.
Showing alternative feasible points and their costs.		
Including the possibility that there is no feasible solution.		
Problems will not be restricted to maximisation problems with ⩽ constraints, eg. blending problems; shortest path problems; stock cutting problems.		Non-linear problems, including integer programming problems.
Candidates will need to be able to distinguish between, and handle, decision nodes and chance nodes.	EMV. Decision nodes: □ Chance nodes: ○	Explicit knowledge of Bayes' theorem.

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DECISION AND DISCRETE COMPUTATION (2622) : A2

Objectives

To give the student experience of modelling with the aid of suitable software and of the use of algorithms in a variety of situations.

To develop modelling skills with the aid of suitable software.

The problems presented are diverse and require flexibility of approach. Candidates are expected to be able to evaluate the success or failure of their modelling, and to appreciate the limitations of their solutions.

Assessment
(IT) 3.1, 3.2, 3.3

Examination (60 marks)

2 hours. Four questions will be set, three to be answered.

The questions cover all three themes. Individual questions are not restricted to a single theme.

Candidates require access to a computer with a spreadsheet program and a linear programming package, and suitable printing facilities throughout the examination.

Computing Resources

Candidates are expected to be thoroughly familiar with a spreadsheet program and an appropriate linear programming package. It is expected that a substantial proportion of the teaching for this module will involve the use of this software. Centres are advised to contact OCR about the latest available software.

Assumed Knowledge

Knowledge of the module *Decision and Discrete Mathematics 1* is assumed. Knowledge of the relevant techniques in *Pure Mathematics 1* is also assumed.

DECISION AND DISCRETE COMPUTATION

Specification		Competence Statements
NETWORK FLOWS		
Using networks to model transmission systems.	DCN1	Be able to model a transmission system using a network.
Maximum flow/minimum cut theorem.	2	Be able to specify a cut and to calculate its capacity.
	3	Understand that if an established flow is equal to the capacity of an identified cut, then the flow is maximal and the cut is a minimum cut.
Flow augmentation.	4	Be able to use flow augmentation and the labelling algorithm to establish a maximum flow in simple transmission networks (directed and undirected).
Linear programming formulation	5	Be able to formulate and solve network flow problems as linear programming problems.
MATCHINGS		
Bipartite graphs.	DCM1	Be able to identify when a bipartite graph is an appropriate model.
	2	Be able to construct a bipartite graph.
The matching algorithm.	3	Be able to construct an alternating path and use it to improve a matching.
	4	Be able to model a matching problem as a network flow problem.
Allocation and transformation.	5	Be able to recognise and formulate allocation and transportation problems.
LP formulation	6	Be able to formulate and solve matching, allocation and transformation problems as linear programming problems.
LINEAR PROGRAMMING		
Problem solving	DCL1	Be able to formulate and solve (using a computer package) a wide variety of problems as linear programming problems.
RECURRENCE RELATIONS		
Use in problem solving.	DCs1	Be able to model appropriate problems by using recurrence relations.
Solving recurrence relations.	2	Be able to solve first and second order homogeneous and non-homogeneous relations.
	3	Be able to produce, manipulate and interpret spreadsheet models of recurrence relations (including second order oscillatory relations).

DECISION AND DISCRETE COMPUTATION

<table>
<tr><th>Notes</th><th>Notation</th><th>Exclusions</th></tr>
<tr><td>Single and super sources and sinks.
Flow in = Flow out for other nodes.</td><td>Source: S
Sink: T</td><td></td></tr>
<tr><td>Describing a cut symbolically.</td><td>eg SA|BCT</td><td></td></tr>
<tr><td></td><td></td><td>Consideration of the complexity of the flow augmentation algorithm.</td></tr>
<tr><td>Maximal matchings; complete matchings.</td><td></td><td></td></tr>
<tr><td>To include the use of dummies as required.</td><td></td><td>The Hungarian and transportation algorithms.</td></tr>
<tr><td>Problems will not be restricted to maximisation problems with $\leqslant$ constraints: eg blending problems; shortest path problems; stock cutting problems.

The variety should be wider than that in earlier modules. eg flows in networks; matchings; allocation problems; transportation problems; set covering; set packing; maximin and minimax problems.

(The allocation problem, and other similar problems require integer solutions, but their structure is such that linear programming guarantees an integer solution.)

