STEP III question 2, part three

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Dadeyemi proved that:

$$\sum_{r=1}^{\infty} \arctan \frac{1}{r^2+r+1} = \frac{\pi}{4}$$

To prove the sum in part three we define

$$H_r = \arctan \frac{1}{r^2-r+1}$$

$$K_r = \arctan \frac{1}{r^2+r+1}$$

We notice that $H_{r+1} = K_r$ so:

$$\sum_{r=1}^{\infty} H_r = H_1 + \sum_{r=2}^{\infty} H_r = \frac{\pi}{4} + \sum_{r=1}^{\infty} K_r = \frac{\pi}{2}$$