Paper Reference(s)

6667

Edexcel GCE

Further Pure Mathematics FP1

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio CFX-9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Prove that $\sum_{r=1}^{n}\left(r^2 - r - 1\right) = \frac{1}{3}(n-2)n(n+2)$.

(5)

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2. $$\mathrm{f}(x) = \ln x - 1 - \frac{1}{x}.$$

(*a*) Show that the root α of the equation f(x) = 0 lies in the interval $3 < \alpha < 4$.

(2)

(*b*) Taking 3.6 as your starting value, apply the Newton-Raphson procedure once to f(x) to obtain a second approximation to α. Give your answer to 4 decimal places.

(5)

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3. Find the set of values of x for which

$$\frac{x}{x-3} > \frac{1}{x-2}.$$

(7)

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4. $$\mathrm{f}(x) \equiv 2x^3 - 5x^2 + px - 5, \quad p \in \mathbb{R}.$$

The equation f (x) = 0 has (1 – 2i) as a root.

Solve the equation and determine the value of p.

(7)

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5. (*a*) Obtain the general solution of the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} - 0.1S = t.$$

(6)

(*b*) The differential equation in part (*a*) is used to model the assets, £S million, of a bank t years after it was set up. Given that the initial assets of the bank were £200 million, use your answer to part (*a*) to estimate, to the nearest £ million, the assets of the bank 10 years after it was set up.

(4)

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6. The curve C has polar equation

$$r^2 = a^2 \cos 2\theta, \qquad \frac{-\pi}{4} \le \theta \le \frac{\pi}{4}.$$

(a) Sketch the curve C.

(2)

(b) Find the polar coordinates of the points where tangents to C are parallel to the initial line.

(6)

(c) Find the area of the region bounded by C.

(4)

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7. Given that $z = -3 + 4\text{i}$ and $zw = -14 + 2\text{i}$, find

(a) w in the form $p + \text{i}q$ where p and q are real,

(4)

(b) the modulus of z and the argument of z in radians to 2 decimal places

(4)

(c) the values of the real constants m and n such that

$$mz + nzw = -10 - 20\text{i}.$$

(5)

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8. (*a*) Given that $x = \mathrm{e}^t$, show that

(i)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t},$$

(ii)
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-2t}\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right).$$

(5)

(*b*) Use you answers to part (*a*) to show that the substitution $x = \mathrm{e}^t$ transforms the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = x^3$$

into

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 3\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = \mathrm{e}^{3t}.$$

(3)

(*c*) Hence find the general solution of

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = x^3.$$

(6)

END