Problems requiring the setting up and use of indicator variables, eg modelling 'fixed cost if used' situations. Simple integer problems, eg the knapsack problem.</td><td></td><td>The theory of integer programming.</td></tr>
<tr><td>Second order homogeneous equations will have constant coefficients. Those to be solved analytically will have real roots.
Candidates will be expected to solve equations of the form $a^x = b$.</td><td></td><td>An extensive knowledge of particular solutions.</td></tr>
</table>

DECISION AND DISCRETE COMPUTATION

Specification		Competence Statements
SIMULATION		
Simulation modelling.	DCZ1	Be able to build and use discrete event/discrete time simulation models.
	2	Be able to use a spreadsheet function to generate uniformly distributed random numbers (discrete and continuous).
	3	Be able to use spreadsheet logic functions to transform random variables.
	4	Be able to determine the number of repetitions needed to obtain a required level of accuracy.
	5	Be able to verify and validate a model.
	6	Be able to interpret results.
Computer modelling	7	Be able to use a spreadsheet package to build, run and interpret simulation models.

DECISION AND DISCRETE COMPUTATION

Notes	Notation	Exclusions
Hand simulations.		Continuous random variables.
Computer simulations using spreadsheets.		
Use of standard error of output variable.		
Verification: checking that the model functions according to specifications.		
Validation: checking that a model adequately reflects reality.		

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NUMERICAL ANALYSIS

Introduction

Numerical analysis exists as a branch of mathematics because so many real-world problems in algebra and calculus cannot be solved analytically. For example, there are many equations for which an exact solution cannot be given, many functions which cannot be integrated exactly, and so on. It is an important element in any mathematician's education to appreciate this and to have some idea of how approximate solutions to such problems can be found.

Once the need for numerical methods is understood, it becomes important to investigate their accuracy and efficiency. For many problems there are 'obvious' approaches to obtaining an approximate solution, frequently based on the ideas of pure mathematics. In practice, however, such pure mathematical techniques are rarely efficient, they can be very inaccurate, and they sometimes fail altogether. The techniques of numerical analysis are developed to avoid these problems, or at the very least to minimise them.

Numerical methods are best carried out on a computer because it is only in that way that realistic problems can be solved to a satisfactory level of accuracy. The emphasis in studying these modules should therefore be on building up a repertoire of techniques which are sound in method and which are fully implemented on a computer. Though any general purpose programming language could be used, a spreadsheet is ideal for the purpose. Spreadsheets have all the mathematical and logical functions needed and they provide output in a tabular form which suits all numerical methods. (In *Numerical Computation* candidates are required to develop solutions using a spreadsheet.)

Coursework
(C) 3.3 (N) 4.1, 4.2, 4.3
(IT) 3.1, 3.2, 3.3

The units *Numerical Methods* and *Numerical Analysis* have coursework amounting to 20% of the total assessment in each case. The principles underlying the coursework for each module are the same, but a somewhat deeper treatment will be expected in the second module.

Candidates are expected to investigate a problem which is suitable for numerical solution using one of the methods in the syllabus. (Problems which have analytical solutions are acceptable only if the analytical solution is too time-consuming to be feasible.) Candidates should use a computer to develop a solution which is both efficient and accurate. In particular, they must show how the desired accuracy has been achieved, either by means of a theoretical analysis of errors, or by sufficient iterations of the numerical process to ensure that accuracy has been achieved.

Since it assumed throughout that numerical methods will be implemented on a computer, the coursework should arise naturally from the work done on each module.

There is no coursework for the unit *Numerical Computation* as candidates are required to develop computer-based solutions in the examination.

MEI STRUCTURED MATHEMATICS

NUMERICAL METHODS, NUMERICAL ANALYSIS (2623/2624)

(Please delete as appropriate)

COURSEWORK ASSESSMENT SHEET

The Numerical Analysis modules aim to develop skills in the areas of problem identification, use of numerical methods and control of error in practice. The coursework assignments should enable candidates to demonstrate a facility with technology and an awareness of the difficulties that can arise when computers are used to do mathematics.

Coursework Title .... Date ....

Candidate Name .... Candidate Number ....

Centre Name .... Centre Number ....

