ALL questions should be attempted.
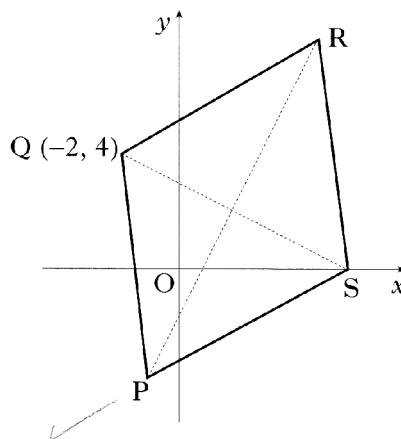
Marks

1. The diagram shows a rhombus PQRS with its diagonals PR and QS.

   PR has equation $y = 2x - 2$.

   Q has coordinates $(-2, 4)$.

   (a) (i) Find the equation of the diagonal QS.

   (ii) Find the coordinates of T, the point of intersection of PR and QS.

   (b) R is the point $(5, 8)$. Write down the coordinates of P.

6

2

2. With reference to a suitable set of coordinate axes, A, B and C are the points $(-8, 10, 20)$, $(-2, 1, 8)$ and $(0, -2, 4)$ respectively.

   Show that A, B and C are collinear and find the ratio AB : BC.

4

3. (a) Calculate the limit as n → ∞ of the sequence defined by $u_{n+1} = 0{\cdot}9u_n + 10$, $u_0 = 1$.

3

   (b) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.

2

4. (a) Write $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a < 360$.

4

   (b) Hence find the maximum value of $5 + \sqrt{3}\sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \le x \le 360$.

2

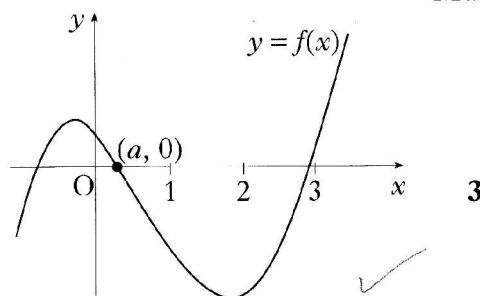5. Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \le x < 2\pi$.

4

[Turn over

Marks

6. The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown.

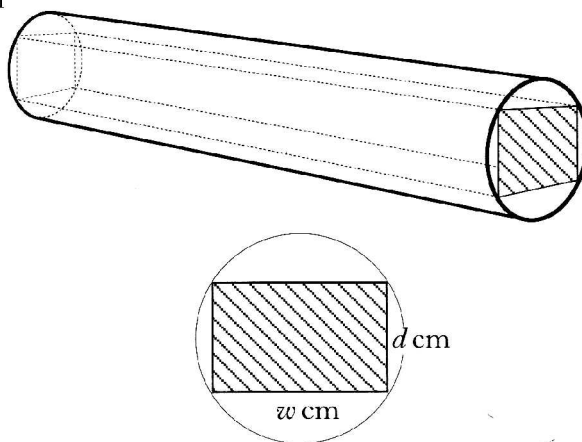   Find the value of a correct to one decimal place.

   3

7. A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.

   The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.

   The strength S of the beam is given by

   $S = 1{\cdot}7\, w\, (400 - w^2)$.

   Find the dimensions of the beam for maximum strength.

   5

8. Find $\displaystyle\int_0^1 \left(\cos(3x) - \sin\left(\tfrac{1}{3}x + 1\right)\right) dx$ correct to 3 decimal places.

   3

9. A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2{\cdot}6)^{0{\cdot}2t}$.

   (a) What was the size of the colony when observations began?

   1

   (b) How long does it take for the size of the colony to be multiplied by 10?

   4

10. The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.
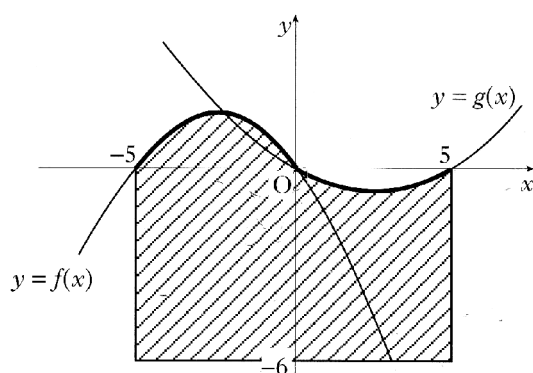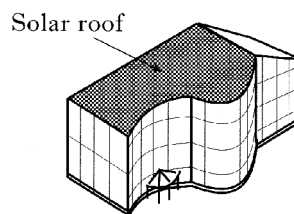
    (a) Find the value of k.

    7

    (b) Deduce the coordinates of the point of contact.

    2

Marks

11. An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.

Solar roof

The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x) = \frac{1}{4}\left(-x^2 - 5x\right)$, $g(x) = \frac{1}{12}\left(x^2 - 5x\right)$ and the lines $x = -5$, $x = 5$ and $y = -6$.

(a) Find the area of the solar roof.   7

(b) Ten square units of solar cells generate a maximum of 1 kilowatt.

What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?   1

[END OF QUESTION PAPER]