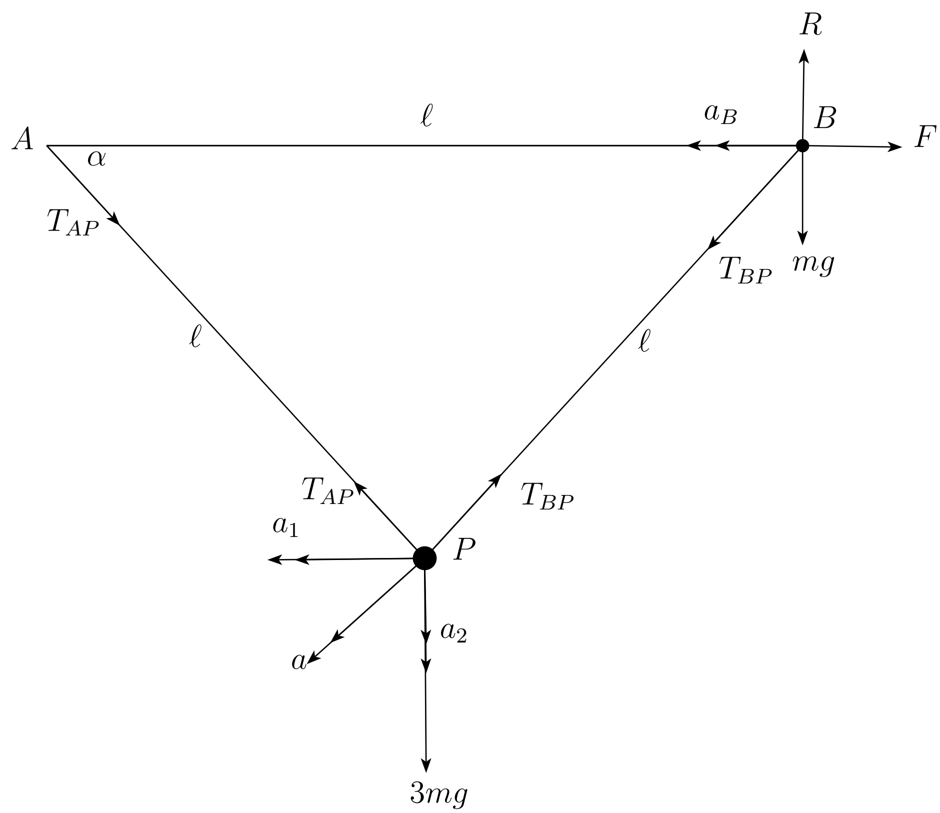
Question 10

A bead B of mass m can slide along a rough horizontal wire. A light inextensible string of length 2ℓ has one end attached to a fixed point A of the wire and the other to B. A particle P of mass $3m$ is attached to the mid-point of the string and B is held at a distance ℓ from A. The bead is released from rest.
Let a_1 and a_2 be the magnitudes of the horizontal and vertical components of the initial acceleration of P. Show by considering the motion of P relative to A, or otherwise, that $a_1 = \sqrt{3}a_2$. Show also that the magnitude of the initial acceleration of B is $2a_1$.

The diagram of the situation is as follows



Consider the acceleration components at P, we have $a_1^2 + a_2^2 = a^2$ and that

$$a\cos\alpha = \frac{1}{2}a = a_1$$
$$a\sin\alpha = \frac{\sqrt{3}}{2}a = a_2$$

since $\alpha = \pi/3$ (for an equilateral triangle), hence $a_1 = \sqrt{3}a_2$.

The acceleration of the system is therefore a, which is $a_1/\cos\alpha = 2a_1 = a_B$.

Given that the frictional force opposing the motion of B is equal to $\left(\sqrt{3}/6\right) R$, where R is the normal reaction between B and the wire, show that the magnitude of the initial acceleration of P is $g/18$.

We need to resolve the forces around B and P individually.

At B:

$$(\rightarrow) : 2a_1 m = \frac{1}{2}T_{BP} - \frac{\sqrt{3}}{6}R \quad (1)$$

$$(\uparrow) : R = mg + \frac{\sqrt{3}}{2}T_{BP} \quad (2)$$

and at P :

$$(\rightarrow) : 3a_1 m = \frac{1}{2}\left(T_{AP} - T_{BP}\right) \quad (3)$$

$$(\uparrow) : 3a_2 m = 3mg - \frac{\sqrt{3}}{2}\left(T_{AP} + T_{BP}\right) \quad (4)$$

$\frac{1}{6}\sqrt{3}(2) + (1)$:

$$\frac{1}{2}T_{BP} = \frac{\sqrt{3}}{6}\left(mg + \frac{\sqrt{3}}{2}T_{BP}\right) + 2a_1 m \Leftrightarrow T_{BP} = 8ma_1 + \frac{2}{3}\sqrt{3}mg \ (5)$$

(4)+$\sqrt{3}$(3):

$$\sqrt{3}T_{BP} = -3m\left(\sqrt{3}a_1 + a_2 - g\right) \Leftrightarrow T_{BP} = \sqrt{3}mg - 4ma_1 \ (6)$$

hence equating (5) and (6) gives

$$12ma_1 = mg\left(\sqrt{3} - \frac{2\sqrt{3}}{3}\right) \Leftrightarrow a_1 = \frac{\sqrt{3}g}{36}$$

and since $a_1 = \sqrt{3}a_2$, hence $a_2 = g/36$. Combining the components together using Pythagoras' theorem gives

$$a = \sqrt{a_1^2 + a_2^2} = \sqrt{\frac{1}{36^2} + \frac{3}{36^2}}g = \frac{g}{18}.$$

as required.