Domain	Mark	Description	Comment	Mark
Problem Specification (2)	1 1	An appropriate problem is identified. There is a correct explanation of why it is appropriate for numerical solution.		
Strategy (2)	1 1	An appropriate procedure is used. The candidate explains why the procedure to be adopted is appropriate to the problem.		
Formula Application (2)	1 1	There is an application of routine algorithms and methods, which is at least partially correct. There is a substantially correct application of routine algorithms and methods.		
Use of technology (2)	1 1	Appropriate use is made of calculator or computer software. There is a clear explanation of the use of technology.		
Error Analysis (3)	1 1 1	There is a competent analysis of errors, but it may be brief. The analysis is developed for the particular problem, either from theoretical formulae or iteration of methods to achieve convergence. There is consideration of the implication for the accuracy of the solution to the problem.		
Interpretation (2)	1 1	The solution is clearly expressed. In addition there is a discussion of the validity of the solution and any limitations are identified.		
Oral Communication (2)	2	Up to 2 marks may be awarded for the results of an interview or discussion. The candidate should be able to talk through the work submitted with enough confidence to enable the assessor to authenticate the work.		
			TOTAL	15

Half marks may be awarded but the overall total must be an integer.

Coursework should be available for moderation by OCR.

Authentication by the Centre

I confirm that this assignment has been assessed and internally standardised (where necessary) in accordance with the given guidelines and that as far as I can tell the work is that of the student only.
Signed
Name

Please report overleaf on any help that the candidate has received beyond the conduct guidelines.

NUMERICAL METHODS (2623) : A2

Objectives

To provide the student with an understanding that many mathematical problems cannot be solved analytically, but require numerical methods.

To develop a repertoire of simple numerical methods and to give experience in using them.

To state or prove theoretical results about the accuracy of these numerical techniques and to demonstrate the control of error in practice.

To implement these numerical methods on computers and to develop an awareness of the difficulties which can arise when computers are used to do mathematics.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes. Candidates answer all four questions.

Component 2: Coursework (15 marks)

One assignment.

Assumed Knowledge

Knowledge of the module *Pure Mathematics 1* and of the relevant techniques in *Pure Mathematics 2* is assumed.

NUMERICAL METHODS

Specification		Competence Statements
SOLUTION OF EQUATIONS		
Bisection Method. False Position (linear interpolation). Secant method. Fixed point iteration. Newton-Raphson method.	NMe1	Understand the graphical interpretations of these methods.
	2	Know how to solve equations to any required degree of accuracy using these methods.
	3	Understand the relative computational merits and possible failure of the methods.
	4	Know that fixed point iteration generally has first order convergence, Newton-Raphson generally has second order convergence.
ERRORS		
Absolute and relative error.	NMv1	Know how to calculate errors in sums, differences, products and quotients.
Error propagation by arithmetical operations and by functions.		
	2	Know how to calculate the error in f(x) when there is an error in x.
	3	Understand the effects on errors of changing the order of a sequence of operations.
Errors in the representation of numbers: rounding; chopping.	4	Understand that computers represent numbers to limited precision.
	5	Understand the consequences of subtracting nearly equal numbers.
NUMERICAL DIFFERENTIATION		
Forward difference method. Central difference method.	NMc1	Know how to estimate a derivative using the forward and central difference methods with a suitable value (or sequence of values) of h.
	2	Have an empirical and graphical appreciation of the greater accuracy of the central difference formula.
NUMERICAL INTEGRATION		
Mid-point rule. Trapezium rule. Simpson's rule.	NMc3	Be able to evaluate a given definite integral to any desired degree of accuracy using these methods.
The relationship between the methods.	2	Know the order of errors of the mid-point and trapezium rules. Understand the development of Simpson's rule from the mid-point and the trapezium rules.
FINITE DIFFERENCES		
Newton's forward difference interpolation method.	NMf1	Be able to use Newton's forward difference interpolation formula to reconstruct polynomials and to approximate functions.

NUMERICAL METHODS

Notes	Notation	Exclusions
Staircase and cobweb diagrams. A greater depth of treatment is expected than that in *Pure Mathematics 2*.		Proofs of orders of convergence.
	Exact value x. Approximate value X. Absolute error $X - x$ Relative error $\frac{X-x}{x}$.	
$f(x + h) \approx f(x) + hf'(x)$.		Functions of more than one variable.
	$f'(x) \approx \frac{f(x+h)-f(x)}{h}$. $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$.	
	M_n : mid-point rule with n strips. T_n : trapezium rule with n strips. S_{2n} : Simpson's rule with $2n$ strips.	
$T_{2n} = \frac{1}{2}(M_n + T$ $S_{2n} = \frac{1}{3}(2M_n + T$ $= \frac{1}{3}(4T_{2n} - T_n)$.		
Functions tabulated at equal intervals.	$\Delta f(x) = f(x + h) - f(x)$.	

