GCE

Edexcel GCE

Core Mathematics C3 (6665)

January 2006

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Mark Scheme (Results)

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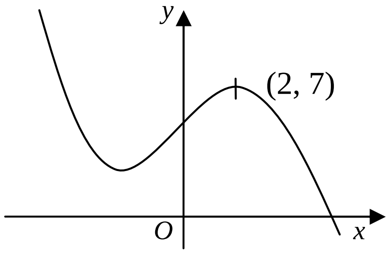
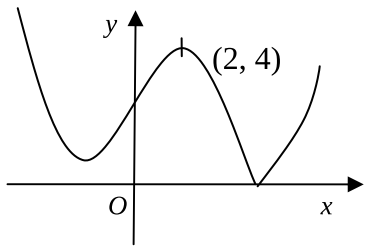
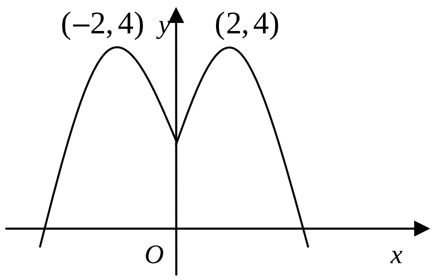
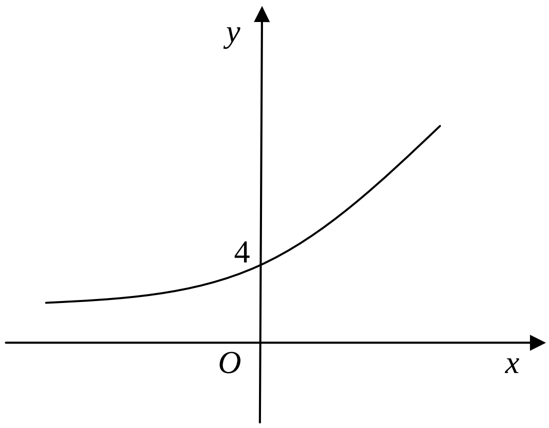
Question Number	Scheme		Marks	
1.	(a) 	Shape unchanged Point	B1 B1	 **(2)**
	(b) 	Shape Point	B1 B1	 **(2)**
	(c) 	Shape (2, 4) (−2, 4)	B1 B1 B1	 **(3)** **[7]**

Question Number	Scheme	Marks
2.	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}=\dfrac{x(2x+3)}{(2x+3)(x-2)}=\dfrac{x}{x-2}$	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{x^2-x-2}=\dfrac{x(x+1)-6}{(x-2)(x+1)}$	M1
	$=\dfrac{x^2+x-6}{(x-2)(x+1)}$	A1
	$=\dfrac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1
	$=\dfrac{x+3}{x+1}$	A1 **(7)**
		[7]
	Alternative method	
	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$(2x+3)$ appearing as a factor of the numerator at any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{(x-2)(x+1)}=\dfrac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1
	$=\dfrac{2x^3+5x^2-9x-18}{(2x+3)(x-2)(x+1)}$ can be implied	A1
	$=\dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(2x+3)(x^2+x-6)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(x+3)(2x^2-x-6)}{(2x+3)(x-2)(x+1)}$ Any one linear factor × quadratic	M1
	$=\dfrac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ Complete factors	A1
	$=\dfrac{x+3}{x+1}$	A1 **(7)**

Question Number	Scheme	Marks
3.	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ accept $\frac{3}{3x}$	M1 A1
	At $x = 3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3} \Rightarrow m' = -3$ Use of $mm' = -1$	M1
	$y - \ln 1 = -3(x-3)$	M1
	$y = -3x + 9$ Accept $y = 9 - 3x$	A1 **(5)**
		[5]
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3x}$ leading to $y = -9x + 27$ is a maximum of M1 A0 M1 M1 A0 =3/5	
4.	(a) (i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x+2}\right) = 3\mathrm{e}^{3x+2}$ $\left(\text{or } 3\mathrm{e}^{2}\mathrm{e}^{3x}\right)$ At any stage	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\,\mathrm{e}^{3x+2} + 2x\,\mathrm{e}^{3x+2}$ Or equivalent	M1 A1+A1
		(4)
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos\left(2x^3\right)\right) = -6x^2\sin\left(2x^3\right)$ At any stage	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-18x^3\sin\left(2x^3\right) - 3\cos\left(2x^3\right)}{9x^2}$	M1 A1 **(4)**
	Alternatively using the product rule for second M1 A1	
	$y = (3x)^{-1}\cos\left(2x^3\right)$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3(3x)^{-2}\cos\left(2x^3\right) - 6x^2(3x)^{-1}\sin\left(2x^3\right)$	
	Accept equivalent unsimplified forms	
	(b) $1 = 8\cos(2y+6)\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\cos(2y+6)$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8\cos(2y+6)}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ $\left(= (\pm)\frac{1}{2\sqrt{\left(16 - x^2\right)}}\right)$	M1 A1 **(5)**
		[13]

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ * cso	A1 **(3)**
	(b) $x_1 = 1.41,\ x_2 = 1.39,\ x_3 = 1.39$	B1, B1, B1
	If answers given to more than 2 dp, penalise first time then accept awrt above.	**(3)**
	(c) Choosing $(1.3915, 1.3925)$ or a tighter interval	M1
	$\mathrm{f}(1.3915) \approx -3\times10^{-3},\ \mathrm{f}(1.3925) \approx 7\times10^{-3}$ Both, awrt	A1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$	
	$\Rightarrow \alpha = 1.392$ to 3 decimal places * cso	A1 **(3)** **[9]**
6.	(a) $R\cos\alpha = 12,\quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12},\quad \Rightarrow \quad \alpha \approx 18.43°$ awrt 18.4°	M1, A1**(4)**
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R}\quad (\approx 0.5534)$	M1
	x + their $\alpha = 56.4°$ awrt 56°	A1
	$= \ \ldots\ ,\ 303.6°$ $360° -$ their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)** **[12]**

Question Number	Scheme	Marks
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1
	$\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ ∗ cso	A1 **(2)**
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ ∗ cso	A1 **(3)**
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1
	$\cos 2\theta = \sin 2\theta$ ∗	A1 **(3)**
	(c) $\tan 2\theta = 1$	M1
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1
	The 4 correct solutions	A1 **(4)**
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.	**[12]**

Question Number	Scheme	Marks
8.	(a) $\quad \mathrm{gf}(x) = \mathrm{e}^{2(2x+\ln 2)}$	M1
	$= \mathrm{e}^{4x}\mathrm{e}^{2\ln 2}$	M1
	$= \mathrm{e}^{4x}\mathrm{e}^{\ln 4}$	M1
	$= 4\mathrm{e}^{4x}$ $\quad$ Give mark at this point, cso	A1 $\quad$ **(4)**
	$\left(\text{Hence } \mathrm{gf} : x \mapsto 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}\right)$	
	(b)  Shape and point	B1 $\quad$ **(1)**
	(c) $\quad$ Range is $\mathbb{R}_+$ $\quad$ Accept $\mathrm{gf}(x) > 0,\ y > 0$	B1 $\quad$ **(1)**
	(d) $\quad \dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{gf}(x)\right] = 16\mathrm{e}^{4x}$	
	$\mathrm{e}^{4x} = \dfrac{3}{16}$	M1 A1
	$4x = \ln\dfrac{3}{16}$	M1
	$x \approx -0.418$	A1 $\quad$ **(4)**
		[10]