This is the equation we are trying to solve,

$$\frac{dy}{dx} - \frac{1}{x}y = xe^x$$

I have simply divided by x because we have to get the equation in the form below

Solving differential equations in the form

$$\frac{dy}{dx} + P(x)y = G(x)$$

Where P(x) and G(x) are functions of x and in this case, $P(x) = \frac{1}{x}$ and $G(x) = xe^x$

The integrating factor is found by

$$e^{\int P(x)dx}$$

So therefore, in this question, the integrating factor is

$$e^{\int -\frac{1}{x}dx}$$

$$\int -\frac{1}{x}dx = -lnx = lnx^{-1} = ln\frac{1}{x}$$

So there fore, the integrating factor is

$$e^{ln\frac{1}{x}}$$

Which simply equals x

So the Integrating Factor (I.F) = $\frac{1}{x}$

The equation isn't solved yet though, this is just a step we have to take to make it solveable.

What we now do is multiply the whole equation;

$$\frac{dy}{dx} - \frac{1}{x}y = xe^x$$

By the integrating factor.

So multiplying by $\frac{1}{x}$ gives us

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = e^x$$

You have to now realise that

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \frac{d}{dx}\left[\frac{1}{x}y\right]$$

Where $\frac{d}{dx}\left[\frac{1}{x}y\right]$ means the differential of $\frac{1}{x}y$

So now you have

$$\frac{d}{dx}\left[\frac{1}{x}y\right] = e^x$$

Now you can integrate both sides of the equation,

$$\int \frac{d}{dx}\left[\frac{1}{x}y\right] = \int e^x\, dx$$

Which therefore gives

$$\frac{1}{x}y = e^x + c$$

To find c, you substitute your values of y=2 and x=1

So,

$$\frac{1}{1}.2 = e^1 + c$$

So,

$$c = 2 - e$$

$$\frac{1}{x}y = e^x + 2 - e$$

And to get y, just multiply by x

$$y = xe^x + 2x - xe$$

Im hoping thats right..

Heres a summary for solving differential equations in the form

$$\frac{dy}{dx} + P(x)y = G(x)$$

-rearrange to get it into that form (like i did with dividing by x)

-find the integrating factor

$$e^{\int P(x)dx}$$

-multiply the equation be the integrating factor

-put the left hand side of the equation in the form $\frac{d}{dx}[(I.F)y]$ where I.F is the integrating factor.

-integrate both sides.

-find constant

This is a tough thing to learn at the start, so don't worry if its hard to understand, if there are any questions, just ask.