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NUMERICAL ANALYSIS (2624) : A2

Objectives

To extend the repertoire of numerical methods developed in *Numerical Methods*, and to give experience in using them.

To state or prove theoretical results about the accuracy of these numerical techniques and to demonstrate the control of error in practice.

To implement these numerical methods on computers and to be aware of the difficulties which can arise when doing so.

Assessment

Component 1: Examination (60 marks)

1 hour 20 minutes. Candidates answer three questions from four set.

Component 2: Coursework (15 marks)

One assignment.

Assumed Knowledge

Knowledge of the modules *Numerical Methods*, *Pure Mathematics 1*, *2* and *3* is assumed.

NUMERICAL ANALYSIS

Specification		Competence Statements
TAYLOR SERIES		
Representations of functions by Taylor polynomials.	NAs1	Know how to obtain the Taylor polynomial of specified degree for a given function.
The error term.	2	Be able to estimate the error of a Taylor approximation over a given interval using the error term.
Truncation errors.	3	Be able to use a Taylor series and its error term to analyse errors in numerical differentiation and integration methods.
Order of convergence	4	Be able to develop the order of convergence of a quadrature method using Taylor series.
	5	Know how to analyse the order of convergence of the different numerical methods using a Taylor series and its error term
INTERPOLATION		
Lagrange's form of the interpolating polynomial.	NAf1	Be able to construct the interpolating polynomial of degree n given a set of n+1 data points.
Newton's *divided* difference interpolation method.	2	Be able to interpolate polynomials and approximate values of functions from tabulated data, using Newton's divided difference formula.
DIFFERENTIAL EQUATIONS		
Taylor series method.	NAc1	Be able to develop a recurrence relation for the solution $y(x)$ of a given first order
	2	differential equation $\frac{dy}{dx} = f(x, y)$ using Taylor series and implicit differentiation. Use this recurrence relation to solve the differential equation.
Orders of convergence.	3	Understand that higher order Taylor series converge more rapidly than lower order Taylor series.
ILL-CONDITIONING		
Absolute ill-conditioning. Relative ill-conditioning.	NAI1	Understand the problems associated with ill-conditioned systems.
Well-conditioned systems. Implications of ill-conditioning.	2	Recognise the effects of ill-conditioning on the solution of simultaneous equations, difference equations and differential equations.

NUMERICAL ANALYSIS

Notes	Notation	Exclusions
	$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots$ Error term $\frac{h^n}{n!} f^{(n)}(\xi)$ $x < \xi < x + h$.	Functions of more than one variable.
Only methods covered in *Numerical Methods.*		
		Knowledge of the error terms.
	$f[x_i] = f(x_i)$. $f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$. $f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$.	
	$y_{n+1} = y_n + hf(x_n, y_n) + \frac{h^2}{2!} f'(x_n, y_n) + \frac{h^3}{3!} f''(x_n, y_n) + \ldots$	

NUMERICAL ANALYSIS

Specification		Competence Statements
FINITE DIFFERENCES		
Relationships between operators.	NAO1	Understand and be able to manipulate operators.
Operator form of Taylor series.	2	Be able to develop and manipulate Taylor series in operator form.
Series method for numerical differentiation.	3	Be able to develop and use numerical differentiation formulae from operators.
	4	Be able to use these formula.
ERRORS		
Functions of two variables	NAU1	Know how to calculate the error in $f(x,y)$ when there is an error in x and an error in y.
SUMMATION OF SERIES		
	NAs1	Be able to use Euler's transformation and integrals to find the approximate sum of an alternating series.
Euler's transformation. Integrals.	2	Be able to use an integral to find the approximate sum of a series whose terms are all of the same sign.

NUMERICAL ANALYSIS

Notes	Notation	Exclusions
	Ef(x) = f($x + h$) Δf(x) = f($x + h$) - f(x) Df(x) = f '(x) E = $e^{h\text{D}}$ $\text{D} = \frac{1}{h}\ln(1+\Delta)$ $= \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \ldots\right)$	
Taylor series for functions of two variables.	f($x + h, y + k$) ≈ $f(x,y) + h\frac{\partial f}{\partial x} + k\frac{\partial f}{\partial y}$	

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NUMERICAL COMPUTATION (2625) : A2

Objectives

To extend the repertoire of numerical methods developed in *Numerical Methods* and, with the aid of suitable software, to give experience in using them.

