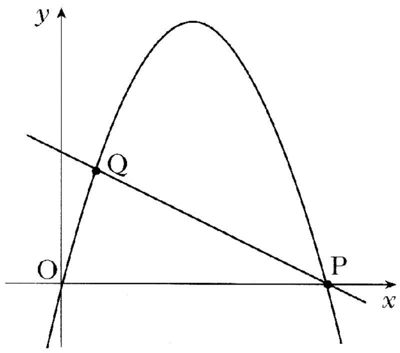
ALL questions should be attempted.

Marks

1. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.

The line PQ has equation $2y + x = 4$.

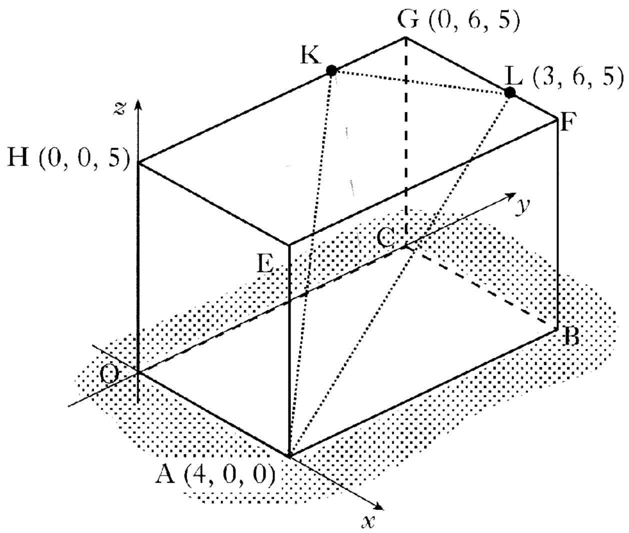
Find the coordinates of P and Q. **5**



2. OABCEFGH is a cuboid.

With axes as shown, O is the origin and the coordinates of A, H, G and L are (4, 0, 0), (0, 0, 5), (0, 6, 5) and (3, 6, 5) respectively.

K lies two thirds of the way along HG, (ie HK:KG = 2:1).



(*a*) Determine the coordinates of K. **2**

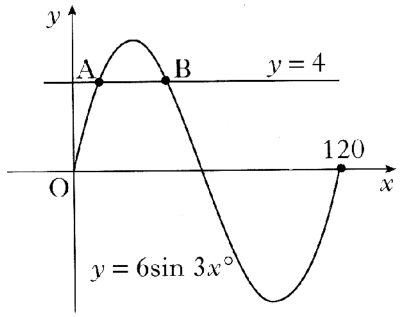
(*b*) Write down the components of $\overrightarrow{AK}$ and $\overrightarrow{AL}$. **2**

(*c*) Calculate the size of angle KAL. **5**

Marks

3. The diagram shows part of the graph of $y = 6\sin 3x$ and the line with equation $y = 4$.

Find the x-coordinates of A and B. **3**



4. Two sequences are defined by the recurrence relations

$$u_{n+1} = 0{\cdot}2u_n + p, \quad u_0 = 1 \quad \text{and}$$
$$v_{n+1} = 0{\cdot}6v_n + q, \quad v_0 = 1.$$

(*a*) Explain why each of these sequences has a limit. **1**

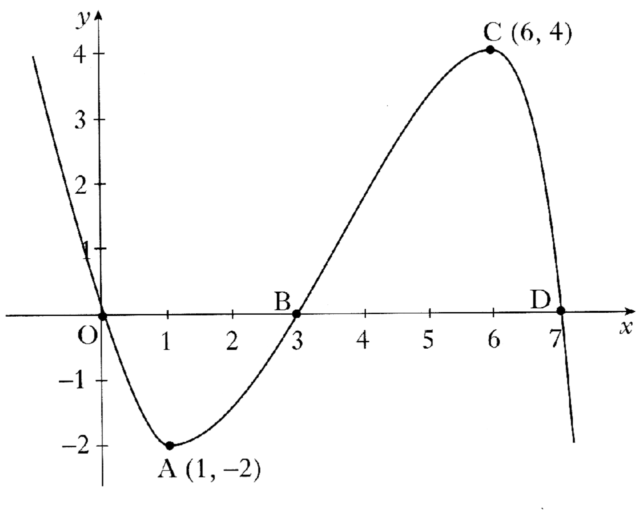
(*b*) If both sequences have the same limit, express p in terms of q. **3**

5. Part of the graph of $y = f(x)$ is shown in the diagram. On separate diagrams, sketch the graphs of

(i) $y = f(x + 1)$

(ii) $y = -2f(x)$.

Indicate on each graph the images of O, A, B, C and D. **5**



Marks

6. The graph of $y = g(x)$ passes through the point (1, 2).

If $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$, express y in terms of x. **4**

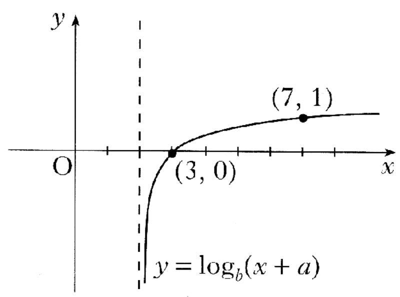
7. The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(*a*) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k. **4**

(*b*) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. **3**

8. The diagram shows part of the graph of $y = \log_b(x + a)$.

Determine the values of a and b. **3**

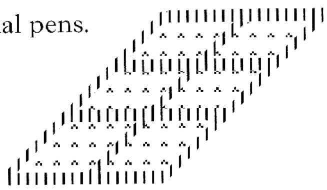


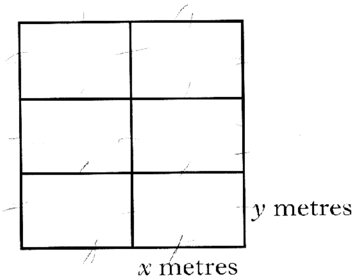
9. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

Marks

10. A zookeeper wants to fence off six individual animal pens.





Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(*a*) (i) Express the total length of fencing in terms of x and y.

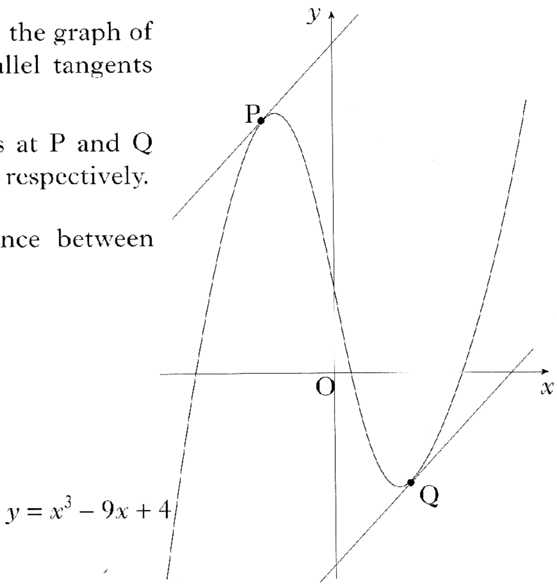
(ii) Given that the total length of fencing is 360 m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. **4**

(*b*) Find the values of x and y which give the maximum area and write down this maximum area. **6**

11. The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.

The equations of the tangents at P and Q are $y = 3x + 20$ and $y = 3x - 12$ respectively.

Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **5**



[END OF QUESTION PAPER]