To state or prove theoretical results about the accuracy of these numerical techniques and, with the aid of suitable software, to demonstrate the control of error in practice.

To implement these numerical methods on computers and to be aware of the difficulties which can arise when doing so.

Assessment
(IT) 3.1, 3.2, 3.3

Examination (60 marks)

2 hours

Candidates require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

Computing Resources

Candidates are expected to be thoroughly familiar with the use of a spreadsheet program to implement numerical techniques. It is expected that a substantial proportion of the teaching for this module will involve the use of spreadsheets.

Assumed Knowledge

Knowledge of the modules *Numerical Methods* and *Pure Mathematics 1, 2* and *3* is assumed.

NUMERICAL COMPUTATION

Specification		Competence Statements
SOLUTION OF EQUATIONS		
Relaxation. Richardson's method. Aitken's delta squared method.	NCe1	Be able to use relaxation and the methods of Richardson and Aitken to accelerate convergence.
NUMERICAL INTEGRATION		
Romberg’s method.	NCc1	Be able to perform Romberg integration on definite integrals.
Gaussian methods	2	Understand the principles of Gaussian methods.
	3	Be able to apply Gaussian methods to the evaluation of integrals.
DIFFERENTIAL EQUATIONS		
The Euler Method. The modified Euler method. (Runge-Kutta order 2).	4	Know how to solve a given first order differential equation $\frac{dy}{dx} = f(x, y)$ with initial conditions to any required degree of accuracy by repeated application of these methods.
	5	Understand that the modfied Euler method converges more rapidly than the Euler method.
Predictor-corrector methods.	6	Be able to solve first order differential equations using predictor-corrector methods.
Runge-Kutta methods.	7	Understand the concepts underlying the Runge-Kutta methods.
	8	Be able to solve first order differential equations using Runge-Kutta methods. Understand that higher order Runge-Kutta methods converge more rapidly than lower order methods.
Finite difference methods	9	Be able to use finite difference methods for solving second order differential equations.

NUMERICAL COMPUTATION

Notes	Notation	Exclusions
		Error terms.
Candidates will be expected to understand the difference between local error and global error in integration formula.	No standard notation will be required for Romberg's method. The notation used will be explained within any examination question.	
	$y_{n+1} \approx y_n + h\mathrm{f}(x_n,y_n)$ $y_{n+1} \approx y_n + \frac{h}{2}(k_1 + k_2)$. $k_1 = h\mathrm{f}(x_n,y_n)$ $k_2 = h\mathrm{f}(x_n + h, y + k_1)$	
Candidates will be expected to know the standard Runge-Kutta method of order 4 but may be asked to work with other methods, given sufficient information.	$k_1 = h\mathrm{f}(x,y)$ $k_2 = h\mathrm{f}\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right)$ $k_3 = h\mathrm{f}\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right)$ $k_4 = h\mathrm{f}(x + h, y + k_3)$ $y_{n+1} \approx y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$	
Central difference formula for $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \approx \frac{\mathrm{f}(x+h) - 2\mathrm{f}(x) + \mathrm{f}(x-h)}{2h^2}$	

NUMERICAL COMPUTATION

Specification		Competence Statements
APPROXIMATIONS TO FUNCTIONS		
Least squares.	NCc7	Be able to approximate data by simple functions (eg. quadratics and cubics) using the least squares method.
Cubic splines.	8	Be able to fit cubic splines to data.
LINEAR ALGEBRA		
Gaussian elimination. Pivoting. Matrix inversion. Determinants.	NCm1	Be able to solve systems of linear equations, invert matrices, find determinants using Gaussian elimination and pivoting strategies.
Gauss-Jacobi method. Gauss-Seidel method.	2	Be able to solve systems of linear equations using iterative methods. Understand and use the condition of diagonal dominance.

NUMERICAL COMPUTATION

Notes	Notation	Exclusions

6. FURTHER INFORMATION AND TRAINING FOR TEACHERS

The specifications are supported by a complete package provided by MEI and OCR.

INSET and teacher support

- A Programme of INSET meetings, including the MEI annual 3-day conference and Training Days around the country.
- MEI branch meetings.
- Help from both MEI and OCR at the end of the telephone.
- Regular newsletters from MEI and OCR.
- Information available on Web-sites.

Examinations

- Past examination papers and mark schemes.
- Examiners' reports.
- Specimen examination papers and mark schemes.

Coursework

- A bank of coursework resource material.
- Exemplar marked coursework.
- Reports to centres from coursework moderators.

Teaching materials

- Students' Handbook.
- Guidance on opportunities for developing the Key Skills.
- Textbooks.
- Practice comprehension questions.
- Videos.

7 FOUNDATIONS OF ADVANCED MATHEMATICS

This module does not carry any credit towards AS or A level qualifications.

The syllabus that follows is valid for assessment from January 2000 until June 2001. It is possible that changes will be made to it for use after that date.

OBJECTIVES

To provide access to AS and A level Mathematics courses for those who are not yet ready to undertake them.

To provide mathematics for students preparing for Higher Education whose studies include a significant numerate element.

To contribute towards GNVQ qualifications.

To contribute towards Key Skills Application of Number qualifications.

To provide a qualification and a worthwhile course in its own right.

ASSESSMENT

The assessment of this module is by examination. The examination will be held in January and June each year. It will last $1^1/_2$ hours and consist of 30 multiple choice questions each worth one mark.

Students using this module as a course in their work towards a GNVQ and/or Key Skills Application of Number qualification must check carefully what they need to do to obtain these awards.

GRADING

The examination result is reported as a grade (A, B, C, D, E or U).

Grade	Typical Raw Mark threshold (/30)
A	24
B	21
C	18
D	15
E	12

The actual grade thresholds will be set making a reasonable allowance for examination performance, and for any features of a particular paper that only become apparent after it is taken.

Grade A	The student is clearly ready to study Mathematics at a higher level.
Grade B/C	The student should benefit from further study in Mathematics.
Grade D/E	The student has benefited from studying this module.

AWARDING BODY

OCR

ASSUMED KNOWLEDGE

Candidates are expected to have knowledge of Level 6 and familiarity with Level 7 of the National Curriculum.

CALCULATING AIDS

Candidates are permitted to use a scientific or graphical calculator in the **examination** for this module. Computers and calculators with computer algebra facilities are not permitted.

The use of computer software (e.g. spreadsheet) is encouraged in **classwork**.

FUNDING

Foundations of Advanced Mathematics is recognised by FEFC as a fundable course (subject to certain conditions) for those seeking access to AS or A level or to Higher Education.

CONTENT

Syllabus

PROBLEM SOLVING

The specification content is structured under the headings Arithmetic, Algebra, Graphs, Statistics and Trigonometry for ease of reference. It is not intended that these areas of work remain discrete since the content forms a coherent syllabus in which one area of work supports another.

In the real world mathematics is used by industry and commerce to solve problems. The mathematics involved is often quite straightforward so that many problems can be solved by confident use of the techniques in this syllabus.

Students should be encouraged to recognise and use mathematics as a tool in problem solving. Each of the following skills is part of the problem solving process. Together they should therefore pervade the whole of the course.

- identifying the mathematics in a situation.
- making simplifying assumptions to allow work to begin.
- selecting appropriate techniques.
- carrying out the activities required by these techniques.
- obtaining results.
- checking the reasonableness of the results.
- giving results to a suitable degree of accuracy.
- presenting results in an appropriate way.
- drawing conclusions.
- considering the validity of results.

Notes

These skills are all involved in the Application of Number Key Skill and so Foundations of Advanced Mathematics provides a suitable course to support that Unit.

FOUNDATIONS OF ADVANCED MATHEMATICS

Syllabus		Competence Statements
ARITHMETIC		
Numerical terms.	FB 1	Know and be able to use vocabulary and notation appropriate to arithmetic at this level.
Numerical techniques.	2	Be able to evaluate expressions.
	3	Be able to work with fractions.
	4	Be able to solve problems involving ratio and proportion.
	5	Understand and be able to work with percentages.
Equivalence.	6	Understand the equivalence of fractions, decimals, percentages and ratios and be able to convert from one of these forms to another.
Indices.	7	Be able to work with numbers in index form.
	8	Be able to use standard form.
Mensuration.	9	Be able to use appropriate units.
	10	Be able to convert from one set of units to another.
	11	Be able to solve problems involving perimeter, area and volume.
	12	Be able to use scale drawings.
Accuracy.	13	Know how to write any number to a specified number of decimal places or significant figures, or to some other level of accuracy.
	14	Be able to give numerical answers to an appropriate degree of accuracy.
	15	Be able to identify the effects of any accumulating errors on calculations.
Reasonable conclusions.	16	Be able to comment on the expected order of magnitude of an answer and to decide if the answer is reasonable.
	17	Be able to establish reasonable upper and lower bounds for the answer to a problem.

FOUNDATIONS OF ADVANCED MATHEMATICS

Notes	Notation	Exclusions
Including integer, factor, multiple, prime, power, root, reciprocal.		
Including negative numbers, brackets, powers; correct order of precedence.		
Including inverse proportion.		
Including percentage change and 'reverse' percentage problems where 100% is required.		
e.g. $^3/_4$, 0.75, 75%, 3:1.		
$7^2 \times 7^3 = 7^5$ etc.		
Both large and small numbers.		
Including compound measures, eg metres per second; density.	ms^{-1}	
Including between metric and Imperial.		eg m^2 into yd^2.
Including cylinders, prisms, cones, pyramids and spheres.		Frustum of a cone
Including nearest 10, nearest 100 etc.		
Accuracy appropriate to context including sensible rounding of calculator displays.		
By making rough checks. By using a context to consider the reasonableness of a result.		

FOUNDATIONS OF ADVANCED MATHEMATICS

Syllabus		Competence Statements
ALGEBRA		
Relationships.	Fa 1	Know and be able to use vocabulary and notation appropriate to algebra at this level.
	2	Be able to express relationships in symbolic form.
	3	Be able to write the rule for a sequence in symbolic form.
Interpretation.	4	Be able to explain the contents of an algebraic expression in words.
	5	Be able to form an algebraic expression from a description in words.
Evaluation.	6	Be able to substitute given numbers into an expression and evaluate it.
Techniques.	7	Be able to simplify basic algebraic expressions.
	8	Be able to multiply out expressions involving brackets.
	9	Be able to use brackets to factorise expressions.
	10	Be able to factorise quadratic expressions.
	11	Be able to add or subtract algebraic fractions in cases when the denominator is a positive integer.
	12	Be able to rearrange simple algebraic formulae.
Solutions.	13	Be able to solve linear equations.
	14	Be able to solve simultaneous linear equations.
	15	Be able to solve quadratic equations.
	16	Be able to formulate linear, simultaneous and quadratic equations.
	17	Be able to solve linear inequalities.
Checking answers.	18	Know how to check answers to equations by substitution.

FOUNDATIONS OF ADVANCED MATHEMATICS

Notes	Notation	Exclusions
Including expression, constant, variable, term, coefficient, equation, identity, factorise, solve, root.		
Including simple relationships involving more than one variable eg $t = 3600h + 60\,m + s$.		
Sequences based on linear (eg $n \rightarrow 2n + 5$), quadratic and exponential functions (eg $P = 3 \times 2^n$).	$n \rightarrow 2n + 5$ or $\mathrm{f}(n) = 2n + 5$.	The use of e.
eg $2(3x + 4y) - (x - 5y) = 5x + 13y$. $(2x + 3)(5x - 4) = 10x^2 + 7x - 12$.		
eg $3x^2 + 12xy = 3x(x + 4y)$;		
eg $\dfrac{5a - 3b}{6} - \dfrac{2a + b}{4}$		
eg make t the subject in $v = u + at$; make r the subject in $A = \pi r^2$.		The subject appearing more than once.
eg $5x - 9y = 13$, $3x - 7y = 7$.		More than 2 unknowns.
By factorisation and by formula.		
eg $2x + 3 > 5x - 6$.		Graphical inequalities

FOUNDATIONS OF ADVANCED MATHEMATICS

Syllabus		Competence Statements
GRAPHS		
Plotting.	Fg 1	Be able to plot data.
	2	Be able to draw graphs by constructing a table of values.
	3	Be able to construct and use conversion graphs.
	4	Be able to use a graphical calculator or a computer package to draw graphs.
Information.	5	Be able to extract information from a graph.
	6	Know how to find the gradient (m) and intercept (c) of a straight line graph and how they are related to its equation ($y = mx + c$).
Solving equations.	7	Be able to solve an equation in one unknown graphically.
	8	Be able to solve a pair of simultaneous equations in two unknowns graphically.
	9	Be able to use the zoom facility on a graphical calculator or computer package.
Gradient.	10	Be able to estimate the gradient of a curve at a point by drawing the tangent.
	11	Be able to interpret gradient.
Area.	12	Be able to estimate the area under a curve.
	13	Be able to interpret area.

FOUNDATIONS OF ADVANCED MATHEMATICS

Notes	Notation	Exclusions
The data may be given or found experimentally.		
$y = ax^3 + bx^2 + cx + d + ex^{-1} + fx^{-2}$ where at least three of the constants, a, b, c, d, e and f, are zero.		
eg currency conversion.		
Including specific value, maximum value, minimum value.		Calculus techniques; completing the square.
Including finding m and c from a graph and from an equation.		
Point(s) of intersection with the x-axis.		
eg solve $x^2 - 5 = 0$ to 3 significant figures.		
Including rate of change.		
By counting squares or approximating as the sum of rectangles and triangles (or trapezia).		Formal application of rules eg the trapezium rule.

FOUNDATIONS OF ADVANCED MATHEMATICS

Syllabus		Competence Statements
TRIGONOMETRY		
Techniques.	Ft 1	Know and be able to use Pythagoras theorem.
	2	Know the meanings of sine, cosine and tangent.
	3	Be able to find sine, cosine and tangent of any angle.
	4	Know the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for all values of x.
	5	Be able to use sine, cosine and tangent to find unknown sides and angles in right-angled triangles.
	6	Be able to find sides, angles and areas in figures involving more than one triangle.
Three dimensions.	7	Be able to interpret drawings of three-dimensional objects.
	8	Be able to draw simple three-dimensional objects.
	9	Be able to calculate lengths and angles in three-dimensional objects.
Vectors.	Fv 1	Know and be able to use vocabulary and notation appropriate to vectors at this level.
	2	Be able to work with vectors.
	3	Be able to solve simple problems involving vector quantities by scale drawing and calculation.

FOUNDATIONS OF ADVANCED MATHEMATICS

Notes	Notation	Exclusions
Including the converse.		
Using degrees.		Radians.
Including forms like $y = 5\sin x$, $y = \cos x + 3$.		$y = \sin 2x$ etc.
Including scalar, vector, modulus, magnitude, direction, component, equal vectors, unit vectors, resultant.	$\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$	
Including addition, subtraction, scalar multiplication.		Vectors in three dimensions.
Graphical representation of vector quantities eg displacement, velocity, force.		

FOUNDATIONS OF ADVANCED MATHEMATICS

Syllabus	Competence Statements
STATISTICS	
Collection.	FS 1 Understand that data collection may require sampling.
	2 Be able to design and carry out a sensible sampling procedure.
	3 Be able to organise data.
Display.	FD 1 Be able to display data using pictograms, pie charts, bar charts, vertical line diagrams, frequency charts, and line graphs.
	2 Be able to construct a cumulative frequency table and plot the corresponding graph.
	3 Know and be able use standard conventions involved in data display.
	4 Recognise techniques and arguments designed to give a misleading impression.
Interpretation.	5 Be able to describe the main features of a displayed data set.
	6 Be able to extract numerical information from a displayed data set.
Central tendency.	7 Know the meanings of mode, median and mean and be able to find their values from a given data set.
	8 Be able to make judgements on the most appropriate measure of central tendency in a given situation.
Spread.	9 Understand the concept of spread.
	10 Be able to find values of range, interquartile range and standard deviation.
Comparison.	11 Be able to compare data sets.
Probability.	Fu 1 Be able to calculate probability theoretically when it is possible to do so.
	2 Be able to estimate probability as relative frequency.
	3 Appreciate that in everyday use probability is often estimated subjectively.
	4 Be able to find the probability of a complementary event.
Combined events.	5 Be able to find the probability of more than one event, distinguishing between situations where it is necessary to add and to multiply.
	6 Be able to use tree diagrams to find probabilities.
	7 Be able to distinguish between situations where the second event is/is not independent of the first.

FOUNDATIONS OF ADVANCED MATHEMATICS

Notes	Notation	Exclusions
Determine appropriate groups/classes.		
Including histograms with equal class intervals.		Histograms with unequal class intervals.
Correct use of scales and axes.		
eg truncation of axes; use of mean of bi-modal data.		
Including trends.		
		Moving averages.
Including the use of statistical facilities on a calculator to find the mean.	Mean = $\bar{x}$	Questions requiring long calculations 'by hand' will not be set.
		Mean deviation.
Including the use of statistical facilities on a calculator to find the standard deviation.	Standard deviation = s	Questions requiring long calculations 'by hand' will not be set.
		Formal tests.
eg situations involving coins, dice or cards.	P(heads) = $^{1}/_{2}$	
Using data.		
eg outcomes of sporting events.		
	A' is the event 'not-A'.	
First-then and either-or situations.		
Tree diagrams with two levels of branching.		
Drawing cards from a pack without/with replacement.		

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